

Cross-Field Plasma Acceleration and Potential Formation Induced by Electromagnetic Waves in a Relativistic Magnetized Plasma

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Abstract

It has been proved theoretically that particle acceleration along and across a magnetic field and electric field across a magnetic field can be induced by nonlinear Landau damping of almost perpendicularly propagating electrostatic waves in a relativistic magnetized plasma.

1. Introduction

Particle acceleration along and across a magnetic field and generation of electric field transverse to the magnetic field, both induced by nonlinear Landau damping (nonlinear wave-particle scattering) of almost perpendicularly propagating electromagnetic waves in a relativistic magnetized plasma have been investigated theoretically on the basis of the relativistic transport equations derived from relativistic Vlasov-Maxwell equations¹⁻⁶. Two electromagnetic waves interact nonlinearly with particles, satisfying the resonance condition of $\omega_k - \omega_{k'} - (k_{\perp} - k'_{\perp})v_d - (k_{\parallel} - k'_{\parallel})v_{\parallel} = m\omega_{cs} / \gamma_d^2$, where v_{\parallel} and $v_d = cE_0 / B_0$ are the parallel and perpendicular velocities of particles, respectively, $\gamma_d = (1 - v_d^2 / c^2)^{-1/2}$ and $\omega_{cs} = e_s B_0 / \gamma_s m_s c$ ($\gamma_s = (1 + p^2 / m_s^2 c^2)^{1/2}$) is the relativistic cyclotron frequency. The relativistic transport equations show that the electromagnetic waves accelerate plasma particles in the \mathbf{k}'' direction ($\mathbf{k}'' = \mathbf{k} - \mathbf{k}'$). As a result, the strong plasma acceleration or transport across a magnetic field appears. Simultaneously the intense cross-field electric field $\mathbf{E}_0 = \mathbf{B}_0 \times \mathbf{v}_d / c$ is generated via the dynamo effect of perpendicular particle drift to satisfy the generalized Ohm's law, that is, the electromagnetic waves can produce the cross-field particle drift that is identical to $\mathbf{E} \times \mathbf{B}$ drift. Moreover, the relativistic transport equations for the relativistic cross-field particle acceleration were derived by means of Lorentz transformation of the relativistic momentum-space diffusion equation in the frame moving with the cross-field drift velocity^{3,5}. They can be applied usefully to the theoretical study of the highly relativistic electron beam acceleration in a magnetized plasma^{5,7,8} as well as the relativistic cross-field particle acceleration which may occur possibly in space plasmas.

2. Relativistic Transport Equations

The relativistic kinetic wave equations and the relativistic transport equations for nonlinear Landau and cyclotron damping of two electromagnetic waves propagating almost perpendicularly in a homogeneous relativistic plasma immersed in uniform magnetic and electric fields $\mathbf{B}_0 = (B_0, 0, 0)$ and $\mathbf{E}_0 = (0, E_0, 0)$ can be derived from the relativistic

Vlasov-Maxwell equations¹⁻⁶. At first those for the nonrelativistic cross-field particle acceleration ($v_d/c \ll 1$) are given as follows:

$$\frac{\partial U_k}{\partial t} = 2\gamma_k U_k + \sum_{j,l,l',l''} \frac{\omega_k}{4\pi} A_{k,k'',k'}^{jl''l'l} E_k^{*j} E_k^l E_{k'}^{*l'} E_{k'}^{l''} , \quad (1)$$

$$\frac{\partial U_{k'}}{\partial t} = 2\gamma_{k'} U_{k'} - \sum_{j,l,l',l''} \frac{\omega_{k'}}{4\pi} A_{k,k'',k'}^{jl''l'l} E_k^{*j} E_k^l E_{k'}^{*l'} E_{k'}^{l''} , \quad (2)$$

$$\frac{\partial U_s}{\partial t} = -2\gamma_k^{(s)} U_k - 2\gamma_{k'}^{(s)} U_{k'} - \sum_{j,l,l',l''} \frac{\omega_{k'}}{4\pi} A_{k,k'',k'}^{(s)jl''l'l} E_k^{*j} E_k^l E_{k'}^{*l'} E_{k'}^{l''} , \quad (3)$$

