

Incentives for and Developments of the Accretion Theory of Spontaneous Rotation*

B. Coppi¹, D.A. D'Ippolito², S.I. Krasheninnikov³, M. Lontano⁴, J.R. Myra², P. Nataf⁵, D.A. Russell²

¹*M.I.T., Cambridge, MA, USA;* ²*Lodestar, Boulder, CO, USA;* ³*U.C.S.D., San Diego, CA, USA;*

⁴*I.F.P. – C.N.R., Milan, Italy; E.N.S., Paris, France*

I. Introduction

Since the “accretion” theory [1,2] of the spontaneous rotation phenomenon in axisymmetric plasmas was introduced, the importance of this phenomenon has been increasingly recognized and the body of experimental observations supporting this theory has grown considerably. Accordingly, angular momentum is ejected from the plasma column by modes excited at the edge of it in a direction associated with their toroidal phase velocity while angular momentum in the opposite direction resulting from the resulting “recoil” force is transported from the outer region of the plasma column toward the center of it. The observed inversion of the rotation velocity in the transition from the H to the L confinement regime [3] is consistent with the predictions of the accretion theory and has confirmed that, during the transition, the rotation propagates from the outside toward the center. In the H-regime the central velocity is in the direction of the ion diamagnetic velocity and the prevailing modes at the edge of the plasma column, according to the theory, have phase velocities in the direction of the electron diamagnetic velocity. In the L-regime the opposite situation occurs. A series of experiments carried out by the JET machine has confirmed these facts, and in particular the inversion of the phase velocities of the modes excited at the edge of the plasma column in the transition from the L to the H regime [4]. Since all of these modes are associated with the energy and particle transport processes characteristic of the plasma column, one of the key features of the “accretion theory” has been the connection that it has predicted between transport and spontaneous rotation as well as the importance to this of the physics of the edge of the plasma column. Thus, a scaling for the rotation velocity observed for different plasma parameters, that has been tentatively formulated, involves the characteristics of the modes excited both in the main body and at the edge of the plasma column.

II. V.T.G. Modes and Transport Equation

In the H-regime, where the highest rotation velocities have been observed, and most of the experiments have been performed, we consider that the inward transport of angular momentum is associated with “VTG modes” that involve the gradients of both the ion velocity and temperature and have a phase velocity in the direction of the ion diamagnetic velocity. For

simplicity, we refer to an equilibrium plane geometry where the plasma has a flow velocity $V_{\parallel} = \mathbf{V} \cdot \mathbf{B}/B$ remembering that the extension of the relevant analysis to the case of an axisymmetric toroidal geometry has to be made judiciously. In this context the VTG modes would not be of the ballooning type as they involve a propagation along the magnetic field. These modes are considered to acquire a saturated state in which the ion thermal conductivity and viscosity associated with the mode itself play an important role and to be represented by the fluctuating electric field $\hat{\mathbf{E}} \simeq -\nabla\hat{\Phi}$ with $\hat{\Phi} \simeq \tilde{\Phi}(x)\exp(-i\omega_0 t + ik_y y + ik_{\parallel} z)$ and $k_{\parallel}^2 \ll k_y^2$. The z-direction is that of the local magnetic field. The longitudinal phase velocities are in the range $V_{thi} \leq \omega/k_{\parallel} < V_{the}$ while $|V_{\parallel}| < V_{thi}$. For simplicity, we neglect the density gradient relative to the ion temperature gradient, define $\omega_{Ti} \equiv k_y c(dT_i/dx)/(eB)$ and consider $\omega_0/\omega_{Ti} > 0$. Thus, when $dV_{\parallel}/dx \neq 0$ the sign of k_{\parallel}/k_y is relevant and two classes of modes can be identified on the basis of the sign of $(k_{\parallel}/k_y)dV_{\parallel}/dx$. We assume $V_{\parallel}(x) > 0$ and indicate as “primary modes” those with $k_{\parallel}/k_y > 0$ that have the prevalent growth rates on the basis of conventional linear theory in the initial phase, when momentum penetrates into the plasma column, when $dV_{\parallel}/dx > 0$ and both dV_{\parallel}/dx and dT_i/dx contribute to the momentum inflow. In the “saturated” state where the momentum has penetrated and $dV_{\parallel}/dx < 0$ the momentum outflow associated with dV_{\parallel}/dx is compensated by the inflow associated with dT_i/dx . The dispersion relation for these modes is

