

Radiation reaction effects in ultrahigh irradiance laser pulse interactions with multiple electrons

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Unprecedented laser powers have been achieved at existing laser facilities [1] or can be achieved at those planned in the near future [2]. By focusing these lasers down to extremely small spot sizes or by using relativistic mirrors [3] ultra-high irradiances can be attained. When such ultrahigh irradiance laser pulses interact with high energy electrons, the emission of radiation from these electrons can significantly affect the motion of the electrons themselves (radiation reaction). In particle-in-cell (PIC) simulations including radiation reaction, a laser pulse interacting with a plasma slab with a radiance $>10^{22}$ W/cm² and duration of 20 fs converted more than 35% of its energy to radiation [4]. Since the spatial grid resolution necessary to self-consistently simulate high intensity laser interactions with electrons via PIC methods is prohibitive [4], in this paper we will describe a particle-particle code to simulate such interactions. Since we directly solve the interaction between the electrons numerically, this allows an accurate description of the dynamics. We will numerically examine the collective effects of two electrons undergoing strong radiation reaction effects.

The equations of motion for electrons have the following form [5]

$$\frac{d\vec{p}}{dt} = -e\{\vec{E} + \vec{\beta} \times \vec{B}\} + \vec{f}_{RD}$$

$$\vec{f}_{RD} = -e\frac{2r_e}{3}\gamma\left(\frac{\partial}{c\partial t} + \vec{\beta} \cdot \vec{\nabla}\right)\left(\vec{E} + \vec{\beta} \times \vec{B}\right) + \frac{2r_e^2}{3}\left\{\vec{E}(\vec{\beta} \cdot \vec{E}) + (\vec{E} + \vec{\beta} \times \vec{B}) \times \vec{B}\right\} - \frac{2r_e^2}{3}\gamma^2\vec{\beta}\left\{(\vec{E} + \vec{\beta} \times \vec{B})^2 - (\vec{\beta} \cdot \vec{E})^2\right\}$$

where \vec{p} is the electron momentum, e is the electron charge, (\vec{E}, \vec{B}) are the electric and magnetic fields, $\vec{\beta} = \frac{\vec{v}}{c}$ where \vec{v} is the velocity, $\gamma = 1/\sqrt{1-\beta^2}$, and $r_e = \frac{e^2}{mc^2}$ is the classical electron radius. These set of equations have been shown to be an accurate description of the radiation damping down to the quantum regime [6]. The fields used in these equations are of the form: $\vec{E}(\vec{x}, t) = \vec{E}_{laser}(\vec{x}, t) + \vec{E}_{ret}(\vec{x}, t)$ where $\vec{E}_{laser}(\vec{x}, t)$ is the laser field and [7]:

$$\vec{E}_{ret}(\vec{x}, t) = \sum_{i=1}^{N-1} q_i \left[\frac{\vec{n} - \vec{\beta}_i}{\gamma^2 (1 - \vec{\beta}_i \cdot \vec{n})^3 R_i^2} \right]_{ret} + \frac{q_i}{c} \left[\frac{\vec{n} \times \{(\vec{n} - \vec{\beta}_i) \times \dot{\vec{\beta}}_i\}}{(1 - \vec{\beta}_i \cdot \vec{n})^3 R_i} \right]_{ret}$$

is the Lienard-Wiechert field contribution where the sum is over all other particles with N being the number of particles and q_i is the charge of particle i . A similar set of equations

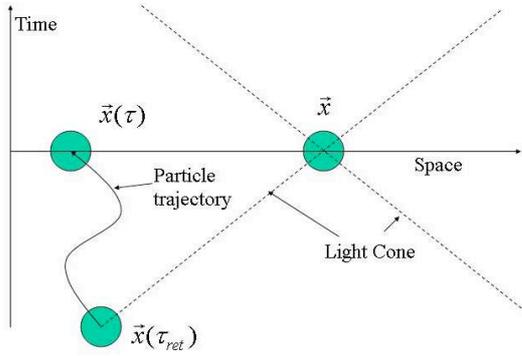


Figure 1 Space Time diagram

exists for the magnetic fields. The *ret* subscript refers to retarded times which are used to calculate the fields as shown in Figure 1. The point in space and time is determined by the intersection of the backward light cone with the world lines of the other electrons. In order to perform such a calculation the trajectory of each electron needs to be saved for a certain

amount of time. The only approximation made in the code is that the spatial and temporal derivatives of the retarded fields ($\vec{E}_{ret}, \vec{B}_{ret}$) in \vec{f}_{RD} are taken to be small and ignored. The equations of motion are integrated forward in time by using an adaptive Runge-Kutta integration scheme [8].

