Trapped electron mode stability in regions of steep gradients

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1 Introduction

The trapped electron mode (TEM) is generally unstable in tokamaks for short poloidal wavelengths, $k_\theta\rho_i \sim 1$, where k_θ is the poloidal wavenumber and ρ_i is the ion Larmor radius. Trapped electrons exist at lower collisionalities, $\nu^* = \nu_{eff}/\omega_{be} < 1$, where $\nu_{eff} = \nu_e/\varepsilon$ with ν_e the electron collision frequency and $\varepsilon = r/R$ is the inverse aspect ratio, where r and R are the minor and major radii of the tokamak, respectively; $\omega_{be} = \varepsilon^{1/2}v_{the}/Rq$ with v_{thj} the thermal speed of species j and q the safety factor, is the bounce frequency of trapped electrons. However the nature of the TEM varies depending on the collisionality. In the dissipative regime, $\nu_{eff} > \omega$, where ω is the mode frequency, there is a robust instability (DTEM) with growth rate $\gamma/\omega_{*e} \sim \varepsilon^{1/2}\ \omega_{*e}/\nu_{eff}$, where $\omega_{*e} = k_\theta\rho_i v_{thi}\tau/L_n$, with L_n the density profile scalelength, is the electron diamagnetic frequency and $\tau = T_e/T_i$. On the other hand, in the absolutely collisionless limit there is an instability, driven by a drift Landau resonance with the magnetic curvature and grad-B drifts of trapped electrons, with $\gamma/\omega_{*e} \sim \varepsilon^{1/2}\ (R/L_n)^{5/2}\exp(-R/L_n)$. In a steep density profile, the collisionless γ is exponentially small and the question is: what dissipative mode exists in the limit $\nu_{eff} < \omega$? Using a simple Krook collision operator there is an unstable mode with $\gamma/\omega_{*e} \sim \nu_{eff}/\omega$ due to finite Larmor radius effects (i.e. finite $b = (k_\theta\rho_i)^2$), provided $\eta_e = L_{ne}/L_{Te}$ with L_{Te} the electron temperature scalelength, is small enough, $< O(b)$ – i.e. instability occurs for longer wavelengths. However this theory predicts a discontinuity in pitch angle, $\lambda = (v_\perp/v)^2/B$, between the passing electron distribution, f_{pe} and that for the trapped electrons, f_{te}; this occurs because g_{pe}, the non-adiabatic part of f_{pe}, ~0, since $\omega < k_{||}\ v_{||e}$, where $k_{||}$ is the parallel wavenumber and $v_{||e}$ is the electron parallel velocity. Thus, at low collisionality it is essential to use a more realistic collision operator that properly represents pitch-angle scattering, producing a narrow boundary layer (of width $\delta\lambda \sim (\nu_e/\omega)^{1/2}/B_{max}$) that smoothly connects f_{pe} and f_{te} [1]. This treatment leads to $\gamma/\omega_{*e} \sim (\nu_{eff}/\omega)^{1/2}$, with instability again confined to longer wavelengths [2]. It is therefore of interest to determine the critical collision frequency for this transition in behaviour as collisionality decreases. We develop an analytic dispersion relation, albeit requiring numerical solution, to investigate this issue. First we develop a 'local' theory for low magnetic shear ($s < (\rho_i Rq/L_n L_*)$ where L_* represents the scalelength of L_n) when the radial structure is determined by the 'well' due to the density profile [3], before considering how to account for magnetic shear [4] and toroidal coupling [5]. Transport coefficients are calculated.

2 Dispersion Relation

Electron and ion distributions, f_j, are written as $f_j = -(e_j\varphi/T_j)F_j + g_jJ_0(z_j)$, where g_j satisfies the gyro-averaged kinetic equation, φ is the electrostatic potential and J_0 is a Bessel Function of argument $z_j = k_\theta v_\perp/\omega_{cj}$, with $v_\perp$ a perpendicular velocity and ω_{cj} a Larmor frequency. For ions, collisions, parallel motion and magnetic drifts are neglected and the perturbed ion density, $\hat{n}_i$ with full FLR effects is calculated [2]. For the electrons we retain collisions, set $z_e \approx 0$, again neglect magnetic drifts, and take ω, ω_{*e}, $\nu_e/\varepsilon \ll k_{||} v_{||e}$. We exploit the rapid parallel motion to average over transit and bounce times, to obtain:

