*33rd EPS Conference on Plasma Phys. Rome, 19 - 23 June 2006 ECA Vol.**30I**, D-5.021 (2006)*

The Helimak transport dynamics: low dimensional chaos or coloured noise?

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Recent theoretical developments indicate that the cross-field transport in the Helimak configuration is mediated by electrostatic flute structures with a fixed perpendicular wave-length [2] and can be modeled as a low-dimensional dynamical system; in the simplest formulation as the Lorenz equations in the diffusionless limit [3].

We investigate this question by analyzing time series of electric potential fluctuations from the edge of the Blaamann device operated in the helimak configuration.

Since we work with experimental data, we use time-delay embedding theorem by [1] in an attempt to reconstruct the phase space of an hypothesised low-dymensional dynamical system. Here we transform our scalar data into m-dimensional vectors:

$$x_i = (x(t_i), x(t_i+\tau), ..., x(t_i+(m-1)\tau)), \qquad (1)$$

where i=1,...,N and τ is an appropriate time lag. Since the dynamics in the core plasma is mostly influenced by unstable flute convective cells, we analyse edge fluctuations, which reflect the fluctuations in the ensuing anomalous plasma flux. For this purpose we employ the Singular Value Decomposition (SVD) analysis as described in [4]. We present some of the results for the first SVD component, denoted as V1, and the second SVD component, denoted as V2, since they reveal different underlying dynamics. The auto-correlation functions of the V1 and V2 components shown in Fig.1 reveal that the decorrelation time for the V1 component (see Fig. 1b) is larger than the decorrelation time for the V2 component (see Fig. 1a).

The standard method for determining the attractor dimension of a dynamical system from a time series of one of its components is the Grassberger-Procaccia (G-P) algorithm [5]. By using this algorithm for the embedding dimensions $2 < m < 7$ and time lag $\tau = 20$, we found that the V2 component converges to a correlation dimension of $D \approx 2.5$ (see Fig. 2a). On the other hand, the correlation dimension for the V1 component does not converge for $2 < m < 7$. It is known, however, that the G-P algorithm can give a spurious correlation dimension $D = 1/H$ if

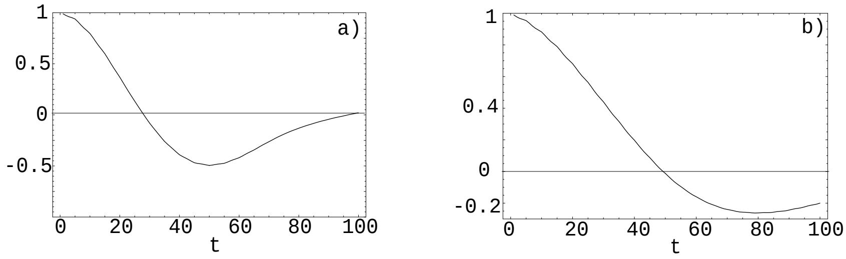

Figure 1: Auto-correlation functions for SVD components: V2 (a), and V1 (b)

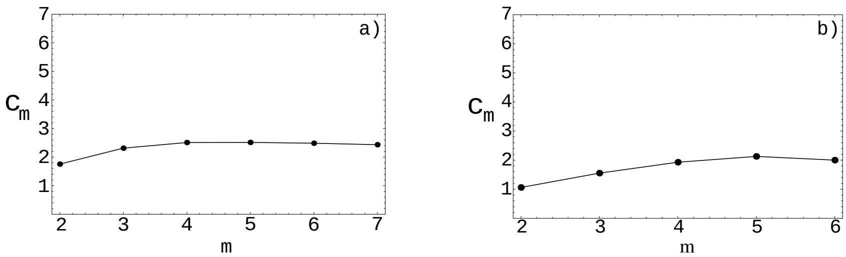

Figure 2: Correlation dimension C_m for embedding dimensions $2 < m < 7$: V2 (a), and V1 (b)

