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On O-X mode conversion in a cold magnetized 2D inhomogeneous plasma in the electron cyclotron frequency range

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Introduction

Nowadays the EBWs having no density cut-offs and effectively damped are considered to provide the auxiliary heating and current drive in a dense plasma of a spherical tokamaks and stellarators. The EBW could be excited via so-called O-X-B scheme, which efficiency, as it was demonstrated in 1D slab model [1], is determined by the efficiency of O to X mode conversion, which can reach 100 percent value at the certain parallel refractive index being constant in slab geometry. Unfortunately, in a real tokamak or stellarator configurations, where the poloidal inhomogeneity of the magnetic field is important and moreover where the parallel refractive index is no longer constant, analysis of the full-wave equations in the frame of 2D model is needed.

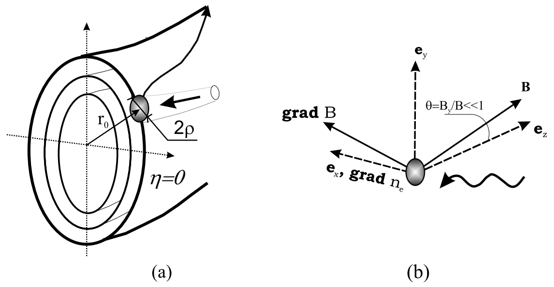

Figure 1: a) Model considered is illustrated, (b) Coordinate system (x,y,z) with the origin on the O-mode cutoff ($\eta = 0$)

The first attempt to consider 2D model of OX transformation has been performed a couple of years ago [2]. The main conclusion provided by authors concerning the absence of the O mode reflection from the cut-off surface, which seems to be quite doubtful, stimulated further investigations. Last half a year a number of papers devoted to this hot topic appear [3], [4]. In [4] the 2D model problem was investigated in the framework of equations [6] under a simplifying supposition of a pure toroidal magnetic field omitting animportant effect of parallel refractive index variation along magnetic field line. In [3] the finite value of poloidal magnetic field and thus the effect of parallel refractive index variation was taken into account and the explicit expressions describing the OX conversion in the realistic experimental geometry had been obtained.

In the present paper the further development of this theoretical model is performed. The solution to a reduced set of the partial differential wave equations derived in [3] and valid in a vicinity of the O-mode cutoff and accounting for the magnetic field 2D inhomogeneity and its properties are considered in detail. As example, the O to X mode conversion coefficient of Gaussian beam incident on the O-mode cutoff surface for MAST conditions is presented. The obtained results and conclusions are compared to those obtained using a simplified model [4].

Physical model

The physical model used in further analysis is as follows. We consider a beam of the ordinary waves incident on the O mode cut-off surface far from the tokamak midplane (Fig. 1). Three effects remain beyond the scope of the present paper. We neglect, first, the curvature of the magnetic field line at the magnetic surface due to its local radius R_f is greater than the beam radius ρ, second, the curvature of the magnetic flux surfaces assuming high localization of the conversion region, third, the magnetic field shear which is not important for OX conversion [5]. We restrict ourselves to the case of not extremely strong plasma density inhomogeneity L_n when the wave conversed through the evanescent region and propagating inward plasma could be associated with WKB X mode. We have used a Cartesian co-ordinate system (x,y,z) with its origin located on the O-mode cut-off surface (Fig. 1) and axis x coinciding with direction of the density gradient, axes y and z imitating the poloidal and the toroidal directions, respectively. To obtain the reduced set of equations in the vicinity of the O mode cut-off one can expands the plasma parameters into the Taylor series at r_0 (Fig. 1) $n \simeq n|_{r0}(1+x/L_n)$ and $|B| \simeq B|_{r0}(1+x/L_{bx}+y/L_{by})$, where $c/(\omega L_n), \rho/L_{by} \ll 1$, $L_n^{-1} = \partial \ln n_e/\partial x|_{r_0}$, $\mathbf{L}_b^{-1} = \partial \ln B/\partial \mathbf{r}|_{r_0}$, $\mathbf{r} = (x,y)$, are the first order quantities and $c/(\omega L_{bx}) \ll 1$ is the second order quantity. Since the components of the dielectric tensor being function of two coordinates, namely, x, y, are not dependent on the coordinate z, we make Fourier transformation $\mathbf{E}(\mathbf{r}) \div \mathbf{E}(x,y,n_z)$ and choose $n_z \simeq n_z^{opt} + \delta n_z$, $n_z^{opt} = 1/(1+1/q_0)^{1/2}$ for which the evanescent region could be transparent only. Assuming that

