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Nonlinear electromagnetic waves in electron-positron plasmas

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1 Introduction

Neutron stars are thought to be surrounded by a relativistic plasma of electrons and positrons, which is penetrated by beams of high energy electrons and positrons traveling along the magnetic field. The pulsar emission mechanism may depend on coherent nonlinear waves arising in the plasma. Previous work has concentrated on nonlinear electrostatic modes [1], but here we analyse possible nonlinear electromagnetic modes arising in the plasma. Imbalanced electron and positron number densities occur in the rotating pulsar magnetosphere, due to the Goldreich-Julian charge, and we allow for unequal electron and positron number densities here. This leads to circularly polarized, rather than linearly polarized, modes propagating along the magnetic field. The linear dispersion relation is discussed, then nonlinear effects are analysed, and the modulational instability of nonlinear plane waves is discussed.

2 The Dispersion Equation

A cold pair plasma consisting of unequal numbers of electrons and positrons has the dielectric tensor components

$$K_{11} = K_{22} = 1 - \frac{\omega_{p+}^2 + \omega_{p-}^2}{\omega^2 - \Omega^2} = 1 - \frac{\omega_p^2}{\omega^2 - \Omega^2}, \tag{1}$$

$$K_{12} = -i\frac{(\omega_{p+}^2 - \omega_{p-}^2)\Omega}{\omega(\omega^2 - \Omega^2)} = -i\eta\frac{\omega_p^2\Omega}{\omega(\omega^2 - \Omega^2)}, \qquad K_{33} = 1 - \frac{\omega_p^2}{\omega^2}, \tag{2}$$

where $\eta = (n_+ - n_-)/(n_+ + n_-)$, Ω is the common cyclotron frequency, and ω_p is the plasma frequency based on the sum of the electron and positron number densities. There is assumed a neutralising background of stationary charge (eg heavy ions or charged dust).

For parallel propagation, the dispersion equation is

$$(\omega^2 - \Omega^2)(\omega^2 - c^2k_z^2) - \omega^2\omega_p^2 \pm \eta\omega_p^2\Omega\omega = 0. \tag{3}$$

In terms of the normalized frequency $f = \omega/\Omega$ and wavenumber $\kappa = ck_z/\Omega$, we have

$$(f^2 - 1)(f^2 - \kappa^2) - f^2h \pm \eta hf = 0, \tag{4}$$

where $h = \omega_p^2/\Omega^2$.

There are 2 solutions for f^2, corresponding to left and right hand polarized waves. This is in contrast to the case of equal electron and positron number densities, when the

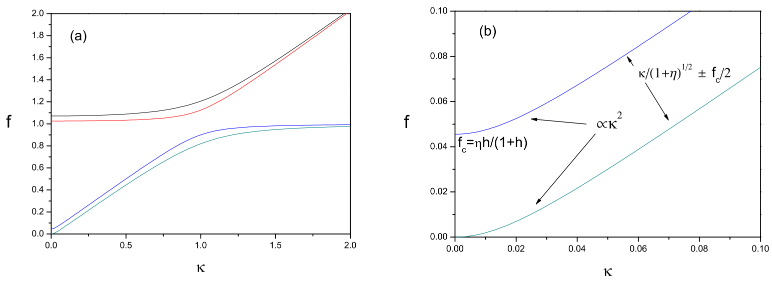

Figure 1: Linear dispersion relation for electromagnetic waves in a pair plasma. (a) Full frequency range, (b) small frequency.

wave modes are linearly polarized. Figure 1(a) shows the dispersion relation for $\eta = 0.5$ and $h = 0.1$. The effect of non-zero η is to split the 2 linearly polarized modes into 4 circularly polarized modes, and produce 3 cutoff frequencies (where $k_z \to 0$) in place of the single cutoff for $\eta = 0$. For example, in the vicinity of $f = 0$, for small h and η, the dispersion relation is shown in Figure 1(b). For $\eta = 0$, the wave has an Alfvén wave type dispersion. For $\eta \neq 0$ a cutoff occurs at $f = |\eta| h/(1+h)$, and the modes behave as shown.

3 Nonlinear waves

Weakly nonlinear waves behave generically determined by the local dispersion relation. The great variation in the dispersion relation shown by the waves in the pair plasma indicate a variety of nonlinear wave behaviour, according to the linear wave dispersion in a particular frequency range of interest. For example, in the range shown in Figure 1(b), dispersion due to $\eta \neq 0$ modifies the Alfvén wave dispersion relation. In a normal plasma, dispersion due to the Hall or cyclotron effect produces a derivative Nonlinear Schrödinger equation (DNLS) describing a weakly nonlinear wave [2]. Here however the dispersive effect is different: for example the initial parabolic dependence on k_z is indicative of a whistler-type wave.

A weakly nonlinear wave of long wavelength can generally be described by a nonlinear Schrödinger equation (NLS) [3]. Such an equation has been derived for a general multi-species plasma by Irie and Ohsawa (2001) [4], and the resulting dispersive and nonlinear coefficients have been calculated. We can adapt their results to our pair plasma situation.