$$g_{te} - \frac{i\nu_e(v)}{\omega\varepsilon}\frac{1}{\hat{\tau}_b(m)}\frac{\partial}{\partial m}\hat{J}_b(m)\frac{\partial g_{te}}{\partial m} = \frac{e<\varphi>}{T_e}R(v), \qquad g_{pe} \approx 0 \qquad (1)$$

where $R(v) = - F_e(x)\{1 - (\omega_{*e}/\omega)[\ 1 + \eta_e(x^2 - 3/2)]\}$, with $x^2 = m_e v^2/2T_e$, for trapped and passing electrons, respectively. Here, $\hat{\tau}_b(m) = 4K(m)$, $\hat{J}_b(m) = 4[E(m)\text{-}(1\text{-}m)K(m)]$, where E and K are complete elliptic integrals and m is a pitch-angle variable: $m = [1 - \lambda B_0(1 - \varepsilon)]/2\varepsilon\lambda B_0$ assuming $B = B_0(1 - \varepsilon\cos\theta)$; $<\varphi>$ is the bounce-averaged φ. The solutions of eqn. (1) must satisfy continuity of number and flux at the trapped/passing interface. The key approximation for TEM modes, ω, ω_{*e}, $\nu_{eff} \ll k_{||} v_{||e}$, that leads to $g_{pe} \approx 0$, implies a discontinuity at the trapped/passing boundary, resolved by a collisional boundary layer. For $\nu_{eff} > \omega$ this engulfs the whole trapped region, but for $\nu_{eff} < \omega$ it leads to a narrow layer at the boundary, predominantly in the trapped region. To obtain a tractable analytic solution for g_{te} we make some further approximations: thus we take $\varphi(\theta)$ to be approximately flute-like and use the small m expansions of E(m) and K(m). Equation (1) now has a solution satisfying the boundary conditions, given by [2]:

$$g_{te}(m) = \frac{e\varphi}{T_e}R(v)\left[1 - \frac{J_0(a\sqrt{m})}{J_0(a)}\right], \qquad (2)$$

so that the perturbed electron density is:

$$\frac{\hat{n}_e}{n} = \frac{e\varphi}{T_e}\left\{1 - \frac{8\sqrt{2\varepsilon}}{\pi^{3/2}}\int_0^\infty x^2 e^{-x^2}dx\left[1 - \frac{2J_1(a)}{aJ_0(a)}\right]\left[1 - \frac{\omega_{*e}}{\omega}\left(1 + \eta_e(x^2 - 3/2)\right)\right]\right\}. \qquad (3)$$

with $a(x)=(1+i)(4\varepsilon\omega/\nu_e(x))^{1/2}$. Quasi-neutrality, $\hat{n}_i = \hat{n}_e$, provides a dispersion relation.

Catto and Tsang [4] have shown how to generalise such a local theory to account for the radial structure required by the variation in $k_{||}$ due to shear, $s = (r/q)dq/dr$, that follows from imposing outgoing-wave boundary conditions. This theory is appropriate to the shorter wavelength, slab-like modes. Since the trapped electron drive occupies only a fraction Δ/x_t of the radial mode width (where $\Delta = 1/k_\theta s$ is the separation of resonant surfaces and $x_t = \rho_i(Rq/sL_n)^{1/2}$ is given by the outgoing-wave transition points) as a result of the mismatch between the direction of the trapped electron bounce-orbit and the pitch of the magnetic field lines, the TEM drive is strongly reduced when $bs > (L_n/Rq)$, i.e. by a factor $(\pi^{1/2}/4)(\Delta/x_t)\ln(x_t/\Delta)$ [4]. However, this theory also introduces shear damping, $\gamma/\omega_{*e} \sim sL_n/Rq$, which readily offsets the feeble TEM drive, noting toroidal coupling is weak in this same limit. For longer wavelengths, $bs < L_n/2R$, toroidal coupling annuls shear damping [5]; also TEM drive occurs at each resonant surface, so $\Delta \sim x_t$.