$$\omega_0^2 + \omega_0 \omega_{Ti} \frac{k_{\parallel}^2 V_s^2}{\omega_0^2 + v_T^2} = -k_y k_{\parallel} D_B \frac{dV_{\parallel}}{dx}, \quad (1)$$

where $k_{\parallel} k_y > 0$, $D_B \equiv c T_e / (eB)$, v_T is the rate of transverse thermal energy transfer, and $V_s^2 \equiv T_e/m_i$, provided $v_{\mu} = (\omega_{Ti}/\omega_0)v_T(k_{\parallel}V_s)^2/(\omega_0^2 + v_T^2)$ where v_{μ} is the rate of viscous momentum transfer.

The corresponding momentum flux that is given by

$$\Gamma_m^k \propto -\frac{v_{\mu}}{v_{\mu}^2 + \omega_0^2} \left[\frac{dV_{\parallel}}{dx} + \frac{k_{\parallel}}{m_i} \frac{dT_i}{dx} \omega_0 \frac{1 + v_T/v_{\mu}}{\omega_0^2 + v_T^2} \right] \left\langle |\hat{v}_{Ex}^k|^2 \right\rangle \quad (2)$$

includes an outgoing component proportional to dV_{\parallel}/dx and an inflow component proportional to dT_i/dx . Here $\hat{v}_{Ex}^k = -(\partial\hat{\Phi}^k/\partial y)c/B$. At saturation, when the dispersion relation (1)

is satisfied, Γ_m^k vanishes. On the basis of Eq. (2) we can propose a variant of the simplified transport equation introduced in Ref. [2] that has been used to simulate the rotation velocity profiles obtained experimentally. This could be $\Gamma_m \simeq -\rho D_J \left[\partial J / \partial r - (J_a / T_i) \partial T_i / \partial r \right]$ where $J = \Omega R^2$, and Ω is the rotation frequency. Near $r = 0$, this equation gives $J(r) \simeq J_0 - J_a r^2 / (2r_T^2)$ where J_0 is taken to be proportional to the source of angular momentum at the edge of the plasma column and $1/r_T^2 \equiv \left[(\partial^2 T_i / \partial r^2) / T_i \right]_{r=0}$. The “secondary” type of modes corresponding to $k_y k_{\parallel} < 0$ are considered to be non-linearly coupled to the primary modes and a simple analytical model for their dispersion equation is given. The relevant Γ_m^k , that vanishes at saturation, has a diffusive component as in Eq. (2) that is cancelled by the appropriate inflow term.

We note that in the absence of a velocity gradient ($dV_{\parallel}/dx = 0$) the spectrum of the ion temperature gradient (ITG) driven modes that are left is expected to be symmetric in $k_y k_{\parallel}$. Thus, no net transport of momentum is produced. Moreover, the prevailing modes should be of the ballooning type, in particular the Toroidal ITG modes [5] that depend strongly on the magnetic curvature, and do not propagate along the magnetic field. If a strong density gradient, represented by the parameter $\eta_i \equiv (d \ln T_i / dr) / (d \ln n / dr)$, is produced both the VTG and the ITG modes would be suppressed and no propagation of the rotation toward the center of the plasma column would occur. This expectation has been confirmed by experiments with the Alcator C-Mod machine on which a strong density peaking process was induced in the center of the plasma column by the appropriate application of ICRH heating [6]. Then a rotation velocity hole was observed [6]. We note that the VTG modes have a key dependence on the electron temperature and this is consistent with the T_e scaling of V_{ϕ} derived from experiments on the DIII-D machine [7] with strong electron cyclotron heating. In addition, V_{ϕ} profiles with a central hole were observed.

