

Numerical simulation of Ion Cyclotron waves in toroidal plasmas

M. Brambilla¹, R. Bilato¹, J.C. Wright², P. Bonoli²

¹ *Max-Planck Institut für Plasmaphysik - Euratom Association, Garching, Germany*

² *Massachusetts Institute of Technology, Cambridge, Massachusetts, USA*

Introduction. The code TORIC [1] solves Maxwell equations in the Ion Cyclotron (IC) range of frequencies in toroidal axisymmetric plasmas. To make possible selfconsistent simulations of IC experiments in tokamaks, TORIC has been coupled with SSFPQL [2], which solves the quasilinear Fokker-Planck (QLFP) equation for ions. The TORIC-SSFPQL package has been described in some details in [3]. Here we will briefly recall the physics included in the codes, and discuss some features of the model related to specifically toroidal effects. Use of this package for parametric studies of IC heating scenarios in ASDEX Upgrade is illustrated in a companion contribution to this Conference [4].

TORIC. The model implemented in TORIC includes propagation and damping of the externally excited fast wave (FW), and of Ion Bernstein (IBW) and Ion Cyclotron (ICW) waves which can be excited by linear mode conversion (LMC) near ion-ion resonances. The absorption channels are fundamental and first harmonic IC heating of ions, and Landau and Transit Time damping by electrons. Optionally, damping of the FW at higher IC harmonics can be simulated, although not simultaneously with mode conversion [5]. The wave equations are derived from Vlasov equation by expanding the e.m. fields in Fourier modes in the toroidal and poloidal angles [6]. While for the FW an expansion to second order in the Larmor radius is adequate, large Larmor radius effects are taken into account to describe IBWs [7], in such a way that the local dispersion relation of the wave equations solved by TORIC is everywhere a good approximation of the full hot-plasma dispersion relation. As a stand-alone, TORIC assumes local thermal equilibrium; for iteration with SSFPQL the coefficients of the wave equations are evaluated (to all orders in the Larmor radius) using the distributions predicted by the quasilinear kinetic equation as explained in [3].

The spectral representation of the hf field allows a rigorous definition of the local wavevector $\vec{k}^{n,m}$ for each mode of toroidal and poloidal numbers n and m . The physical components of $\vec{k}^{n,m}$ depend on position only on the scale of the MHD equilibrium, and, therefore, play much the usual role in determining the plasma response, in particular in defining particle-waves resonances. There are, nevertheless, specifically toroidal effects which cannot be neglected.

The first is toroidal broadening of IC resonances (in addition to the usual Doppler width) due to the finite length of the stationary-phase region of individual ions. In [8] it has been shown

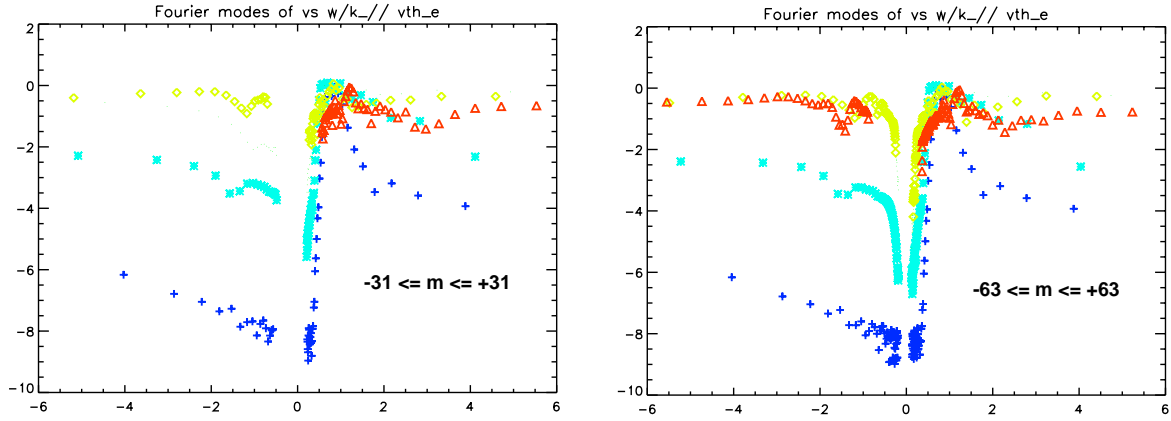


Figure 1: 1 – 6% H in D, ASDEX Upgrade: logarithmic plot of $|\text{mod } E_+(m)|$ versus $\omega/k_{\parallel}v_{\text{the}}$ on 5 equidistant magnetic surfaces, toroidal mode $n = 12$.

that this can be taken into account by “correcting” the local parallel wavevector with a factor which depends on the parallel gradient of the static magnetic field. This approximation is quite accurate, and guarantees that TORIC predicts that in a stable plasma power is everywhere (and not only on average) transferred from the waves to the resonant ions.

