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Test ion acceleration in the field of expanding planar electron cloud

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Abstract

New exact results are reported for relativistic acceleration of test positive ions in the electric field of a planar plasma layer, whose electrons have initially been boosted to one and the same velocity v_0.

One of the important latest achievements in laser-plasma interaction has been a demonstration of efficient proton acceleration by illumination of high-Z foils with ultra-intense laser pulses [1, 2, 3]. Theoretical evaluation of the maximum energy of accelerated protons (or other test particles, like π^+ mesons for example) is usually based on a quasi-static Boltzmann relation for the electron distribution — which, however, becomes inadequate for a sufficiently high energy of laser-heated electrons. Then, of considerable help for gaining insight into the problem of ion acceleration under such conditions may be the solution to the following problem.

Consider a uniform plasma foil of thickness l_0 with an initial density of free electrons n_0. At time $t = 0$ all free electrons are set in motion with the same initial velocity v_0 perpendicular to the foil (see Fig. 1). At later times $t > 0$ the motion of electrons, treated as a collisionless charged fluid, is governed by the electric field $E(t,x)$ arising due to charge separation in the evolving plasma cloud. Our goal is to calculate the motion of a test ion of charge $+eZ_p$ and mass m_p placed initially at the foil surface $x = 0$. The bulk foil ions are assumed to be infinitely heavy and staying at rest. Earlier this problem was addressed by Bulanov *et al.* [4]. Here the results of Ref. [4] are partially corrected and significantly expanded.

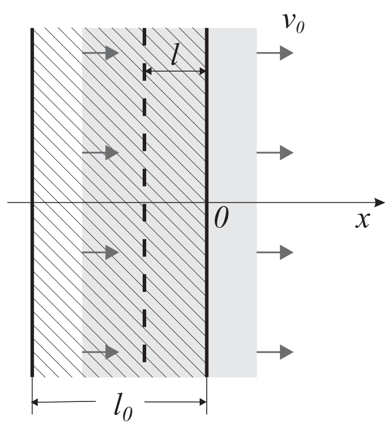

Figure 1: Plasma foil with motionless bulk ions (hatched area) and electrons (grey area) boosted to a velocity v_0.

It is easy to understand that there are three independent dimensionless parameters

$$\mu = \frac{m_e Z_p}{m_p}, \quad \Lambda = \frac{l_0 \omega_0}{v_0 \sqrt{\gamma_0}}, \quad \gamma_0 = (1-\beta_0^2)^{-1/2}, \tag{1}$$

which govern our problem; here m_e is the electron mass,

$$\omega_0 = \left(\frac{4\pi e^2 n_0}{m_e}\right)^{1/2} \tag{2}$$

is the initial plasma frequency, e is the positive elementary charge, and $\beta_0 = v_0/c$.

Evolution of the expanding electron cloud is determined by the values of Λ and γ_0. Analytical results are most readily obtained in the limit of $\Lambda \ll 1$, and then generalized to $\Lambda \gtrsim 1$. After some time the electron cloud is divided into two zones: the inner "relaxation" zone, where the trajectories of collisionless electrons cross each other, and the outer "laminar" zone, where the electron trajectories do not intersect (see Fig. 2). The boundary between the laminar and the relaxation zones grows linearly in time: $x_{rel} = \alpha_{rel} v_0 t$, with $\alpha_{rel} = 0.25 \pm 0.02$ for $\Lambda = 0$ and $\gamma_0 = 1$. Generally, the relaxation zone shrinks with the increasing γ_0, and expands with the increasing Λ. Analytical analysis of the ion acceleration is possible either in the laminar zone, or at the inner quasi-equilibrium core of the relaxation zone, where the Boltzmann relation can be used.

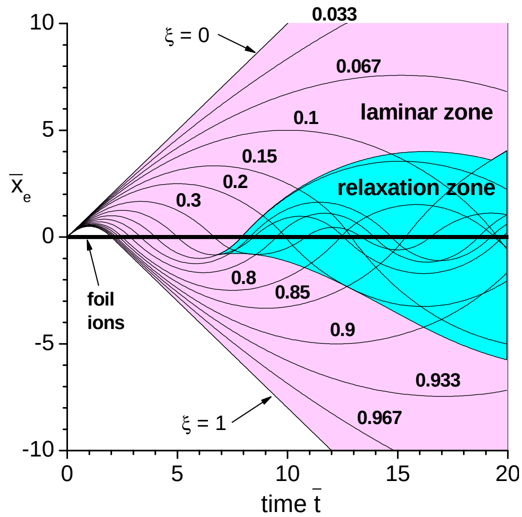

Figure 2: Electron trajectories in the tx-plane for $\Lambda = 0$ and $\gamma_0 = 1$.