3 Results and Interpretation

Numerical solution of the local quasi-neutrality equation shows that there is a stable window for an increasing range of long wavelength modes as $\hat{\nu}$ falls below a critical value. As seen in Fig. 2 this is aided by larger values of η_e, although increasing η_i has the reverse, though a weaker, effect. These results can be understood in part analytically. Asymptotic evaluation of eqn. (3) for $\nu_e/\omega \ll 1$, when $b \ll 1$, yields [2]

$$\frac{\gamma}{\omega_{*e}} = \frac{2\Gamma(3/4)}{\pi^{3/2}}\sqrt{\frac{2\nu_e(v_{the})}{\omega}}\left\{(\eta_i + 1 + \tau)\,b - \frac{3\eta_e}{4}\right\} \qquad (4)$$

However this would imply all modes below $b = b_{crit} = (3\eta_e/4)/(1 + \tau + \eta_i)$ would be stable, in contrast to Fig.1 (note $b_{crit} = 0.25$, as shown by the vertical line in Fig. 1). This discrepancy can be traced to the fact that even if $\nu_e/\omega << 1$, there will still be a population of low energy particles that are more collisional since $\nu_e(v) \propto v^{-3}$. An asymptotic analysis which accounts for this produces an additional destabilising term in eqn.(4), $\sim \nu_e/\omega$, which reproduces the right-side narrowing of the stable spectrum as $\hat{\nu}$ increases [2], also shown in Fig. 1. The narrowing from the left-side can be understood as the transition to the unstable dissipative regime, $\nu_{eff}/\omega >> 1$ because ω becomes small at low k_θ. For the slab-like mode [4], the reduction in TEM drive and shear damping effectively stabilise it; thus the validity condition for this theory, $bs > L_n/Rq$, provides a stability criterion.

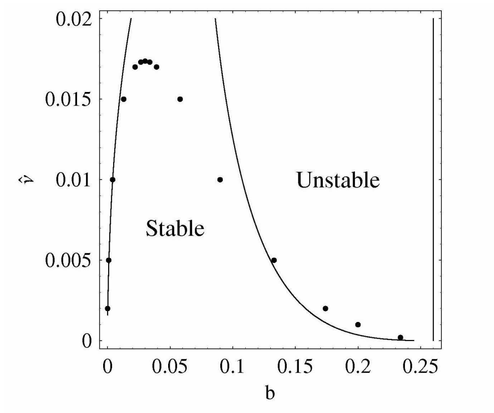

Fig 1: Critical $\hat{\nu}$(b); numerical (dots), asymptotic limits (lines); vertical line is eqn.(4)

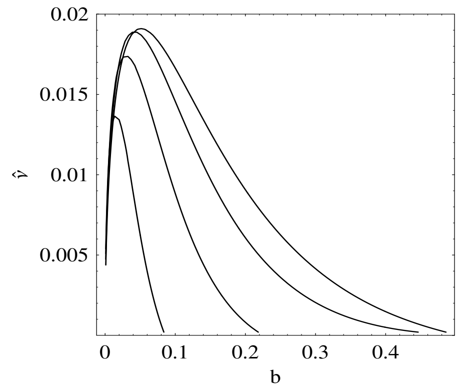

Fig.2: Narrowing of stable region as η_e increases from left to right (η_e=0.5,1,1.5,2)

4 Quasi-linear fluxes

Quasi-linear fluxes can be constructed based on the linear characteristics of the mode and some prescription for the saturated spectrum of fluctuations. A number of models exist for the latter, but we consider a simple mixing-length estimate, $e\varphi/T_e \sim 1/k_\theta L_n$, An expression for the particle diffusion coefficient can then be obtained as $D = \Sigma_k(\gamma_k/k_\perp^2)$, restricting the sum over k to the unstable long wavelength spectrum, $b \leq 1$, where neglect of ion sound is justified (the results are insensitive to the precise upper limit). Figs. 3 and 4 show the dependence of $\hat{D}$ on $\hat{\nu}$ and η_e (where $D = \hat{D}\,D_0$, with $D_0 \propto r\rho_s c_s/qL_n$).

