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A current driven instability in parallel, relativistic shocks

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Abstract

Recently, Bell [2] has reanalysed the problem of wave excitation by cosmic rays propagating in the pre-cursor region of a supernova remnant shock front. He pointed out a strong, nonresonant, current-driven instability previously overlooked in kinetic treatments [1, 5] and suggested that it can substantially amplify the ambient magnetic field. Magnetic field amplification is also an important issue regarding the formation and structure of relativistic shock fronts, particularly in relation to models of gamma-ray bursts [4]. We have generalised the linear analysis to apply to this case, assuming a relativistic background plasma and a monoenergetic, unidirectional incoming proton beam. We find essentially the same nonresonant instability noticed by Bell, and show that also under GRB conditions, it grows much faster than the resonant waves. We quantify the extent to which thermal effects in the background plasma limit the maximum growth rate.

Introduction

The acceleration of cosmic rays at the outer shock front of a supernova remnant is thought to proceed via the diffusive first-order Fermi mechanism [3]. In the shock precursor waves can grow as a result of interacting with the energetic particles at their cyclotron resonance. In turn, the waves provide the pitch-angle scattering essential for the acceleration process [5].

Using a hybrid MHD/kinetic approach Bell [2] has identified a non-resonant mode that is strongly driven by the current induced in the plasma by the streaming cosmic rays. Simple analytic estimates and MHD simulations both suggest that the nonlinear evolution of this instability leads to very strong amplification of the ambient magnetic field. In turn, this may result in a cosmic-ray acceleration rate that is much more rapid than previously thought [2].

Gamma-ray bursts drive highly relativistic outflows with $\Gamma \sim 100$ or more. On interacting with the surrounding medium, a shock front forms, but the mechanism by which this happens is controversial [8]. Observations of GRB afterglows suggest that the ambient magnetic field must be amplified substantially at the shock front. The relativistic Weibel instability has been investigated in this connection [6], but appears to saturate at a relatively low amplitude [4, 9].

In this paper we analyse the growth rate of the relativistic analogue of the nonresonant mode discovered by Bell[2]. We find it grows much faster than the resonant mode, but is quite sensitive to damping by thermal effects once the background plasma is heated.

Dispersion relation

The linear dispersion relation for the propagation of transverse waves parallel to the magnetic field in a plasma made up of components labelled by j is:

$$n_{\parallel}^2 - 1 - \sum_j \chi_j(k,\omega) \quad = \quad 0 \tag{1}$$

where $n_{\parallel} = ck/\omega$ is the refractive index and $\chi_j(k,\omega)$ is the susceptibility of the j'th component for wavenumber k and frequency ω (> 0). The three components in the case we consider are: (1) Protons that make up the background thermal distribution, with number density $n_{\rm p}$ and a thermal distribution with (dimensionless) temperature $\Theta_{\rm p} = k_{\rm B}T_{\rm p}/m_{\rm p}c^2$. (2) Electrons that also have a thermal distribution with lab. frame density $n_{\rm e}$ and temperature $\Theta_{\rm e}$. (3) Protons of the upstream medium that form a monoenergetic, unidirectional incoming beam along the magnetic field. Their lab. frame density is $n_{\rm b}$ and their drift Lorenz factor $\Gamma_{\rm b}$.

Following [1], we impose overall charge neutrality and zero net current:

$$\sum_j \omega_{{\rm p}j}^2/\omega_{{\rm c}j} = 0 \quad \text{and} \quad \sum_j \beta_j \omega_{{\rm p}j}^2/\omega_{{\rm c}j} = 0 \tag{2}$$

where $\omega_{{\rm p}j}$, denotes the plasma frequency of the j'th component, $c\beta_j$ its drift speed, and $\omega_{{\rm c}j}$, its cyclotron frequency. In the present case $m_{\rm b} = m_{\rm p}$ is the proton mass and $q_{\rm b} = q_{\rm p} = -q_{\rm e} = e$ is the electronic charge. For parallel propagation, the susceptibility χ_j is given by [10]:

$$\omega^2 \chi_j \quad = \quad \omega_{{\rm p},j}^2 \int \frac{{\rm d}^3 u}{\gamma} f_j(\vec{u}) \frac{-\omega\gamma + cku_{\parallel}}{D\left(u_{\parallel}\right)} - \frac{u_{\perp}^2}{2} \frac{(c^2k^2 - \omega^2)}{D^2\left(u_{\parallel}\right)} \tag{3}$$

