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Self-similar solutions of unsteady ablation flow

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The stability of ablative flows is of importance in inertial confinement fusion (ICF). Here we exhibit a family of exact self-similar solutions of gas dynamics equations with nonlinear heat conduction for semi-infinite slabs of perfect gases. Such self-similar solutions arise for particular but realistic time-increasing boundary pressure and heat flux laws, and are representative of the early stage of an ICF capsule irradiation, where a shock-wave front propagates upstream of an ablation front. Linear stability analyses of such time dependent solutions are performed by solving an initial and boundary value problem for linear perturbations [5]. Our goal here is to study these self-similar ablation flows, which are computed using a highly accurate numerical method, namely a dynamical multidomain Chebyshev spectral method [4, 7].

Self-similar mean flow

For one-dimensional motions in the x-direction of an inviscid heat-conducting fluid, obeying a perfect gas equation of state

$$p = \rho R T, \qquad \mathscr{E} = T R/(\gamma - 1), \tag{1}$$

where ρ is the fluid density, p the fluid pressure, $\mathscr{E}$ the internal energy, T the fluid temperature, R the gas constant and γ the fluid adiabatic exponent, the equations of motion, written in Lagrangian form, are

$$\begin{cases} \partial_t (1/\rho) - \partial_m v_x = 0, \\ \partial_t v_x + \partial_m p = 0, \\ \partial_t \left({v_x}^2/2 + \mathscr{E} \right) + \partial_m (p v_x + \varphi_x) = 0, \end{cases} \tag{2}$$

where m is the Lagrangian coordinate ($\mathrm{d}m = \rho \mathrm{d}x$), v_x the velocity and φ_x the heat flux, of expression

$$\varphi_x = -\kappa \partial_x T = -\kappa \rho \, \partial_m T = -\chi \left(\frac{\rho}{\rho_\mathrm{i}} \right)^{-\mu} \left(\frac{T}{T_*} \right)^{\nu} \rho \, \partial_m T, \tag{3}$$

and χ, μ, ν are fluid constants to be chosen such that $\chi \geq 0$, $\mu \geq 0$ and $\nu \neq 1$, ρ_i and T_* are characteristic density and temperature of the flow.

Solutions of self-similar type for system (2) have been investigated by several authors [8, 1, 3, 11, 12]. Here we consider time-dependent pressure and heat-flux boundary conditions [10]. At $t = 0$, the fluid of uniform density ρ_i is assumed to occupy the half-space $m \geq 0$, while a heat flux starts being applied along the plane $m = 0$. By choosing initial conditions to be

$$\rho = \rho_\mathrm{i}, \qquad v_x = 0, \qquad T = 0, \qquad \text{for } m \geq 0, \tag{4}$$

and boundary conditions of the form

$$p = p_* \, (t/t_*)^{2(\alpha-1)}, \qquad \varphi_x = \varphi_* \, (t/t_*)^{3(\alpha-1)}, \qquad \text{for } m = 0, \tag{5}$$

system (2) admits a self-similar formulation. Here p_*, φ_* and t_* are characteristic pressure, heat flux and time. For convenience, we choose a dimensionless formulation of the equations based on the quantities ρ_i, R, χ, t_*. Applying the Π-theorem [2], the seven parameters (ρ_i, R, γ, χ, t_*, p_*, φ_*) lead us to retain the three dimensionless numbers:

$$\gamma, \qquad B_p = p_* t_* R/\chi, \qquad B_\varphi = \varphi_* \sqrt{\rho_\mathrm{i} \, (t_* R/\chi)^3}. \tag{6}$$

Henceforth, all the quantities are replaced by their dimensionless equivalents, while keeping the same notations. Introducing the self-similar variable $\xi = m/t^\alpha$, with $\alpha = (2\nu-1)/(2\nu-2)$, and time power-law dependencies for the physical variables

$$\begin{cases} \rho = G(\xi), & T = t^{2(\alpha-1)} \, \Theta(\xi), \\ v_x = t^{\alpha-1} \, V(\xi), & \varphi_x = t^{3(\alpha-1)} \, \Phi(\xi), \end{cases} \tag{7}$$

the dimensionless system obtained from Eqs. (1,2,3) reduces to a system of ODEs:

$$\frac{\mathrm{d}}{\mathrm{d}\xi} \begin{pmatrix} G \\ V \\ \Theta \\ \Phi \end{pmatrix} = \begin{pmatrix} G^2 N/D \\ \alpha\xi \, N/D \\ F \\ (\alpha\xi F - 2(\alpha-1)\Theta)/(\gamma-1) - \alpha\xi G\Theta \, N/D \end{pmatrix} \tag{8}$$

with:

$$N = (\alpha-1)V + GF, \qquad D = \alpha^2\xi^2 - G^2\Theta, \qquad F = -\Phi G^{\mu-1}\Theta^{-\nu}. \tag{9}$$

The corresponding dimensionless form of equations (4) and (5) are

$$G(\xi \to +\infty) = 1, \qquad V(\xi \to +\infty) = 0, \qquad \Theta(\xi \to +\infty) = 0, \tag{10}$$

$$(G\Theta)\,(\xi = 0) = B_p, \qquad \Phi(\xi = 0) = B_\varphi. \tag{11}$$

Any solution of (8) satisfying Eqs. (10,11) necessarily includes the singularity $D = 0$. This singularity corresponds to an isothermal characteristic curve, say $m/t^\alpha = \xi_s$, of the (m,t)-plane, which is circumvented by introducing, as part of the solution, an isothermal shock wave at $\xi = \xi_s$. Henceforth the boundary conditions (10) are replaced by the Rankine-Hugoniot conditions, at $\xi = \xi_s$, for a non-isothermal shock wave with uniform upstream state given by (10), thus defining, along with Eq. (11), a nonlinear eigenvalue problem for system (8).

