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Excitation of Beta induced Alfvèn Eigenmodes in the presence of a magnetic island

S.V. Annibaldi, F. Zonca and P. Buratti

Associazione EURATOM-ENEA, CR ENEA-Frascati, Via E. Fermi 45, 00044 Frascati, (Rome) Italy

Introduction

When a large magnetic island is present, waves with frequencies much higher than those of tearing modes have been observed in FTU and Textor [1, 2]. We show that these modes are Beta induced Alfvèn Eigenmodes (BAE), i.e. modes located in the low frequency β-induced gap in the shear-Alfvèn continuous spectrum, which is caused by finite plasma compressibility [3, 4]. In fact the mode frequencies obtained from a dispersion relation (DR) for BAE in the FTU regime fit very well the experimental results. We then write a more general DR which includes also resistivity [5, 6], and reduces to previous results in limiting cases.

Explanation of FTU observations

In FTU, modes at BAE frequency are observed in the presence of a large (-2,-1) magnetic island. In Fig. (4) of [2] the experimental frequency of those modes is compared with an estimate of the corresponding continuum accumulation point:

$$f_{ACP} = \frac{1}{2\pi R_0}\sqrt{\frac{2T_i}{m_i}\left(\frac{7}{4}+\frac{T_e}{T_i}\right)}; \tag{1}$$

where T_s is the temperature and m_s the mass of the species s ($s = i(e)$ for ions (electrons)). The resulting frequencies overestimate the experimental ones by nearly a factor 3, that is mostly due to the use of temperatures evaluated at the plasma centre.

We write a more precise DR by asymptotic matching the solution of the eigenmode equation in the thin inertial layer, localised around the rational surface where the island develops, and the corresponding field in the external region where ideal magnetohydrodynamics (MHD) applies. In the ballooning representation, with θ the extended poloidal variable, the inertial layer corresponds to $|\theta| \gg 1$.

From [7] the vorticity equation in the inertial layer, including toroidicity, finite Larmor radius (FLR) and finite orbit widths (FOW) effects, expanded to the fourth order in $k_\vartheta \rho_{Li}$ ($k_\vartheta = m/r$, m is the poloidal mode number, r is the minor radial variable, and $\rho_{Ls} = (m_s c\sqrt{T_s/m_s})/(e_s B)$ is the Larmor radius, with c the speed of light, e_s the electric charge and B the magnetic field), is:

$$\frac{\partial^2}{\partial\theta^2}\Psi+\Lambda^2\Phi-\theta^2 Q^2\Phi=0. \qquad (2)$$

Here, $k_\perp = k_\vartheta\sqrt{1+s^2\theta^2}$, with s the magnetic shear,

$$\Lambda^2(\omega)=\frac{\omega^2}{\omega_A^2}\left(1-\frac{\omega_{*pi}}{\omega}\right)+q^2\frac{\omega\omega_{ti}}{\omega_A^2}\left[\left(1-\frac{\omega_{*ni}}{\omega}\right)F(\omega/\omega_{ti})-\frac{\omega_{*Ti}}{\omega}G(\omega/\omega_{ti})-\frac{N^2(\omega/\omega_{ti})}{D(\omega/\omega_{ti})}\right]; \qquad (3)$$

and

$$Q^2(\omega)=s^2k_\vartheta^2\rho_{Li}^2\frac{\omega^2}{\omega_A^2}\left[\frac{3}{4}\left(1-\frac{\omega_{*pi}}{\omega}-\frac{\omega_{*Ti}}{\omega}\right)+q^2\frac{\omega_{ti}}{\omega}S(\omega)\right]; \qquad (4)$$

where ω is the BAE pulsation, $\omega_A = v_A/(qR_0)$ is the Alfvèn pulsation, q is the safety factor, R_0 the major radius, $v_A = B/\sqrt{4\pi n m_i}$, n is the density, equal for electrons and ions, $\omega_{*ps} = \omega_{*ns} + \omega_{*Ts}$, $\omega_{*ns} = [(T_s c)/(e_s B)](\mathbf{k}\times\mathbf{b})\cdot(\nabla n)/n$, $\omega_{*Ts} = [(T_s c)/(e_s B)](\mathbf{k}\times\mathbf{b})\cdot(\nabla T_s)/T_s$, $\mathbf{b} = \mathbf{B}/B$, $\omega_{ti} = \sqrt{2T_i/m_i}/(qR_0)$ and F,G,N,D,S can be found in [7]. The fields appearing are the potential perturbation, $\Phi = (k_\perp/k_\vartheta)\delta\phi$, and $\Psi = (k_\perp/k_\vartheta)\delta\psi$, related to the parallel vector potential $\delta A_\parallel$ by $\delta A_\parallel \equiv -i(c/\omega)\mathbf{b}\cdot\nabla\delta\psi$. Equation (2) is closed by the quasi-neutrality condition, relating $\delta\phi$ to $\delta\psi$. We assume that the resistivity, η, is negligible, and so