Here,$f_j(\vec{u})$ is the distribution function of particles of four velocity $c(\gamma, \gamma\vec{u})$, normalised such that $\int {\rm d}^3 u f_j = 1$, the components of $\vec{u}$ parallel and perpendicular to the magnetic field direction are $u_{\parallel}$ and $u_{\perp}$, and the resonant denominator is $D\left(u_{\parallel}\right) = \varepsilon\omega_{{\rm c}j}\left[1 + Z\left(u_{\parallel}\right)\right]$ where

$Z\left(u_{\parallel}\right) = (\omega\gamma - cku_{\parallel})/\varepsilon\omega_{{\rm c}j}$ and $\gamma = \sqrt{1 + u_{\perp}^2 + u_{\parallel}^2}$. The waves are circularly polarised, with $\varepsilon = +1(-1)$ corresponding to left(right)-handed waves, for $k > 0$.

The waves we consider do not resonate with the electrons and background protons. Furthermore, these components are "magnetized", in the sense that for all relevant values of $u_{\parallel}$, the resonant denominator $D\left(u_{\parallel}\right)$ can be expanded for small parameter $Z\left(u_{\parallel}\right)$. The susceptibilities χ_j describe the currents induced by the wave field in each plasma component. In the plasma we consider, the susceptibility of the beam is

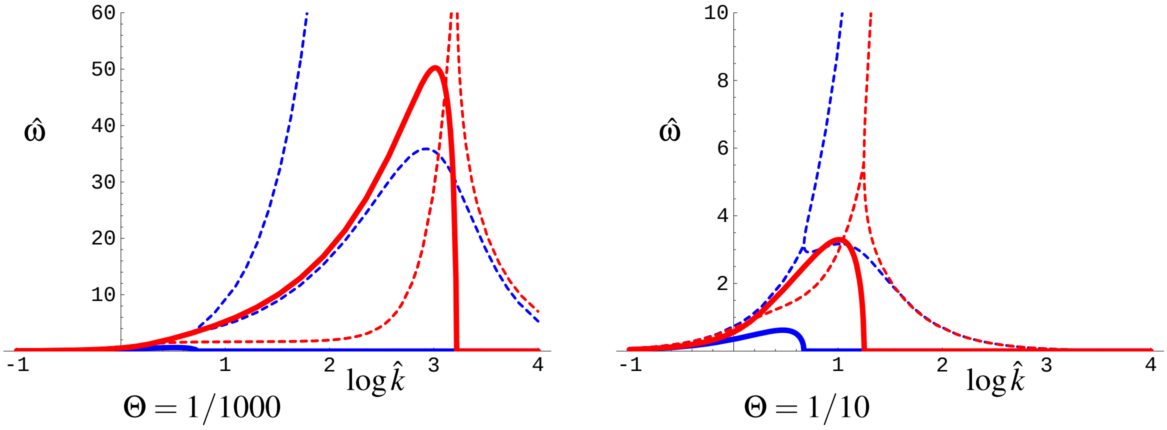
$$\omega^2 \chi_{\rm b}(k,\omega) \quad = \quad \frac{\omega_{\rm pb}^2\left(c\beta_{\rm b}k - \omega\right)}{\varepsilon\omega_{\rm cb} - \Gamma_{\rm b}\left(c\beta_{\rm b}k - \omega\right)} \tag{4}$$

If $\Gamma_b c \beta_b |k| \ll |\omega_{cb}|$ the denominator can be expanded and the plasma is fully compensated. However, waves with short wavelength, such that $|\omega_{cp,e}|/\Theta_{p,e} \gg c|k| \gg |\omega_{cb}|/\Gamma_b$ are unable to induce a compensating current in the beam particles. Inserting all terms into Eq. (1), using Eq. (2) and $\langle u_\parallel/\gamma\rangle_j = \beta_j$, where $\langle \ldots \rangle_j = \int d^3u \ldots f_j(\vec{u})$, the overall susceptibility can then be written:

$$\omega^2 \chi \quad \approx \quad \frac{\omega_{pb}'^{\,2}\omega'}{\varepsilon\omega_c} - \frac{\omega_{pb}'^{\,2}\omega'}{\varepsilon\omega_c + \omega'} + \frac{c^2\omega^2}{v_A^2} + \frac{\omega_p^2\omega}{\varepsilon\omega_c^3}\left(c^2k^2 - \omega^2\right)\left\langle u_\perp^2 \right\rangle_p \tag{5}$$

where we have neglected the electron response, except for its contribution to the overall current and to v_A, the non-relativistic Alfvén speed: $v_A = c\left(\sum_{p,e} \frac{\omega_{pj}^2}{\omega_{cj}^2}\langle\gamma\rangle_j\right)^{-1/2}$ Eq. (5) describes the waves in a reference frame in which the background protons have zero drift and are assumed to have an isotropic distribution. All primed quantities are in the beam's reference frame.