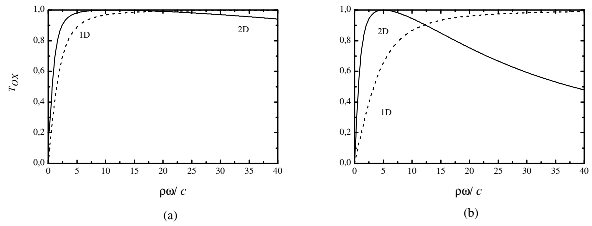

Figure 2: T_{OX} calculated according 1D (dashed line) and 2D (solid line) model for MAST, (a) H - mode ($L_n = 5$ cm, $L_{by} = 60$ cm), (b) L- mode ($L_n = 15$ cm, $L_{by} = 60$ cm).

$$B_y/B \ll 1 \tag{1}$$

and keeping in Maxwellian wave equations terms being the first order quantity, one obtains the reduced set of equations [3]

$$\begin{aligned} \left(\partial_+ - \theta n_z^{opt}\right) E_\xi \quad &+ \quad \left(x - ay'\right) F = 0 \\ \left(\partial_- + \theta n_z^{opt}\right) F \quad &- \quad x E_\xi = 0, \end{aligned} \tag{2}$$

where new notations appear $\partial_\pm = \partial/\partial x \pm i\partial/\partial y'$, $y' = y - b/a$, $2^{1/4}/L_n^{1/2}/q_0^{1/4} \cdot x, y' \to x, y'$, $F = -iE_+/\sqrt{1+q_0}$, $a = L_n/L_{by} \cdot q_0/(1+q_0)$, $b = 2^{5/4} L_n^{1/2} q_0^{1/4} (1+q_0)^{1/2} \delta n_z$ and $\mathbf{r} \to \mathbf{r}\omega/c$. Prime mark at y' we will omit further. The components of the electric field (E_+, E_-) are associated with ones rotating in the ion and electron directions, respectively. The component E_- is small compared to two others $E_+, E_\xi \sim (c/(\omega L_n) E_-)$. It should be mentioned that the set of equations derived in [4] looking very similar to (2) is correct for plasma without poloidal magnetic field, ignoring as it does the important fact of absence the conservation of the parallel wave vector. Therefore, using the model set out in [4] to describe OX conversion in spherical tokamaks and stellarators straightforwardly seems to be mistaken. As it was shown in [3] the observance of inequality (1) is necessary to obtain the reduced set of equations (2) which is solvable analytically.

Solution to the set of equations

We would like to construct solutions of the system (2) in order to study the properties of the waves in the mode conversion region. To this end we look to adopt the method [6] of seeking the functional substitution reducing the set of equations to single one to 2D case inhomogeneous plasma. As it was shown in [3] we can introduce new dependent functions $\left(F, E_\xi\right) = \exp\left(-i\theta n_z^{opt} y\right) \exp\left(iS\right) \left(\partial_+, -i\sqrt{1-ia}\partial_-\right) \left[\exp\left(-iS\right)W\right]$, where $S = -((2-ia)x^2/2 - axy - iay^2/2)/(2\sqrt{1-ia})$, and new variables $x = M(u\cos\psi + v\sin\psi)$, $y = M(1-N^2)^{1/4}(v\cos\psi - u\sin\psi)$, where $M = (1-N^2)^{1/4}$, $\cos\psi = (1+N^2)^{-1/2}$ and N being defined as $N = \pm(\sqrt{1+a^2} - 1)/|a|$. Here $N >, < 0$ correspond to poloidal position of $\mathbf{r}_0$ above or below plasma mid-plane, respectively. As result the set of equations may be reduced to the equation for the function W