The highly accurate numerical method we have devised [7] consists of a finite-difference shooting procedure followed by a relaxation process coupled to an adaptive multidomain Chebyshev spectral method [9]. This algorithm introduces two numerical parameters ξ_f and ξ_s (ξ_s represents the shock wave front, ξ_f is defined such that $[\xi_f, \xi_s]$ is the conduction-negligible region), adjusted by the shooting method to obtain the boundary value parameters B_p and B_φ.

Results

Similarity solutions depend on five dimensionless flow parameters (γ, μ, ν, B_p, B_φ), which characterize the material, non-linear conduction and external constraints (heat flux and pressure). Given a LMJ-configuration ablation flow [6], we obtain the five dimensionless parameters

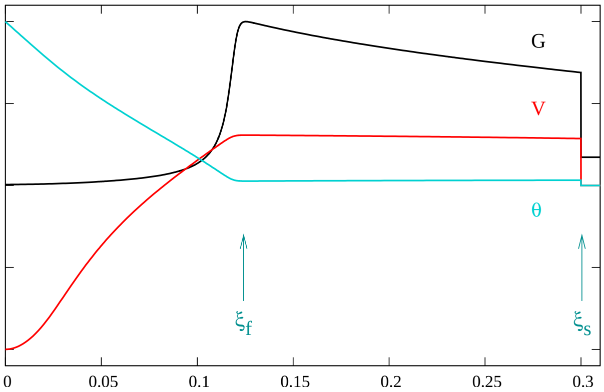

Figure 1: Reduced function profiles of density, velocity and temperature for a LMJ-configuration ablation flow [6].

of a self-similar solution, which is then computed. Fig 1 represents reduced function profiles of density, velocity and temperature. One recognizes *(i)* the undisturbed fluid region ($\xi > \xi_s$), *(ii)* the ablation layer (steep density and temperature gradients), *(iii)* the quasi-isentropic compression region between the ablation layer and the shock-wave front, and *(iv)* the conduction-dominated region extending from the origin up to the ablation layer. Self-similar solutions for various values of the boundary value parameters B_p and B_φ, and of the fluid adiabatic exponent γ have been successfully computed (see Fig. 2 for $\gamma = 5/3$). Variations in laser intensity, fluid density and heat conductivity are accessible with Eq. (6) through proper choices of the boundary value parameters B_p and B_φ. The knowledge of the parameter variation domain allows us to define solutions that could be reached in experiments.

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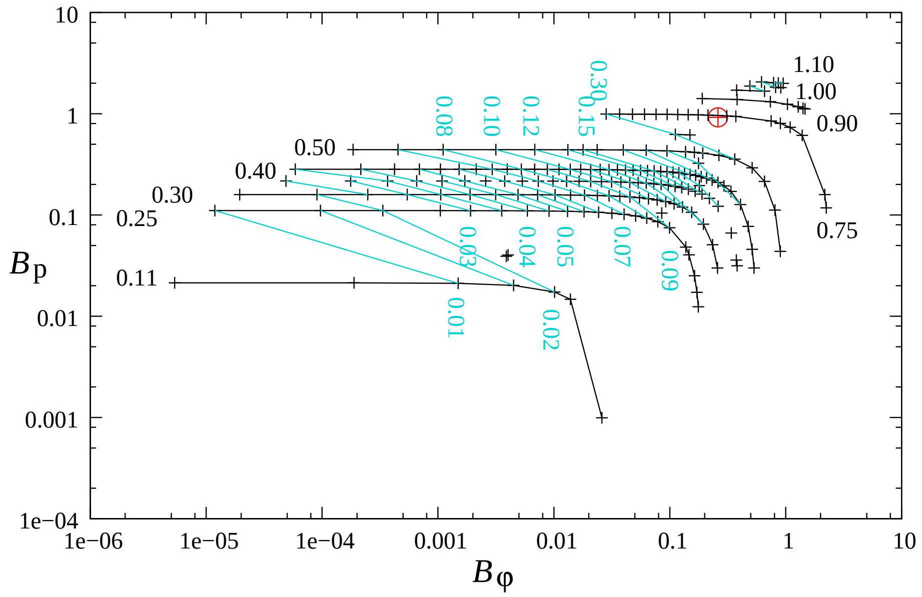

Figure 2: Reachable values of B_p and B_φ, for $\gamma = 5/3$ and $(\mu, \nu) = (0, 5/2)$. Symbol $\oplus$ indicates an LMJ reference solution (see Fig. 1). Black lines correspond to ξ_s = cste, and blue lines to ξ_f = cste.

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