$$\frac{\partial \mathbf{P}_s}{\partial t} = -\frac{2\gamma_k^{(s)} \mathbf{k}}{\omega_k} U_k - \frac{2\gamma_{k'}^{(s)} \mathbf{k}'}{\omega_{k'}} U_{k'} - \sum_{j,l,l',l''} \frac{\mathbf{k}''}{4\pi} A_{k,k'',k'}^{(s)jl''l'l} E_k^{*j} E_k^l E_{k'}^{*l'} E_{k'}^{l''} , \quad (4)$$

where $\mathbf{E}_k = (E_k^x, E_k^y, E_k^z)$ ($j, l, l', l'' = x, y, z$) is the wave electric field, $\mathbf{E}_{-k} = \mathbf{E}_k^*$, $\omega_{-k} = -\omega_k^*$, $\omega_{k''} = \omega_k - \omega_{k'}$, $\mathbf{k} = (k_\perp, 0, k_\parallel)$, $\mathbf{k}' = (k'_\perp, 0, k'_\parallel)$ and the wave energy density U_k , the energy and momentum densities of particles of species s , U_s and \mathbf{P}_s are given by $U_k = (1/8\pi) \mathbf{E}_k^* \cdot [\partial((\boldsymbol{\varepsilon}' - \mathbf{N}_k)\omega_k)] \cdot \mathbf{E}_k$, $N_{jl} = (c^2/\omega_k^2)(k^2 \delta_{jl} - k_j k_l)$, $U_s = \int d\mathbf{p} w_s g_s$, $\mathbf{P}_s = \int d\mathbf{p} \mathbf{p} g_s = (P_{s\perp}, 0, P_{s\parallel})$, $P_{s\perp} = n_s p_{ds}$, $p_{ds} = \gamma_s m_s v_d$, $P_{s\parallel} = n_s \gamma_s m_s v_{s\parallel}$, $w_s = n_s \gamma_s m_s c^2$, $\mathbf{p}_s = n_s \mathbf{p}$ and $g_s = g_{s0} \left\{ \left[(p_x - p_{ds})^2 + p_y^2 \right]^{1/2}, p_\parallel, t \right\}$ ($g_{s0} = g_{s0}(p_\perp, p_\parallel, t)$) is the background relativistic momentum distribution function of particles of species s containing the fluctuation-induced cross-field drift velocity $v_d = \int d\mathbf{p} v_x g_s$ ($\mathbf{v}_d = (v_d, 0, 0) = c\mathbf{E}_0 \times \mathbf{B}_0 / B_0^2$, $v_d/c \ll 1$). The linear damping rates $\gamma_k = \text{Im} \omega_k$ and $\gamma_k^{(s)}$ are shown by $\gamma_k = \sum_s \gamma_k^{(s)}$ and $\gamma_k^{(s)} = -(\omega_k/8\pi U_k) (\mathbf{E}_k^* \cdot \boldsymbol{\varepsilon}_k^{(s)} \cdot \mathbf{E}_k)$, the dielectric tensor $\boldsymbol{\varepsilon}_k = \sum_s \boldsymbol{\varepsilon}_k^{(s)}$ is described in Ref. 6, and the relativistic nonlinear wave-particle coupling coefficients $A_{k,k'',k'}^{jl''l'l}$ are represented by

$$A_{k,k'',k'}^{jl''l'l} = \sum_s A_{k,k'',k'}^{(s)jl''l'l}, \quad A_{k,k'',k'}^{(s)jl''l'l} = -P_{AH} (C_{k,k'',k'}^{(s)jl''l'l} + D_{k,k'',k'}^{(s)jl''l'l}) , \quad (5)$$

$$P_{AH} C_{k,k'',k'}^{(s)jl''l'l} = \frac{\pi \omega_{ps}^2 e_s^2}{\omega_k^2 \omega_{k'} \omega_{-k'}} \int d\mathbf{p} \delta(\xi_m) \Omega_{k,k'}^{jl''} \Lambda_{k,k'}^{l'l} g_{s0} , \quad (6)$$