Fig.1., $v_A = 2\times 10^{-5}c$, $\Gamma = 10$, $n_b/n_p = 1/3$, $\varepsilon = -1$, $\varepsilon = +1$, *Im*$\hat{\omega}$ solid, *Re*$\hat{\omega}$ dashed



Wave modes

For a cold background plasma, and for low frequency modes such that $\omega' \approx -\Gamma_b\beta_b ck$ and $|\omega'| \gg |\omega_{cb}|$, one recovers a simple form analogous to that discussed by Bell [2]:

$$\omega^2 \quad = \quad \frac{v_A^2}{c^2 + v_A^2}\left(c^2k^2 + \frac{\beta_b\omega_{pb}^2 ck}{\varepsilon\omega_c}\right) \tag{6}$$

which gives a purely growing mode for the right-handed polarisation $\varepsilon = -1$, provided the beam-induced driving current is sufficiently strong. The mode reaches a maximum growth rate

$$\mathrm{Im}\,\omega = \frac{1}{2}\frac{n_b}{n_p}\beta_b\omega_{pp} \quad \text{at} \quad k_{max} = \frac{1}{2}\frac{n_b}{n_p}\beta_b\frac{\omega_{pp}}{v_A} \tag{7}$$

for $v_A \ll c$, which is independent of the magnetic field strength.

The thermal effects (arising from the term containing $\langle u_\perp^2\rangle$) are easily analysed in the case of weak magnetic field $v_A \ll c$, $\omega^2 \ll c^2k^2$ and $\omega' \approx -\Gamma_b\beta_b ck$. The dispersion relation is then

$$\hat{\omega}^2 \quad = \quad \frac{v_A^2}{c^2}\hat{k}\left(\hat{k} + \frac{\Gamma_b\beta_b^2\omega_{pb}^2}{\varepsilon\omega_c^2} - \hat{k}\frac{\left\langle u_\perp^2\right\rangle\omega_{pp}^2}{\varepsilon\Gamma_b\beta_b\omega_c^2}\hat{\omega}\right) \tag{8}$$

with dimensionless units $\hat{k} = \Gamma_b \beta_b ck/\omega_c$ and $\hat{\omega} = \Gamma_b \beta_b \omega/\omega_c$. The $\varepsilon = -1$ mode reaches a maximum growth rate,

$$\mathrm{Im}\omega = \frac{\sqrt{3}}{2}\left(\frac{n_b}{n_p}\right)^{\frac{2}{3}}\left(\frac{v_A}{c}\right)^{\frac{2}{3}}\left(\frac{\omega_{pp}}{\omega_c}\right)^{\frac{2}{3}}\left(\frac{\beta_b^2}{\langle u_\perp^2\rangle}\right)^{\frac{1}{3}}\omega_c \tag{9}$$

The complete solution to Eq. (1) for wavenumbers $|\hat{k}| \ll \Gamma_b \beta_b/\Theta$, far from resonance with the background protons, is plotted for low frequency modes in Fig.1.

Discussion

From Fig.1 it can be seen that the $\varepsilon = 1$ mode resonates with the beam particles when k is close to their inverse gyro-radius, i.e., $\hat{k} \approx 1$. However, for low to moderate values of the background temperature, its growth rate is very much lower than that of the non-resonant, current-driven mode. [2] and [7] have discussed the non-linear evolution of the nonrelativistic case. Saturation can be expected when the currents associated with the growing waves become comparable to the current induced by the beam. In terms of the amplitude B_w of the magnetic field of the perturbations, this can be written

$$\left|\vec{k} \wedge \vec{B}_w\right| \approx q_b n_b c\beta \quad \text{or} \quad \frac{B_w^2}{8\pi} \approx \frac{1}{2} n_b \Gamma m_p c^2 \tag{10}$$

Compared to the upstream energy density, this field is stronger than that generated by the Weibel instability in the case of a relativistic shock in a pair plasma [8]. The Weibel instability is not thought to be effective in mediating shocks in the electron/proton plasma considered here [4].

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