For $\Lambda = 0$, the electron trajectories in the upper laminar zone are given by

$$\bar{x}_e = \frac{\bar{t}(2-\xi\bar{t})}{1+\sqrt{1-\beta_0^2\xi\bar{t}(2-\xi\bar{t})}}, \tag{3}$$

where $0 < \xi < 1$ is the Lagrangian coordinate for the electron fluid, and $\bar{t}$ and $\bar{x}$ are time and length normalized to new units $[t] = m_e v_0 \gamma_0 / 4\pi e^2 n_0 l_0$, $[x] = v_0[t]$. Equation (3) yields the electric field in the laminar zone,

$$E(\bar{t},\bar{x}) = 8\pi e n_0 l_0 \frac{\bar{t}-\bar{x}}{\bar{t}^2 - \beta_0^2 \bar{x}^2}, \tag{4}$$

and allows the equations of motion of the test ion to be cast in the form

$$\frac{d\chi}{d\zeta} = \frac{\cosh\chi}{\cosh\eta}\sinh(\eta-\chi), \tag{5}$$

$$\frac{d\eta}{d\zeta} = 2\mu\frac{\cosh\chi}{\cosh\eta}\sinh(\eta_0-\chi), \tag{6}$$

where $\zeta = \ln\bar{t}$, and the position of the test ion, $x_p(t) = ct\tanh\chi$, and its velocity, $v_p(t) = c\tanh\eta$, are expressed in terms of new "hyperbolic" variables χ and η; η_0 is defined as $\beta_0 =$

$\tanh\eta_0$. Since Eqs. (5) and (6) do not contain ζ explicitly, the key features of the ion motion can be elucidated by investigating the integral curves of the phase equation

$$\frac{d\eta}{d\chi} = 2\mu\,\frac{\sinh(\eta_0-\chi)}{\sinh(\eta-\chi)}, \tag{7}$$

and, in particular, the properties of its singular point at $\eta = \chi = \eta_0$.

For $\mu > \frac{1}{8}$ the singular point $\eta = \chi = \eta_0$ is a focus, which means that test ions with $\mu > \frac{1}{8}$ inevitably overtake the electron front, represented by the value $\chi = \eta_0$, and acquire the final velocity $v_{p\infty} > v_0$. For $\mu < \frac{1}{8}$ the singular point $\eta = \chi = \eta_0$ is a node with one direction of general approach, $\eta - \eta_0 \to \frac{1}{2}(1+\sqrt{1-8\mu})(\chi-\eta_0)$, and a solitary integral curve (separatrix), $\eta - \eta_0 \to \frac{1}{2}(1-\sqrt{1-8\mu})(\chi-\eta_0)$, which separates two different regimes of test ion acceleration. The boundary $\mu = \mu_{cr}(\gamma_0)$ [or, alternatively, $\gamma_0 = \gamma_{cr}(\mu)$, where $\gamma_{cr}(\mu)$ is the inverse function of $\mu_{cr}(\gamma)$] in the μ^{-1},γ_0 parametric plane between these two regimes is shown in Fig. 3. When $\Lambda \neq 0$, μ_{cr} generally depends on Λ as well.

Only sufficiently light ions with $\mu > \mu_{cr}(\gamma_0)$ [or when $\gamma_0 > \gamma_{cr}(\mu)$] can catch up with the electron front and reach the final velocity $v_{p\infty} > v_0$. In the non-relativistic limit $\gamma_0 \to 1$ we have $\mu_{cr} = \frac{1}{8}$ (for $\Lambda = 0$). In the opposite ultra-relativistic limit, when $\gamma_{cr}(\mu) \gg 1$ and $\mu_{cr}(\gamma_0) \ll 1$, one derives the following asymptotic expression

$$\gamma_{cr}(\mu) = \frac{1}{2}\mu\exp(\mu^{-1}-1). \tag{8}$$

Note that for protons with $\mu = 1/1836$ the corresponding value of $\gamma_{cr} = 2.3\times10^{793}$ is far beyond any realistic value. For the final energy $\mathscr{E}_{p\infty} = m_p c^2\gamma_{p\infty}$ of ions with $\mu_{cr}(\gamma_0) < \mu \ll 1$, which overtake the electron front and have $\gamma_{p\infty} > \gamma_0$, we derive an asymptotic formula