$$P_{AH} D_{k,k'',k'}^{(s)jl''l'l} = \frac{\pi \omega_{ps}^2 e_s m_s}{\omega_k \omega_{k'} \omega_{k''}} \sum_{j'} (\beta_{k,-k'}^{j'l'l} + \beta_{-k',k}^{j'l'l}) \int d\mathbf{p} \delta(\xi_m) \Omega_{k,k'}^{jl''} X_{k'',m}^{j'}(\mathbf{p}) g_{s0} , \quad (7)$$

$$\beta_{k,-k'}^{j'l'l} + \beta_{-k',k}^{j'l'l} = \sum_s \frac{\pi \omega_{ps}^2 e_s}{\omega_{k''} \omega_k \omega_{-k'}} \int d\mathbf{p} \delta(\xi_m) \left(\boldsymbol{\kappa}_{k''} \cdot [(\mathbf{u}_{k''}^*(\mathbf{p}) + \mathbf{p}_{ds}) J_m(\mu_{k''})] \right)^{j'} A_{k,k'}^{l'l} g_{s0} , \quad (8)$$

$$\begin{aligned} \boldsymbol{\varepsilon}_{k''}^{(s)jl} &= P_{AH} \boldsymbol{\varepsilon}_{k''}^{(s)jl} \\ &= -\frac{\pi \omega_{ps}^2}{\omega_{k''}^2} \int d\mathbf{p} \delta(\xi_m) \frac{1}{\gamma_s} \left[u_{k''}^{*j}(\mathbf{p}) J_m(\mu_{k''}) + s_j p_{ds} J_m(\mu_{k''}) \right] \\ &\quad \times \left[u_{k''}^l(\mathbf{p}) J_m(\mu_{k''}) + s_l p_{ds} J_m(\mu_{k''}) \right] U_m(\mathbf{k}'') g_{s0} , \end{aligned} \quad (9)$$

$$\Omega_{\mathbf{k},\mathbf{k}'}^{jl''} = \eta_j \eta_{l'} J_m(\mu_{\mathbf{k}'}) + \sum_{q=-\infty}^{\infty} \left\{ \frac{1}{(k_{\parallel} v_{\parallel} + k_{\perp} v_d - \omega_{\mathbf{k}} + q\omega_{cs})^2} \left([a_{\mathbf{k},q}^{*j} J_q(\mu_{\mathbf{k}})] [a_{\mathbf{k}',q-m}^{l''} J_{q-m}(\mu_{\mathbf{k}'})] \right. \right. \\ \left. \left. - [a_{\mathbf{k},q}^{*j} J_q(\mu_{\mathbf{k}})] [a_{\mathbf{k}',q-m}^{l''} J_{q-m}(\mu_{\mathbf{k}'})] \right) + \frac{[b_{\mathbf{k}}^{*j} J_q(\mu_{\mathbf{k}})] [b_{\mathbf{k}'}^{l''} J_{q-m}(\mu_{\mathbf{k}'})]}{(k_{\parallel} v_{\parallel} + k_{\perp} v_d - \omega_{\mathbf{k}} + q\omega_{cs})^2 - \omega_{cs}^2} \right\}, \quad (10)$$

$$A_{\mathbf{k},\mathbf{k}'}^{l''} = \eta_l \eta_{l'} \frac{k_{\perp}''}{\gamma_s \omega_{cs}} J_m(\mu_{\mathbf{k}'}) \Psi_{\mathbf{k}',m}^l + \sum_{r=-\infty}^{\infty} \left\{ \frac{V_{\parallel\mathbf{k},\mathbf{k}'}^{l''}}{\gamma_s (k_{\parallel} v_{\parallel} + k_{\perp} v_d - \omega_{\mathbf{k}} + r\omega_{cs})^2} \right. \\ \left. + \sum_{\zeta=\pm 1} \frac{\zeta}{2\gamma_s \omega_{cs}} \frac{V_{\perp\mathbf{k},\mathbf{k}'}^{l''}}{k_{\parallel} v_{\parallel} + k_{\perp} v_d - \omega_{\mathbf{k}} + (r-\zeta)\omega_{cs}} + \frac{V_{\mathbf{k},\mathbf{k}'}^{l''}}{k_{\parallel} v_{\parallel} + k_{\perp} v_d - \omega_{\mathbf{k}} + r\omega_{cs}} \right\}, \quad (11)$$