Secondly, the parallel component of $\vec{k}^{n,m}$, and thus the parallel phase velocity, is different for different Fourier modes. Because of the strong coupling between modes in toroidal geometry, the resulting “toroidally induced broadening” of Cherenkov resonances is by no means a small effect. In fig. (1) of [3] it is shown that even for the FW, whose poloidal number spectrum is limited by the relatively large wavelength, the range of parallel phase velocities for a single toroidal mode extends from infinity to well below the electron thermal speed. It follows that the spectral representation is essential for an accurate estimate of absorption by the electrons: the approximation $k_{\parallel} \sim n/R$, for example, can be at best qualitatively correct.

Numerical resolution of IBWs and ICWs excited by LMC requires a quite broad range of poloidal Fourier modes. We can distinguish two very different situations, roughly corresponding to the minority and the mode-conversion regimes, respectively.

Fig. 1 compares the amplitude of modes with different parallel phase velocities in two runs with different resolution of a typical minority scenario [9] on 5 equidistant magnetic surfaces, from $r/a = 0.1$ (deep blue) to $r/a = 0.9$ (red). In this case, lack of convergence only affects the representation of IBWs in the plasma periphery. Remarkably, the low-order poloidal modes are essentially unaffected when new modes are taken into account: the global power balance and the radial deposition profiles are already accurate to better than 0.5% in a run with only 31 modes ($-15 \leq m \leq +15$), which strictly speaking should be sufficient only to resolve the FW.

The situation is less favourable in simulations of mode conversion scenarios, particularly

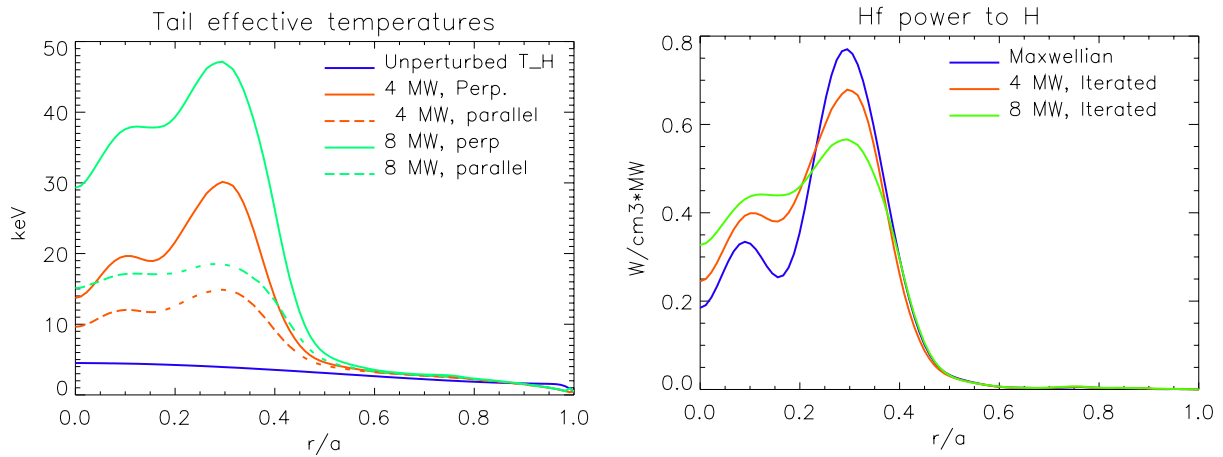


Figure 2: 2 – 6% H in D, ASDEX Upgrade: Effective minority tail temperatures, and quasilinear power deposition profiles in the minority (per MW coupled), at 4 and 8 MW.

when the minority fundamental IC resonance is ‘isolated’, i.e. does not coincide with the first harmonic of the majority ions. Convergence requirements are then particularly severe, because of the rapid spatial variations of the plasma dielectric properties near the MC layer. Reproducing numerically the sharp minimum of E_+ due to screening by resonant ions requires the destructive interference of a sufficiently large number of Fourier modes. Lack of convergence can result in overestimating $|E_+|^2$, and therefore power absorption by the minority ions, by quite a large factor. Indeed, satisfactory simulations in this regime, such as those reported in [4], have been made possible only by the parallelization of the code [10].