III. Edge Modes and Blobs

The H – confinement regime is characterized by the formation of a pressure pedestal at the edge of the plasma column with a steep density gradient. Therefore, we consider the best candidates for excitation in the edge region to be resistive ballooning modes [8] that are represented by the following comprehensive dispersion relation

$$(\omega - \omega_{di})(\omega + i\nu_{\perp\mu}^i) \simeq \frac{k_{\parallel}^2 V_A^2}{1 + iD_m k^2 / (\omega - \omega_{*e}^T)} - \gamma_I^2 \quad (3)$$

where $\gamma_I^2 = 2(dp/dx)/(\rho R_c)$, models the destabilizing effect of the magnetic field curvature $1/R_c$ and of the local pressure gradient, $\omega_{di} \equiv k_y c (dp_i/dx)/(eBn)$, $V_A^2 \equiv B^2/(4\pi\rho)$, $\omega_{*e}^T \simeq -k_y [c(dp_e/dx)/(eBn)] [1 + \alpha_T (d \ln T_e/dx)/(d \ln p_e/dx)]$ $\alpha_T \simeq 0.7$ and $D_m = \eta_{\parallel} c^2/(4\pi)$ is the resistive diffusion coefficient. From this we see that in the case where $D_m k^2$ is considerably smaller than $|\omega_{*e}^T|$, as appropriate around the surface where dp_e/dr is maximum within the “hot” pedestal, $\omega \simeq \omega_{*e}^T + i\gamma$ where $\gamma \simeq D_m k^2 \gamma_I^2 / k_{\parallel}^2 V_A^2$. Thus, the relevant phase velocity is in the direction of the electron diamagnetic velocity consistently with the relevant observations made by the JET machine [7], [4]. In the case of a colder edge with smaller electron pressure gradients, that we may consider to be characteristic of the L-regime, the ratio $D_m k^2 / |\omega_{*e}^T|$ can have moderate values and, correspondingly, the relevant unstable mode acquires a phase velocity in the direction of the ion diamagnetic velocity [4].

The most attractive idea that is being pursued in order to specify how angular momentum is ejected to the wall is the formation of filamentary coherent structures, “blobs”, at the edge of the plasma column. These blobs, whose velocity of rotation is related to and is in the same direction as that of the (edge) modes that produce them, convect radially toward the wall. Thus, momentum is transferred from the modes to the blobs and lost from the core plasma, providing a recoil force that can rotate the core. In particular, the blobs may be viewed as emerging from the non-linear evolution of the edge modes that we have considered.

*Sponsored in part by the US DOE and by the CNR of Italy

- [1] B. Coppi, *Nucl. Fusion* **42**, 1 (2002) and B. Coppi, Paper IAEA-CN-94-TH/P1-02, (Lyon, France, 2002) and MIT-RLE Report PTP02/05 (2002).
- [2] B. Coppi, *et al.*, Paper IAEA-F1-CN-TH3/7 (Yokohama, Japan, 1998).
- [3] J.E. Rice, *et al.*, Paper IAEA-CN-116/EX/6-4Ra, (Vilamoura, Portugal, 2004).
- [4] A. Sharapov and A. Fasoli (Private Communication, 2003).
- [5] B. Coppi and F. Pegoraro, *Nucl. Fusion* **17**, 5 (1977).
- [6] E.S. Marmar, *et al.*, Paper IAEA-CN-94-OV/4-1 (Lyon, France, 2002).
- [7] J.S. deGrassie, *et al.*, paper IAEA-CN-116/EX/6-4Rb, (Vilamoura, Portugal, 2004).
- [8] B. Coppi and M.N. Rosenbluth, *Plasma Phys. Control Fus. Res.* **1**, 617 (1966).
- [9] P. Smeulders, *et al.*, *Plasma Phys. Control. Fusion* **41**, 1303 (1999).