*33rd EPS Conference on Plasma Phys. Rome, 19 - 23 June 2006 ECA Vol.**30I**, D-4.003 (2006)*

Landau Damping of Dust Acoustic Waves in a Lorentzian Plasma

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I. INTRODUCTION

Non-maxwellian plasmas are very common in space. Plasmas in space or laboratories can contain a substantial component of high energy particles which are effectively modeled by Lorentzian (kappa) distribution [1,2]. In reality, most of plasmas coexist with the dust grains which is not neutral in general. Nevertheless, many of the work in this area have been devoted to the Lorentzian electron-ion plasma system. Among many wave phenomena, the Landau damping is one of the most well known physical processes in plasmas [3]. For a bulk plasma, the Landau damping in a Lorentzian electron-ion plasma was reported previously [4,5]. The Landau damping of the dust acoustic waves is also well known for a Maxwellian distribution [6]. However, the Landau damping of dust acoustic waves in a dusty Lorentzian plasma have not been investigated yet. In this paper, an effort is given to obtain the kinetic modes of electrostatic dust acoustic waves propagating in a dusty plasma that has a Lorentzian distribution for electrons and ions and a Maxwllian distribution for dust grains.

II. DUST ACOUSTIC DISPERSION RELATION

In order to obtain a dust acoustic wave dispersion relation for Lorentzian electron-ion plasmas and Maxwellian dust grains, we consider a collisionless, unbounded, unmagnetized dusty plasma. The unperturbed state is neutral overall, so its internal electric field is zero. We assume that the curl of electric field vanishes (*i.e.*, electrostatic), and the perturbation about the equilibrium state is weak. The basic equation for the analysis is the familiar Vlasov equation for a species α with charge q_α and mass m_α given by

$$\frac{\partial f_\alpha}{\partial t} + \mathrm{v}\bullet\frac{\partial f_\alpha}{\partial \mathrm{r}} + \frac{q_\alpha}{m_\alpha}\left(\mathrm{E} + \frac{1}{c}\mathrm{v}\times\mathrm{B}\right)\bullet\frac{\partial f_\alpha}{\partial \mathrm{v}} = 0 \qquad (1)$$

where α(= *e, i, d*) denotes for electron (*e*), ion (*i*) or dust particle (*d*). Now, we shall let our electrons and ions obey the Lorentzian distribution with spectral index κ (> 3/2) which is given by

$$f_{\kappa,\alpha} = n_\alpha \left(\frac{m_\alpha}{2\pi\kappa E_{\kappa,\alpha}} \right)^{3/2} \frac{\Gamma(\kappa+1)}{\Gamma(k-1/2)} \left(1 + \frac{\frac{1}{2} m_\alpha v^2}{\kappa E_{\kappa,\alpha}} \right)^{-(\kappa+1)} \quad (\alpha = e,\, i) \tag{2}$$

and dust grains obey the Maxwellian distribution. Here, n_α is the number density, $E_{\kappa,\alpha} = (1-3/2\kappa)\, T_\alpha$ is a characteristic energy, and Γ is the gamma function. The equilibrium charge neutrality condition satisfies $q_d n_d + q_i n_i - e n_e = 0$ where $q_d = Z_d e$ and $q_i = Z_i e$ are the dust particle and the ion charges, respectively. Throughout this paper, we shall assume hydrogen ions, i.e., $Z_i = +1$. Solving the Vlasov equation requires Poisson's equation

$$\nabla^2 \phi = -4\pi \sum_{\alpha=e,i,d} q_\alpha n_\alpha \, . \tag{3}$$

We shall limit our investigation to small perturbation from the equilibrium conditions by writing $f_\alpha = f_{\alpha 0} + f_{\alpha 1}$ and $n_\alpha = n_{\alpha 0} + n_{\alpha 1}$ where the subscripts 0 and 1 denote the equilibrium and perturbation, respectively. The perturbed quantities are assumed small comapered to the equilibrium quantities. No dust charge fluctuation is considered, i.e., $\partial q_d / \partial t = 0$ is assumed. If we perform Fourier transform for Eqs. (1) and (3) and combine them, we obtain $\varepsilon_\kappa(k,\omega)\phi = 0$ where the dielectric permittivity is given by

$$\varepsilon_\kappa(k,\omega) = 1 + \sum_{\alpha=i,e,d} \chi_\alpha \, . \tag{4}$$