As an example we calculate the interaction of two electrons with a linearly polarized Gaussian laser pulse. The laser pulse is traveling in the +x direction and of the form:

$$\begin{aligned}\vec{E}_{laser} &= \hat{z}E_0h(\phi)\sin(\phi) \\ \vec{B}_{laser} &= -\hat{y}E_0h(\phi)\sin(\phi)\end{aligned}$$

where $\phi = \omega_0(t - x/c)$, ω_0 is the laser frequency,

$$h(\phi) = \exp\left[-\left(\frac{\phi}{\omega_0\tau}\right)^2\right],$$

and τ is the pulse duration. In order to enhance the effects of radiation damping we choose a very short wavelength of 10^{-8} cm and a pulse duration of 0.66 as with a normalized

amplitude of $a_0=10$ where $a_0 = \frac{eE_0}{mc\omega_0}$. The effective peak irradiance is 1.38×10^{28} W/cm²,

which could be achieved via relativistic mirrors [3]. The two electrons are initially traveling in the -x direction with energies of 212 keV ($\gamma_0=1.414$ where $\gamma_0=(1-\beta_0^2)^{-1/2}$). With the current parameters the interaction of these electrons with the laser pulse is in the classical regime by the condition [9]:

$$\sqrt{1+a_0^2}\gamma_0(1+\beta_0) < \left(\frac{mc^2}{\hbar\omega_0}\right)\frac{4\pi}{0.87}.$$

So using the classical equations of motion to describe the dynamics of the electrons are justified. Figure 2 shows a trace of the x- γ positions of the electrons during the interaction with the laser pulse, where x is normalized by the laser wavelength, λ_0 . One electron which is initially at $x/\lambda_0=-0.1$ is represented by the solid line and the other electron which is initially at $x/\lambda_0=0$ is represented by the dotted line. It can be seen that the front electron reaches a lower peak energy than the back electron. After

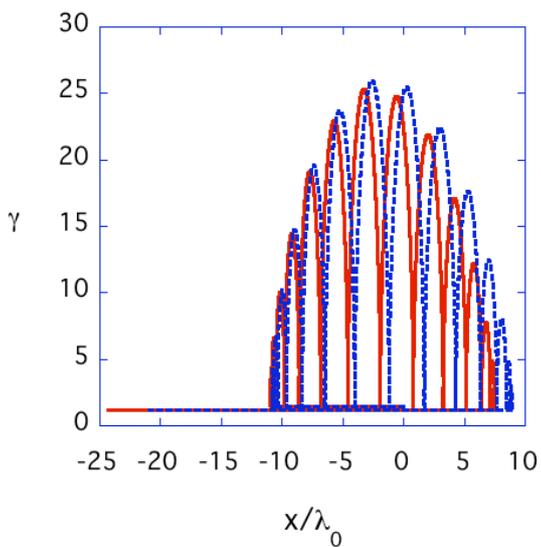


Figure 2 x-y trace of the electron positions

the interaction the difference in the x positions between the electrons has increased to $3.3\lambda_0$. Both electrons have lost energy during the interaction due to radiation damping (front electron 17%, back electron 15.6%). Figure 3 shows that the point in time at which the electron turns around is later (dotted line) than in the case with no damping (solid line) for the back electrons. The electron with damping included reaches a higher peak energy ($\gamma=25.96$) due to the radiation pressure than in the case with no damping ($\gamma=22.56$). Using the same parameters, but with an initial separation in the y direction of $\Delta y/\lambda_0=1$, but no separation in x, we get the results shown in