where $\xi_m = k_{\parallel}'' p_{\parallel} + k_{\perp}'' p_{ds} - \gamma_s m_s \omega_{\mathbf{k}''} + m\gamma_s m_s \omega_{cs}$, $\mathbf{\kappa}_{\mathbf{k}''} = -P_{AH} \boldsymbol{\chi}_{\mathbf{k}''} = \boldsymbol{\chi}_{\mathbf{k}''}^{\dagger} \cdot \boldsymbol{\varepsilon}_{\mathbf{k}''}'' \cdot \boldsymbol{\chi}_{\mathbf{k}''}$, $\boldsymbol{\chi}_{\mathbf{k}''} = (\boldsymbol{\varepsilon}_{\mathbf{k}''} - \mathbf{N}_{\mathbf{k}''})^{-1}$, and $\boldsymbol{\chi}_{\mathbf{k}''}^{\dagger}$ is the complex conjugate transpose matrix of $\boldsymbol{\chi}_{\mathbf{k}''}$. The differential operators are defined as

$$V_{\parallel\mathbf{k},\mathbf{k}'}^{l''} = -k'_{\parallel} [a_{\mathbf{k},r}^{l'} J_r(\mu_{\mathbf{k}})] X_{\mathbf{k}',r-m}^{*l''}(\mathbf{p}) + k_{\parallel} [a_{\mathbf{k}',r-m}^{*l''} J_{r-m}(\mu_{\mathbf{k}'})] X_{\mathbf{k},r}^l(\mathbf{p}) \\ + (k'_{\parallel} v_{\parallel} + (r-m)\omega_{cs}) [X_{\mathbf{k},r}^l(\mathbf{p}) \gamma_s m_s] X_{\mathbf{k}',r-m}^{*l''}(\mathbf{p}) - (k_{\parallel} v_{\parallel} + r\omega_{cs}) [X_{\mathbf{k}',r-m}^{*l''}(\mathbf{p}) \gamma_s m_s] X_{\mathbf{k},r}^l(\mathbf{p}), \\ V_{\perp\mathbf{k},\mathbf{k}'}^{l''} = -k'_{\perp} [b_{\mathbf{k}}^{l'} J_r(\mu_{\mathbf{k}})] [\xi_{\mathbf{k}'}^{*l''} \mu_{\mathbf{k}'} J_{r-m}(\mu_{\mathbf{k}'})] \Psi_{\mathbf{k}',r-m-\zeta}^{l''} + k_{\perp} [b_{\mathbf{k}'}^{*l''} J_{r-m}(\mu_{\mathbf{k}'})] [\xi_{\mathbf{k}}^l \mu_{\mathbf{k}} J_r(\mu_{\mathbf{k}})] \Psi_{\mathbf{k},r-\zeta}^l, \\ V_{\mathbf{k},\mathbf{k}'}^{l''} = m_s [\mu_{\mathbf{k}} \xi_{\mathbf{k}}^l J_r(\mu_{\mathbf{k}})] [\mu_{\mathbf{k}'} \xi_{\mathbf{k}'}^{*l''} J_{r-m}(\mu_{\mathbf{k}'})] [\Psi_{\mathbf{k},r}^l \Psi_{\mathbf{k}',r-m}^{l''} - \Psi_{\mathbf{k}',r-m}^{l''} \Psi_{\mathbf{k},r}^l], \\ X_{\mathbf{k},r}^l(\mathbf{p}) = [\mu_{\mathbf{k}} \xi_{\mathbf{k}}^l J_r(\mu_{\mathbf{k}})] \Psi_{\mathbf{k},r}^l, \quad \Psi_{\mathbf{k},r}^x = \frac{k_{\parallel}}{k_{\perp}} (r\omega_{cs} + k_{\perp} v_d) \frac{\partial}{\partial p_{\parallel}} + r\omega_{cs} \frac{\omega_{\mathbf{k}} - k_{\parallel} v_{\parallel}}{k_{\perp} v_{\perp}} \frac{\partial}{\partial p_{\perp}}, \\ \Psi_{\mathbf{k},r}^y = \omega_{cs} \left(\frac{k_{\parallel}}{k_{\perp}} \frac{\partial}{\partial p_{\parallel}} + \frac{\omega_{\mathbf{k}} - k_{\perp} v_d - k_{\parallel} v_{\parallel}}{k_{\perp} v_{\perp}} \frac{\partial}{\partial p_{\perp}} \right), \quad \Psi_{\mathbf{k},r}^z = (\omega_{\mathbf{k}} - k_{\perp} v_d - r\omega_{cs}) \frac{\partial}{\partial p_{\parallel}} + r\omega_{cs} \frac{p_{\parallel}}{p_{\perp}} \frac{\partial}{\partial p_{\perp}}, \\ a_{\mathbf{k},q}^x = \frac{k_{\parallel}}{k_{\perp}} (q\omega_{cs} + k_{\perp} v_d), \quad a_{\mathbf{k},q}^y = ik_{\parallel} v_{\perp} \frac{d}{d\mu_{\mathbf{k}}}, \quad a_{\mathbf{k},q}^z = \omega_{\mathbf{k}} - k_{\perp} v_d - q\omega_{cs}, \\ b_{\mathbf{k}}^x = \omega_{\mathbf{k}} - k_{\parallel} v_{\parallel}, \quad b_{\mathbf{k}}^y = i\omega_{cs} \frac{d}{d\mu_{\mathbf{k}}} \mu_{\mathbf{k}}, \quad b_{\mathbf{k}}^z = k_{\perp} v_{\parallel},$$