5 Discussion and Conclusions

The stability results show that there is stabilisation of long wavelength modes, but only if the collisionality parameter $\hat{\nu} = \nu_e L_n / v_{thi}$ lies below a critical value, $\hat{\nu}_{crit} \sim 2\text{-}3\times10^{-2}$, interestingly a value typical of an ITB in MAST (Fig. 5). As $\hat{\nu}$ falls below this value an increasingly broad band of the long wavelength spectrum is stabilised. This effect is greater for larger values of η_e, but this is partly off-set if η_i also increases. Since $\hat{\nu}$ is proportional to L_n, a feedback mechanism for ITB formation is suggested: a reduction in the diffusion coefficient resulting from mode stabilisation reduces L_n further, leading to a steeper density gradient. Indeed the quasi-linear flux reduces with $\hat{\nu}$ for values similar to

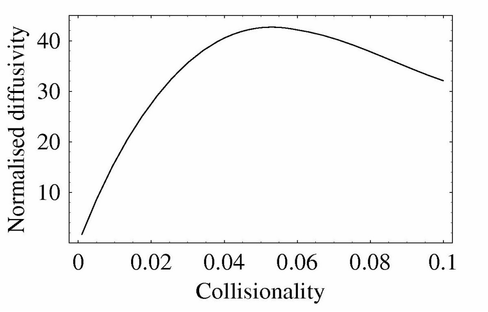

Fig 3: Variation of $\hat{D}$ with collisionality, $\hat{\nu}$; $\hat{D}$ reduces as the stable region of $\hat{\nu}$ (b) is encountered

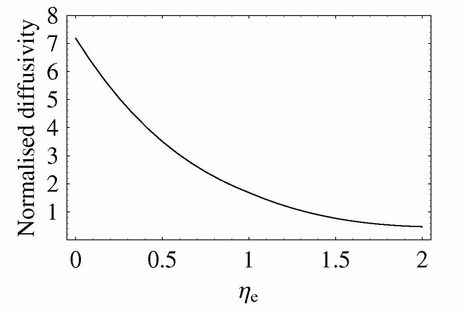

Fig 4: Variation of $\hat{D}$ with increasing η_e implying inward thermo-diffusion ($\hat{\nu}$ =10^{-3})

or less than the critical one (for higher values, the dissipative, DTEM, scaling holds). However if this reduction is achieved by reducing L_n, one should note the compensating inverse dependence of D_0 on L_n. The almost linear reduction in the transport coefficients with η_e shown in Fig. 4 implies an inward thermo-diffusion and density peaking; however the net flux must remain positive since the individual contributions in the k_θ sum are proportional to γ_k and are therefore positive. The trapped electron response f_{te} in eqn. (3) can also be used to calculate the transport due to other modes, say ITG modes.

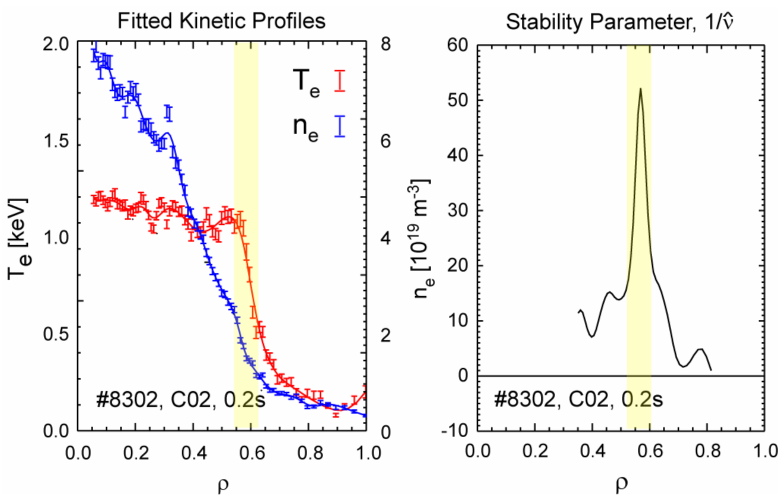

Fig 5: Experimental values of $1/\hat{\nu}$ in a MAST ITB

Acknowledgements

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