Figure 4. In this figure, traces of the two electrons in time, $\omega_0 t$, versus the y position, y/λ_0 , are shown. It can be seen that after the interaction with the laser pulse the electrons diverge in y due to their mutual Coulombic repulsion. Figure 5 shows a comparison of the damping (dotted line) and non-damping (solid line) cases. In this case also the time at which the repulsion occurs is later in time for the damping case due to the longer interaction with the laser pulse. At the end of the interaction the final energy ($\gamma=1.18$) is

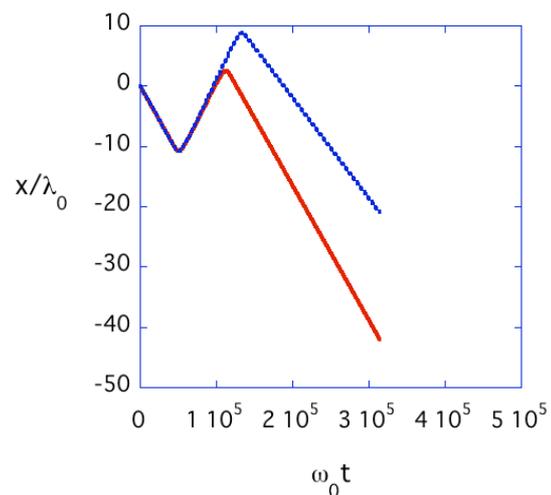


Figure 3 t-x trace of the electrons with (dotted line) and without (solid line) damping.

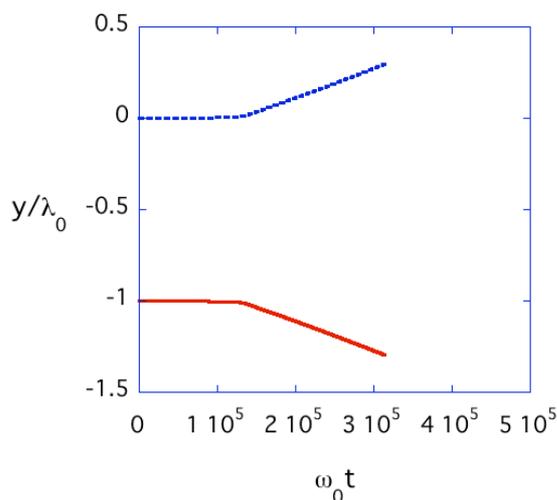


Figure 4 t-y trace of the electrons
less than in the case without damping ($\gamma=1.41$).
As a result the mutual repulsive kick is larger
and the electrons with damping included are scattered to larger angles.

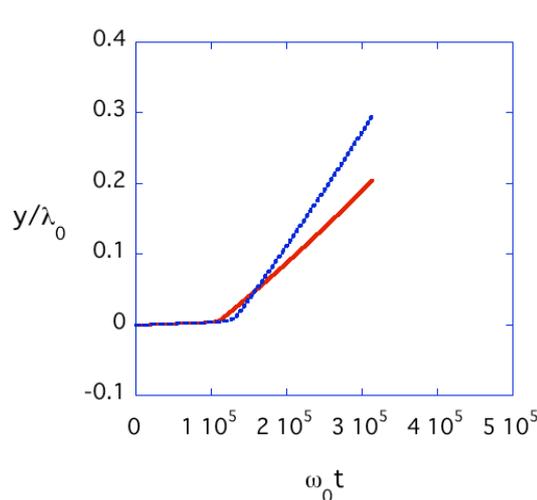


Figure 5 t-y trace of the electrons with
less than in the case without damping ($\gamma=1.41$).
As a result the mutual repulsive kick is larger
and the electrons with damping included are scattered to larger angles.

In conclusion we have developed a self-consistent code which directly calculates the interactions between electrons via Lienard-Wiechert potentials and includes the radiation damping force. The self-consistent calculation has been performed with two electrons interacting with a laser pulse and has shown that difference in the motion of the electrons occurs when damping is and is not included. Due to the computational cost of this calculation only a few particles can be simulated. However, this may be applied to small particle systems such as clusters in the future.

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