The plasma dielectric susceptibility χ_α for the one-dimensional wave propagation in x-direction can be written in the form

$$\chi_{\alpha(=i,e)} = \frac{\omega_{p\alpha}^2}{k^2 n_{\alpha 0}} \int \frac{\partial f_{\kappa,\alpha 0}/\partial v_x}{\omega/k - v_x} d^3 v \quad \text{and} \quad \chi_d = \frac{\omega_{pd}^2}{k^2 n_{d0}} \int \frac{\partial f_{d0}/\partial v_x}{\omega/k - v_x} d^3 v \, . \tag{5}$$

Our concern is the investigation of dust acoustic modes and the nonthermal effects on the damping of waves. We consider plasma waves in the range where the phase velocity far exceeds the dust thermal velocity but is much less than those of electrons and ions, i.e., $kv_d \ll \omega \ll kv_i,\, kv_e$. Then, using the distribution given by Eq. (2) for plasmas and the Maxwellian distribtution for dusts, after some algebra, we can find the following dispersion relation for dust acoustic waves as the condition of the longitudinal dielectric permittivity be zero,

$$1 + \sum_{\alpha=e,i} \frac{1}{k^2 \lambda_\alpha^2} \left(\frac{2\kappa-1}{2\kappa-3} \right) \left[1 - \left(\frac{2\kappa+1}{2\kappa-3} \right) \frac{2\omega^2}{k^2 v_\alpha^2} + i \sqrt{\frac{2\pi}{2\kappa-3}} \frac{\kappa\Gamma(\kappa)}{\Gamma(\kappa+1/2)} \frac{\omega}{k v_\alpha} \right] - \frac{\omega_{pd}^2}{\omega^2} \left(1 + \frac{3k^2 v_d^2}{2\omega^2} \right) = 0 \, . \tag{6}$$

III. LANDAU DAMPING

To obtain the Landau damping rate of a dust acoustic wave, we let $\varepsilon_\kappa = \varepsilon_\kappa^{re} + i\varepsilon_\kappa^{im}$ where ε_κ^{re} and ε_κ^{im} are the real and imaginary parts of the frequency, respectively. We also let $\omega = \omega_r + i\gamma$. We assume that the imaginary parts are small compared to the real parts. Then, the dispersion relation of the dust acoustic wave is obtained as

$$\omega_r^2 = \frac{\omega_{pd}^2 k^2 \lambda_D^2}{k^2\lambda_D^2 + \left(\frac{2\kappa-1}{2\kappa-3}\right)} + \frac{3}{2}k^2 v_d^2 \tag{7}$$

where $\lambda_D = \left(\lambda_i^{-2} + \lambda_e^{-2}\right)^{-1/2}$ is the dust plasma Debye length. The Landau damping of the dust acoustic wave is obtained as

$$\gamma = -\frac{\sqrt{\pi}\mu_\kappa \kappa\Gamma(\kappa)\omega_{pd}^2 k^2\lambda_i^2\left(1+\sqrt{\frac{m_e T_i^3}{m_i T_e^3}}\frac{1}{\delta}\right)}{2\Gamma(\kappa+1/2)(\kappa-3/2)^{1/2}\left(\mu_k k^2\lambda_i^2 + 1 + \frac{T_i}{T_e}\frac{1}{\delta}\right)^2 k v_i} \qquad (<0) \tag{8}$$

where $\delta = n_{i0}/n_{e0}$ is introduced to replace the ion-electron density ratio and the κ-dependent factor μ_κ is defined as $\mu_k = (2\kappa-3)/(2\kappa-1)$. Again, as the spectral index κ goes to infinity, γ reduces to the well known Landau damping rate of Maxwellian plasmas. If we assume the dust is negatively charged and $\delta >> 1$ (high Z_d), the damping rate is simplified to