$$\Psi=\left[1+\frac{\omega_A^2}{\omega^2}\Lambda^2\frac{T_e}{T_i}k_\perp^2\rho_{Li}^2\left(1-\frac{\omega_{*ne}}{\omega}\right)^{-1}\right]\Phi; \qquad (5)$$

then the solutions of Eq. (2) are parabolic cylinder functions.

On the other hand, for tearing symmetry, i.e. odd modes, the ideal region solution is [5]:

$$\Psi_{\rm ID}=C\left[m\pi\theta+(\Delta'\theta)/(s|\theta|)\right]; \qquad (6)$$

where C is a constant and $\Delta' = r_s[\mathrm{d}\ln A/\mathrm{d}r]_{r_s^-}^{r_s^+}$, while r_s is the radius of the rational surface where the island develops.

Matching asymptotically Ψ, resulting from Eq. (2), for $|\Lambda\theta|\to 0$, to $\Psi_{\rm ID}$, we get the DR for BAE with tearing symmetry, taking into account toroidicity, FLR and FOW:

$$-2\sqrt{\hat{Q}}\frac{\Gamma\left(\frac{3}{4}-\frac{\Lambda^2}{4\hat{Q}}\right)}{\Gamma\left(\frac{1}{4}-\frac{\Lambda^2}{4\hat{Q}}\right)}=\frac{sm\pi}{\Delta'}; \qquad (7)$$

with

$$\hat{Q}^2=Q^2+\left(\frac{\omega_A^2}{\omega^2}\Lambda^2\frac{T_e}{T_i}k_\perp^2\rho_{Li}^2\right)\left(1-\frac{\omega_{*ne}}{\omega}\right)^{-1}. \qquad (8)$$

The table below shows some of the most representative cases displayed in [2] with their experimental frequencies, f_{exp}, and complex BAE frequencies, $(f,\gamma)=\omega/(2\pi)$, evaluated by solving Eq. (7) with a NAG routine. We assume $T_e = T_i$, $\Delta' = -2$, $s = 1$ and the other quantities are all taken from experimental observations at the mode location. In the last column we

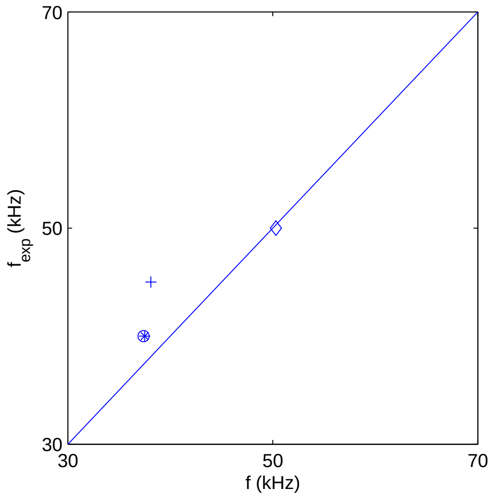

Figure 1: f_{exp} versus f for the shots shown in the table, with relative symbols. The full line plots $f_{exp} = f$.

also report f_{ACP} as calculated from Eq. (1) with the local T_e, to show that the big deviation from the analytical expression, previously found in [2], was mainly due to the use of central temperatures; still, Eq. (7) further improves the results, since, for example f_{ACP} only depends on temperature.

Figure (1) plots f_{exp} versus f for the above mentioned shots: the fitting between the frequencies is very good. It is then a neat improvement with respect to Fig. (4) of [2], and an evidence that the observed modes are BAE. The fact that $\gamma < 0$ shows that the modes are damped, and they need appropriate excitation mechanisms, that in this case are provided by the presence of the island, via nonlinear couplings that are still to be investigated.