$$\frac{\partial^2 W}{\partial u^2} + \frac{\partial^2 W}{\partial v^2} + \left(u^2 - N^2 v^2 - N + i\right) W = 0 \tag{3}$$

The solution to Eq.3 describing incident O-mode conversion to X-mode is given by

$$W = \sum_{p=0}^{\infty} W_p\left(u\right) \phi_p\left(v\right), W_p = B_p D_{i\gamma_p/\pi}\left(\sqrt{2}\exp\left(i\pi/4\right)u\right) \tag{4}$$

$$\phi_p(v) = \left(\frac{|N|}{\pi}\right)^{1/4} \exp\left(-\frac{|N|v^2}{2}\right) \frac{H_p\left(\sqrt{|N|}v\right)}{\left(2^p p!\right)^{1/2}}, \int_{-\infty}^{\infty} \phi_p(v)\phi_k(v)dv = \delta_{pk}$$

with $D_{i\gamma_p/\pi}$ and H_p being parabolic cylinder function and Hermitian polynomial. The scale of an evanescence region $\gamma_p = \pi|N|\left(p + 1/2 - 1/2 \cdot N/|N|\right)$ depends critically on the number of the mode ϕ_p, and sign of N (i.e. the position of r_0 upper and below mid-plane). Using the asymptotic representations of $D_{i\gamma_p/\pi}$ for large argument, which matches the WKB solutions on the both sides of the O mode cut off, and comparing it for $u \to -\infty$ and $u \to \infty$ [7] we obtain the

OX conversion coefficient for each spectral component n_z

$$T_{OX} = \frac{1}{P}\sum_{p=0}^{\infty} |B_p|^2 \exp\left(-2\gamma_p\right), P = \sum_{p=0}^{\infty} |B_p|^2, B_p = \int_{-\infty}^{\infty} A\left(x(u,v), y(u,v)\right)\phi_p\left(v\right)dv, \qquad (5)$$

where A is beam's amplitude. For the reverse XO process the expression for the conversion coefficient T_{XO} possesses the same form as (5), however depending on the different parameter $\gamma_p = \pi|N|\left(p + 1/2 + 1/2 \cdot N/|N|\right)$, i.e. in 2D plasma $T_{OX} \neq T_{XO}$. This property of the conversion coefficient was firstly mentioned in [4]. The reciprocity theorem for T_{OX} could be formulated as $T_{OX}\left(\mathbf{B}\right) = T_{XO}\left(-\mathbf{B}\right)$, where the argument $\mathbf{B}$ of T_{OX}, T_{XO} is expressed explicitly. For typical conditions of experiments $T_{OX} \simeq T_{XO}$. To illustrate the analytical results obtained we consider two demo examples. In (Fig. 2) T_{OX} calculated in the frame of 1D and 2D models for the Gaussian beam with transversal distribution $A = \left(\pi\rho^2\right)^{-1/4}\exp\left(-y^2/(2\rho^2)\right)\delta\left(n_z - n_z^{opt}\right)$ incident on the conversion region are compared. Considerable distinction especially in H-regime is demonstrated.

Conclusions

In this communication the OX conversion for quasi-tokamak 2D configuration was considered. The explicit expressions for the conversion coefficients T_{OX} and T_{XO} were obtained. The importance of 2D effects increasing the conversion rate, was demonstrated by analytic formula and illustrated in the particular case of MAST experiment.

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