The NLS equation is written, in terms of the stretched coordinates $\xi = \varepsilon(x - v_g t)$ and $\tau = \varepsilon^2 t$, and with $B_{\perp 1} = B_y \pm i B_z$, [4]

$$i\frac{\partial B_{\perp 1}}{\partial t} + \beta \frac{\partial^2 B_{\perp 1}}{\partial \xi^2} + \frac{\alpha}{B_0^2}(|B_{\perp 1}|^2 - |B_{\perp 10}|^2)B_{\perp 10} = 0, \quad (5)$$

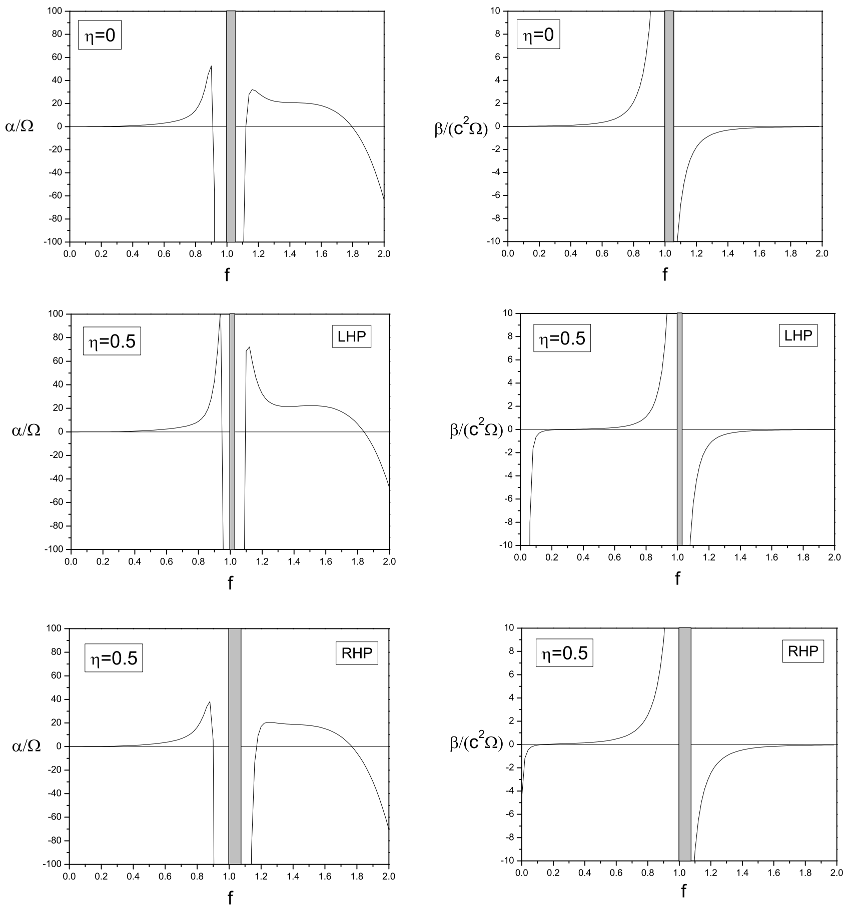

Figure 2: The nonlinear (α) and dispersive (β) coefficients of the NLS equation, as a function of frequency, for $\eta = 0$, and for $\eta = 0.5$ and left-hand polarized waves, and for $\eta = 0.5$ and right-hand polarized waves.

with $B_{\perp 10}$ the upstream value of $B_{\perp 1}$, and with the dispersive coefficient $\beta = (1/2)\partial^2\omega/\partial k^2$ and the nonlinear coefficient

$$\alpha = -\frac{k}{4v_g}\left[-\frac{2\omega^2}{k^2} + \frac{4\omega v_g}{k} - v_g^2 + \left(\frac{\omega^3}{k^2 v_g^2} - \frac{3\omega^2}{k v_g}\right)\frac{\partial v_g}{\partial k} + \frac{\omega^3}{k v_g^3}\left(\frac{\partial v_g}{\partial k}\right)^2 - \frac{\omega^3}{3k v_g^2}\frac{\partial^2 v_g}{\partial k^2} - \frac{c^2(k v_g + \omega)^2}{\sum_j \omega_{pj}^2}\right], \tag{6}$$

with v_g the group velocity.

Upon substituting the pair-plasma dispersion relation, we calculate the graphs of α and β against $f = \omega/\Omega$. Fig 2 shows the cases $\eta = 0$, ie the single linearly polarized wave case, $\eta = 0.5$ and the left hand circularly polarized wave, and $\eta = 0.5$ and the right hand circularly polarized wave. The frequency stop-band (the grey region) is narrowed for the $\eta > 0$ and LHP case, while it is broadened for the $\eta > 0$ and RHP case.

Modulational Instability A solution of the NLS equation is a plane wave with wavenumber K and frequency W, and $B_{\perp 1} = B_{\perp 10}$ [4]. However it can be modulationally unstable [5, 6], if $\alpha\beta > 0$ and $0 < K < (2\alpha|B_{\perp 10}/B_0|^2/\beta)^{1/2}$. The growth rate is $\Gamma = |\beta K|(2\alpha|B_{\perp 10}/B_0|^2/\beta - K^2)^{1/2}$.

Discussion The dispersive and nonlinear coefficients for the $\eta \neq 0$ cases differ both quantitatively and qualitatively from the $\eta = 0$ (linearly polarized) case. The scale size and speed of soliton solutions are determined by α and β. We see from Fig 2 that for $\eta = 0$ a modulational instability of the plane wave solution occurs ($\alpha\beta > 0$) for $f \lesssim 0.95$, but that the wave is stable for f greater than the upper cutoff, until f approaches 2. When there is an imbalance in the electron and positron number densities ($\eta > 0$), the instability criterion is largely unchanged near $f = 1$, but at low frequencies both the LHP and RHP modes become stable (where β becomes negative. The left-hand polarized wave has a cutoff frequency at $k = 0$, and is stable for a small range of frequencies above the cutoff. The right-hand polarized wave has no cutoff at $k = 0$, and remains stable for a small range of frequencies above zero. If the electron number density is greater than the positron number density ($\eta < 0$), the polarizations of the modes discussed above become reversed.

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