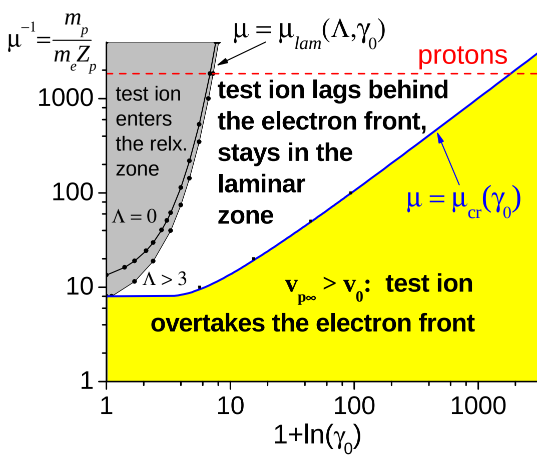

Figure 3: Different regimes of test ion acceleration in the μ^{-1},γ_0 parametric plane.

$$\gamma_{p\infty} = \mu\gamma_0\left(1+\ln\frac{2\gamma_0}{\mu}\right), \tag{9}$$

which differs significantly from the corresponding value $\gamma_{p\infty} = 2\gamma_0^2$ given by Eq. (35) in Ref. [4].

If, however, $\mu < \mu_{cr}$ (or $\gamma_0 < \gamma_{cr}$), a test ion in the laminar zone approaches the electron velocity v_0 only asymptotically, in the limit of $t \to \infty$, but on an enormously long time scale of $t_{ac} \approx \omega_0^{-1}\exp(1/2\mu)$ for $\mu \ll 1$. The latter means that in reality protons (and similar test ions) never reach the limiting electron velocity v_0, and their energy $\mathscr{E}_p = m_p c^2\gamma_p$ at a realistic time of observation t can be evaluated by using the following intermediate asymptotical expression

$$\frac{\beta_p\gamma_p}{\beta_0\gamma_0} = \mu \ln\frac{t\omega_0}{\sqrt{\gamma_0}} \left\{ 1 + \left[1 + (\gamma_0+\beta_0\gamma_0-1)\,\mu\ln\frac{t\omega_0}{\sqrt{\gamma_0}} \right]^{-1} \right\}, \tag{10}$$

which one derives from Eq. (6) for $\mu \ll 1$ and $\omega_0^{-1}\sqrt{\gamma_0} \ll t \ll \omega_0^{-1}\sqrt{\gamma_0}\exp\left[(2\mu)^{-1}\right]$.

Although the function $\mu_{cr}(\Lambda,\gamma_0)$ is most easily calculated for $\Lambda = 0$, in practical situations one usually has $\Lambda > 1$ (for example, 1 μm of solid gold ionized to $z = 50$ with $\gamma_0 = 100$ corresponds to $\Lambda = 10$). One can prove that the entire process of test ion acceleration and the values of $\mu_{cr}(\Lambda,\gamma_0)$ cease to depend on Λ for $\Lambda > \Lambda_{vg} = \sqrt{2\gamma_0/(\gamma_0+1)} \le \sqrt{2}$, when no vacuum gap is formed between the ejected electrons and the motionless foil ions. Numerical solution of the test ion equations of motion reveals that the two opposite extremes $\mu_{cr}(0,\gamma_0)$ and $\mu_{cr}(\sqrt{2},\gamma_0)$ never differ by more than 6%, i.e. the dependence of $\mu_{cr}(\Lambda,\gamma_0)$ on Λ would be hardly noticeable in Fig. 3 and can be ignored. Note that Eq. (10) applies to the case of $\Lambda > \Lambda_{vg}$.

Strictly speaking, our analysis applies to situations where the trajectory of a test ion lies entirely in the laminar zone. This is the case when the parameter μ is sufficiently large, i.e. for $\mu \ge \mu_{lam}(\Lambda,\gamma_0)$. To calculate the values $\mu_{lam}(\Lambda,\gamma_0)$, one has to simulate the evolution of the electron cloud with a full account of interpenetration of different elements of the collisionless electron fluid. The results of such calculations are shown in Fig. 3 as two extreme positions of the $\mu_{lam}(\Lambda,\gamma_0)$ curve in the μ^{-1},γ_0 parametric plane, obtained respectively for $\Lambda = 0$ and for $\Lambda > 3$. It is seen that there is a relatively wide window in the parameter space where our results for ion acceleration in the laminar zone do indeed apply, especially for highly relativistic electrons with $\gamma_0 \gg 1$. For protons with $\mu = 1/1836$ this window opens at $\gamma_0 > 348$ for $\Lambda \ll 1$, and at $\gamma_0 > 537$ for $\Lambda > 3$.

In a non-relativistic electron cloud with $\beta_0 \ll 1$, only very light ions with $\mu > 0.0745$ are accelerated entirely within the laminar zone at $\Lambda = 0$, and the laminar zone disappears at all for $\Lambda > 5.45$. As it can be inferred from Fig. 3, acceleration of protons in a non-relativistic electron cloud takes place deeply inside the relaxation zone, where one can expect the usual Boltzmann relation to be a good approximation.

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