the attractor of the signal is coloured noise with a scaling exponent $H > 0$ (fractional Brownian motion). This problem can be circumvented by a recurrence plot analysis which allows us to determine whether the low dimensionality of the V2 component arises from correlations in space due to the recurrence of the phase space trajectory to the same regions, or from self-affinity of the trajectory due to correlations in time. This plot marks points in an $i - j$ diagram for which the distance $|x_i - x_j|$ is smaller than the given radius r. Recurrence plot analysis was performed for the embedding dimension $m = 3$ and time lags $\tau = 50$ and $\tau = 20$ for the V1 and V2 components, respectively. In Fig. 3b, the V1 component behaves like coloured noise, i.e. neighbouring points in the state space are also close in time and hence points are grouped along the diagonal, because the data are highly autocorrelated (see also [6]). On the other hand, recurrence plot for the V2 component (see Fig. 3a) shows lines parallel to the diagonal, which implies that a part of the trajectory comes closer to an earlier part of the trajectory and, hence, shows spatial correlations typical for a chaotic attractor. Also, the calculation of the largest Lyapunov exponent for the V2 component gave us a small, but positive value, which is also consistent with chaos.

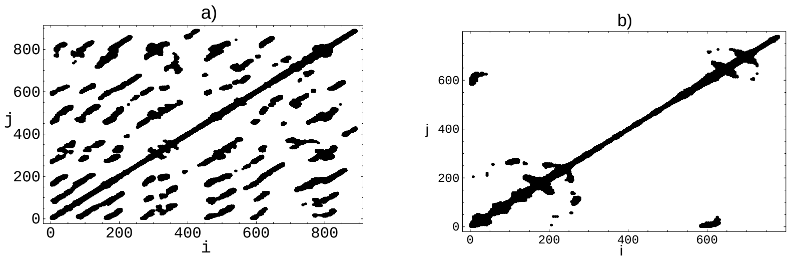

Figure 3: Reccurence plot for SVD components: V2 (a), and V1 (b)

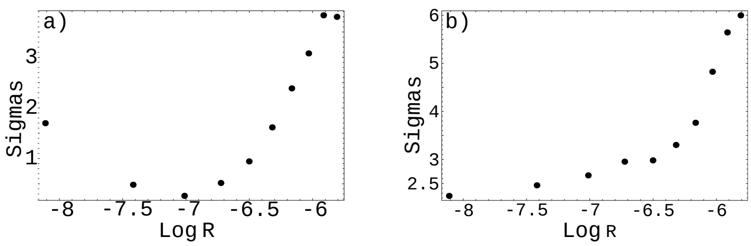

Figure 4: Nonlinear test for V1 component (a), and V2 component (b)

At the end, we run a nonlinearity test for the V2 and V1 components. In the case of coloured noise processes the convergence of the correlation dimension is forced mainly by the shape of the power spectrum, while for low-dimensional dynamics, phase correlations play an essential role. We perform a test as described in [7], i.e. we construct a new time series , which has the same power spectrum, but not the same phase correlations as the original V2 (or V1) component. Thus, if the correlation dimension of a new time series is approximately the same as for the original one, we do not have low-dimensional dynamics, and vice versa. In order to improve statistics, we calculate the correlation dimensions for ten new time series and compare them with a correlation dimension of the original V2 (V1) component as follows:

$$Sigma = \frac{|C_o - < C_{sur} > |}{\sigma_{sur}}, \quad (2)$$

where σ_{sur} and $< C_{sur} >$ are the standard deviation and the average of the correlation integrals of a new time series for some radius r; C_o is the correlation integral for the original time series. When Sigma takes values higher than $2-3$, the probability that the original time series is nonlinear is higher than 0.95-0.99, correspondingly. This is confirmed for the V2 component (Fig. 4b), while the V1 component shows some nonlinearity only for bigger r values (Fig. 4a).

From this brief analysis, we conclude that the V1 component follows the behaviour of coloured noise, while the V2 component reveals low-dimensional, nonlinear dynamics.

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