$a_{\mathbf{k},q}^x = q\omega_{cs} \omega_{\mathbf{k}} / ck_{\perp} + k_{\parallel} v_{\parallel} v_d / c$, $a_{\mathbf{k},q}^y = i(\omega_{\mathbf{k}} - k_{\perp} v_d) v_{\perp} / c$, $a_{\mathbf{k},q}^z = (\omega_{\mathbf{k}} - k_{\perp} v_d) v_{\parallel} / c$, P_{AH} indicates the anti-Hermitian part of the tensor, and $U_r(\mathbf{k})$, $a_{\mathbf{k},q}^j$, $b_{\mathbf{k}}^j$, $\xi_{\mathbf{k}}^j$, $\mu_{\mathbf{k}}$, s_j , η_j and ω_{ps} are defined in Ref. 6.

It is found easily that Eqs. (1)-(4) lead to the conservation laws for the total energy and momentum densities of two electromagnetic waves and plasma particles,

$$\frac{\partial}{\partial t} \left[\sum_s U_s + U_{\mathbf{k}} + U_{\mathbf{k}'} \right] = 0 \quad , \quad \frac{\partial}{\partial t} \left[\sum_s \mathbf{P}_s + \frac{\mathbf{k}}{\omega_{\mathbf{k}}} U_{\mathbf{k}} + \frac{\mathbf{k}'}{\omega_{\mathbf{k}'}} U_{\mathbf{k}'} \right] = 0 \quad . \quad (12)$$

The transport equations and the conservation laws (3), (4), (12) predict obviously that the electromagnetic waves can generate strong particle acceleration or transport along and across the magnetic field via nonlinear Landau and cyclotron damping of electromagnetic waves.

Next the relativistic transport equations for the relativistic cross-field particle acceleration^{3,5} ($v_d/c \lesssim 1$) are provided by the same way as the previous work described in Ref. 6. The relativistic transport equations in the laboratory (stationary) frame of reference with an electric field and $\mathbf{E} \times \mathbf{B}$ drift can be derived by means of Lorentz transformation for the momentum-space integration of the relativistic momentum-space diffusion equation in the frame of reference moving with $\mathbf{E} \times \mathbf{B}$ drift velocity \mathbf{v}_d . The details are described in Ref. 6.

3. Conclusion

It was verified theoretically that relativistic particle acceleration along and across the magnetic field and electric field transverse to the magnetic field can be induced by nonlinear Landau damping of almost perpendicularly propagating electromagnetic waves in a relativistic magnetized plasma. The relativistic transport equations show that the electromagnetic waves produce strong particle acceleration. Simultaneously the intense cross-field electric field is created via the dynamo effect of the cross-field particle drift. The obtained transport equations can be available for the theoretical investigation of the relativistic and nonrelativistic cross-field particle acceleration and transport which may occur possibly in space and fusion plasmas. It can be available usefully to the acceleration of the highly relativistic electron beam^{5,7,8}.

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