Shot n^o	$B(T)$	$r_s(m)$	$n/10^{20}(m^{-3})$	$T_e(keV)$	$f_{exp}(kHz)$	$f;\gamma$	f_{ACP}
25298 +	5.9	0.16	0.48	0.5	45	38.1;-0.88	61.5
26702 o	5.2	0.18	0.41	0.3	40	37.4;-0.91	47.7
25239 $\star$	7.1	0.18	1.49	0.3	40	37.5;-0.80	47.7
26644 $\diamond$	7.1	0.13	0.37	0.9	50	50.3,-1.16	82.6

Dispersion relation of BAE with resistivity

The results shown in the previous should be generalised by the inclusion of resistivity.

By introducing quasineutrality in Ohm's law we get a relation between the $\delta\psi$ and $\delta\phi$ [6]:

$$\left(1 - \frac{\omega_{*ne}}{\omega}\right)\frac{\partial}{\partial\theta}(\delta\phi - \delta\psi) - \frac{i}{\omega\tau_R}\frac{k_\perp^2}{k_\vartheta^2}\frac{\partial}{\partial\theta}\delta\psi + \frac{\omega_A^2}{\omega^2}\Lambda^2\frac{T_e}{T_i}\rho_{Li}^2\frac{\partial}{\partial\theta}(k_\perp^2\delta\phi) = 0; \qquad (9)$$

where $\tau_R = 4\pi/(\eta c^2 k_\vartheta^2)$ is the resistive time. Substituting Eq. (9) in the θ derivative of Eq. (2), we get an equation for the perturbed field Ψ in the inertial layer:

$$\left(\frac{\partial}{\partial\theta} - \frac{1}{\theta}\right)\left\{\frac{\partial^2}{\partial\theta^2} + \Lambda^2 - \left[\hat{Q}^2 - \frac{i\Lambda^2 s^2}{\omega\tau_R(1-\omega_{*ne}/\omega)}\right]\theta^2\right\}\Psi = \frac{2i\Lambda^2 s^2\theta}{\omega\tau_R(1-\omega_{*ne}/\omega)}\Psi. \qquad (10)$$

It can be easily shown that Eq. (10) reduces to Eq. (2), with $\Phi = \Psi$, when $\tau_R \to \infty$, and to Eq.(30) of [5], that is the equation for modes in the collision - dominated regime, for $\hat{Q} = 0$.

To get a DR for the modes we are interested in, Eq. (10) has to be solved numerically and the resulting $\Psi(\theta)$ has to be matched asymptotically, for $|\Lambda\theta| \to 0$, to $\Psi_{\rm ID}$ of Eq. (6). The frequencies found by this procedure are the more general BAE frequencies for tearing symmetry modes, taking into account toroidicity, FLR, FOW and resistivity.

Conclusions and future work

In this paper we wrote a DR for BAE in the FTU low - collisional regime, which includes toroidicity, FLR and FOW effects, and showed that the resulting frequencies are in very good agreement with those of the modes observed in FTU in the presence of an $m = -2$, $n = -1$ magnetic island. Consequently we prove that the observed modes are BAE. Since no fast ions are present, we will concentrate on the nonlinear excitation of BAE modes by the large magnetic islands. Experimental observations [2, 1] and theory suggest three wave interactions as the best candidate for explaining these nonlinear behaviours.

The present DR for BAE modes, which includes finite Larmor radius and magnetic drift orbit width effects, can be further generalised to incorporate finite resistivity. This implies solving a third order equation in the inertial layer that has to be integrated numerically and then matched to the ideal MHD region, with tearing symmetry.

References

[1] P. Smeulders at al., Proc. 29th EPS Conf. on Plasma Phys. and Contr. Fusion, D5.016. (2002); P. Buratti et al., Nucl.Fus. **45**, 1446 (2005)

[2] P. Buratti et al., Proc. 32nd EPS Conf. on Plasma Physics and Contr. Fusion, P5-055 (2005).

[3] W.W. Heidbrink et al., Phys. Rev. Lett **71**, 855 (1993); A.D. Turnbull et al., Phys. Fluids B **5**, 2546 (1993)

[4] F. Zonca, L. Chen and R.A. Santoro, Plasma Phys. Control. Fusion **38**, 2011 (1996)

[5] F. Pegoraro and T.J. Schep, Plasma Phys. Control. Fusion, **28**, 647 (1986)

[6] F. Pegoraro et al., Phys. Fluids B, **1**, 364 (1989)

[7] F. Zonca et al., Plasma Phys. Control. Fusion, **40**, 2009 (1998)