TORIC has successfully reproduced IC heating results in several tokamaks, and shows good agreement with the AORSA code [11], which solves for the same physics (to all orders in the Larmor radius) with a different numerical approach. Perhaps the best benchmark has been the accurate simulation of mode conversion to ICW, a toroidal effect first observed in Alcator C-Mod [12].

SSFPQL. The code SSFPQL [2] evaluates the steady-state distribution function of ions heated at the fundamental and first harmonic IC resonance by balancing the Kennel-Engelmann quasilinear operator (QLO) [13] with the linearized collision operator. The form of the bounce-averaged QLO is simplified by neglecting the acceleration of ions in the resonance region. The solution is represented as a series in even Legendre polynomials including typically 5 to 8 terms; a special Bessel function identity is used to guarantee that the truncated expansion of the QLO is positive definite up to a sufficiently high energy, about 7 times the unperturbed temperature. Due to this approach, SSFPQL is very fast (less than 20 sec for two ion species on 100 magnetic surfaces on a laptop).

Fig. 2 shows the tail effective temperatures, estimated from the logarithmic slopes of the tails in the distribution function calculated by SSFPQL, and the power absorption profile of the minority for the same ASDEX Upgrade shot as in fig. 1, at two power levels, after 3 iterations with TORIC. Of 4 MW, the power effectively coupled in the experiment, about 30% is predicted to heat the electrons, 10% directly from the wave, and 20% by collisional transfer, in good agreement with the experimental observations [9]. 8 MW is the power level which would have been necessary for half of the power to end up in the electrons. As expected, increase in $T_{\text{eff}\parallel}$ broadens the minority absorption profile, but increase in $T_{\text{eff}\perp}$ decreases the peak heating rate. Even at the relatively high collisionality of this experiment the anisotropy of the suprathermal tail is large, and increases with power, so that the second effect dominates, and the fraction of power absorbed by the minority ions decreases with increasing total power.

SSFPQL is not adequate to determine the fate of the most energetic ions. For the exponentially more numerous ions at intermediate energies, which absorb the bulk of the IC power, it nevertheless provides sufficiently accurate distribution functions for coupling with TORIC. Important for this purpose are a good estimate of the anisotropy of the suprathermal tails, and the inclusion of large Larmor radius corrections in the coefficients of the wave equations. The major source of inaccuracy is the fact that SSFPQL, like all otherwise more sophisticated QLFP solvers based on surface averaging, neglects the finite width of the banana orbits of fast ions.

References.

- 1 M. Brambilla, *Plasma Phys. Contr. Fusion* **41**, 1, (1999).
- 2 M. Brambilla, *Nucl. Fusion* **34**, 1121, (1994).
- 3 M. Brambilla, R. Bilato, *Nucl. Fusion* **46** (2006), in press.
- 4 R. Bilato, M. Brambilla, J.R. Wright, P.T. Bonoli, this Conference.
- 5 M. Brambilla, *Plasma Phys. Contr. Fusion* **44** 2423 (2002).
- 6 M. Brambilla, *Plasma Phys. Contr. Fusion* **41** 775 (1999).
- 7 M. Brambilla, *Nucl. Fusion* **38**, 1805, (1998).
- 8 M. Brambilla, *Phys. Letters A* **188**, 376, (1994).
- 9 M. Brambilla, R. Bilato, C. Maggi, et al *16th Topical Conf. on Radio Frequency Power in Plasmas*, AIP Conf. Proc. 787 (2005), p. 38.
- 10 J.C. Wright, P. Bonoli, M. Brambilla, et al, *Phys. Plasmas* **11**, (2004) 2473.
- 11 E.F. Jaeger, L.A. Berry, E.D'Azevedo, et al *Phys. Plasmas* **8** 1573 (2001).
- 12 E. Nelson-Melby, M. Porkolab, P.T. Bonoli, et al, *Phys. Rev. Letters* **90** 155004 (2003).
- 13 C.F. Kennel, F. Engelmann, *Phys. Fluids* **9**, 2377, (1966).