$$\gamma \approx -\sqrt{\frac{\pi}{8}}\frac{M_k \omega_{pd} k\lambda_i}{\left(\mu_k k^2 \lambda_i^2 + 1\right)^2}$$

where M_κ is defined as $0 < M_\kappa = \mu_\kappa \kappa\Gamma(\kappa)\Big/\left[\Gamma(\kappa+1/2)(\kappa-3/2)^{1/2}\right] \le 1$. For $\delta = 100$, $T_i/T_e = 0.1$, $m_i/m_d = 10^{-5}$, and $n_i/n_d = 10^5$, the normalized Landau damping rate, γ/ω_{pd}, as a function of normalized wavenumber, $k\lambda_i$, is depicted in Fig. 1. One can see that the Landau damping increases linearly with $k\lambda_i$ in the long wavelength limit, but it decreases as $(k\lambda_i)^{-3}$ in the short wavelength limit. One can observe the tendency that the damping is enhanced as the nonthermality of distribution increases. The maximum damping rate can be obtained by taking the derivative of γ with respect to the wave number. The result is simple in the case of $\delta >> 1$, $\gamma_{\max} \approx -0.203 M_k \omega_{pd}\big/\sqrt{\mu_k}$ where the κ-dependent factor has the range of $1 < M_k\big/\sqrt{\mu_k} < 1.33$. The maximum value of damping rate also moves to larger region of $k\lambda_i$ as the nonthermality increases.

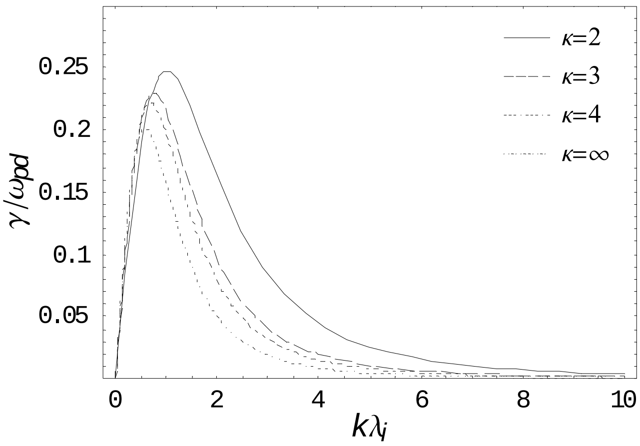

Fig. 1. Landau damping rate of dust acoustic waves in a dusty Lorentzian plasma. (δ= 100, Ti/Te = 0.1, $m_i/m_d = 10^{-5}$, and $n_i/n_d = 10^{5}$)

IV. CONCLUSIONS

We have kinetically investigated the dust acoustic wave modes propagating in a dusty plasma modeled by a Lorentzian (kappa) distribution for electrons and ions and by a Maxwellian distribution for dust grains. The Landau damping rate of dust acoustic mode is derived and investigated for various values of κ. The damping is found to be enhanced by increase of nonthermality, i.e., by decrease of κ. The maximum damping rate also is derived in terms of the spectral index κ and found to be approximately $0.2M_k\omega_{pd}\big/\sqrt{\mu_k}$.

REFERENCES

[1] A. Hasegawa, K. Mima, and M. Duong-van, Phys. Rev. Lett. 58, 2608 (1985).

[2] D. Summers and R. M. Thorne, Phys. Fluids B 3, 1835 (1991).

[3] L. Landau, J. Phys. 10, 25 (1946).

[4] R. M. Thorne and D. Summers, Phys. Fluids B 3, 2117 (1991).

[5] M.-J. Lee, J. Korean Phys. Soc. 46, 1258 (2005).

[6] P. K. Shukla and A. A. Mamun, *Introduction to Dusty Plasma Physics* (IOP Publishing, Bristol. 2002).