NUMERICAL INVESTIGATION OF DENSE SHEET Z-PINCHES

A. Muravich, K. Takasugi and T. Miyamoto

1-8, Atomic Energy Research Institute, Nihon University
Kanda-Surugadai, Chiyoda-ku, Tokyo 101, Japan

Abstract

Recently, dense z-pinches with noncircular cross section are proposed in order to improve stability of conventional fiber z-pinches with a circular cross section [1,2]. The noncircular z-pinches, for example sheet Z-pinches, can exist transiently or under influence of magnetic field of the return current or/and external current circuits. Curvature of central part of plasma can be zero for the sheet produced between two flat return current conductors and negative for some kind of return current chamber.

Introduction

Z-pinches are well known as a method of producing hot, dense plasma. Since cross-section is circular in the equilibrium state of most Z-pinches (for example, of ordinary fiber pinch), the plasma boundary (and accordingly magnetic field lines) has essentially positive curvature. Such a magnetic field configuration causes fast growth of instabilities of interchange mode. The positive curvature can be avoided in most part of Z-pinch, if it’s cross-section is sheet-like ([1], [2]). Such configurations are of considerable interest from the point of view of stability properties in comparison with traditional circular column Z-pinch. Similar concept was studied in the inverse pinch that is a kind of low-density Z-pinch [3]. In the present paper we study noncircular plasma equilibrium of dense Z-pinches that can be produced under the influence of the magnetic field of return current conductors.

Results of numerical modeling

In the present calculations we assume that the plasma has homogeneous temperature and spread infinitely in Z-direction, so there is only Z-component of plasma current density $i_z$. Shape of the plasma and return current conductors is symmetric in XZ and YZ planes. The plasma boundary shape $y = f(x)$ is found as simultaneously satisfying the pressure equilibrium equation:

$$\nabla p = \tilde{i} \times (\tilde{B}_p + \tilde{B}_r)$$  \hspace{1cm} (1)

and full energy balance equation:

$$\int_S P_{rad} dS = \int_S \frac{i^2}{\sigma} dS$$  \hspace{1cm} (2)

where $p$ is plasma pressure, $B_p$ is the plasma magnetic field, $B_r$ is the magnetic field of return current, $P_{rad}$ is radiated power density and $\sigma$ is plasma conductivity.
If one suppose bremsstrahlung radiation losses for $P_{rad}$ and Spitzer expression for conductivity $\sigma$, the plasma shape is mainly determined by only the current density $i$ and shapes of the return current conductors $r_1(x)$ and $r_2(x)$. Other plasma parameters affect only through Coulomb logarithm. Replacing $p$ and $i$ by values $p^* = pZ^2/(Z+1)^2$ and $i^* = iZ/(Z+1)$ equations (1-2) become non-depending on ion charge $Z$, so $Z$ does not affects on shape. It is convenient to introduce the parameter $r_{PB} = (I_{PB}/(\pi i))^1/2$, where $I_{PB}$ is Pease-Braginskii current of ordinary Z-pinich with circular cross-section [4]. Thus $r_{PB}$ has univalent correspondence with $i$ and means radius of equilibrium isolated pinch.

We distinguish two types of return current conductors: configurations with open ends and closed ends (chambers). Different shapes of conductors were tested for the both types. Here we consider only two of them, because the others give similar results. A pair of flat parallel conductors ($r_1(x) = d = const, r_2(x) = -d, -l \leq x \leq l$) is an example of “open ended” type, and configurations with hyperbolically curved central part and elliptic ends is an example of “chambers”.

In the case of parallel flat return current conductors the plasma boundary shape is critically depends only on two non-dimensional parameters: $r_{PB}/d$ and $l/d$. While $r_{PB}/d$ is much smaller than unity, the influence of the magnetic field of return current is relatively small, and pinch cross-section hardly differs from circular. When $r_{PB}/d$ increase up to order of unity, the width of plasma sheet $b$ stops growth because of the influence of the return current. In turn length of sheet $a$ increase dramatically (up to order of $l$), so the pinch cross-section becomes far from circular (see Fig. 1a-1d, where plasma density profiles are shown for the increasing parameter $r_{PB}/d$ at $l/d = 2$). When $l/d >> 1$, solutions exists only if $a < a_{max}$, $a_{max} \approx l - 1.5d \pm 0.5d$. The ambiguity in numerical determination of $a_{max}$ is caused by

![Plasma density profiles](image)

**Fig. 1.** Plasma density profiles for $l/d = 2$ at the different values of $r_{PB}/d$: 
(a) $r_{PB}/d = 0.377$, (b) $r_{PB}/d = 0.539$, (c) $r_{PB}/d = 0.702$, (d) $r_{PB}/d = 0.890$
existence of critical maximum value of \( r_{PB}/d \). At this value the derivatives of \( a \), total current \( I \) and some other plasma parameters become infinity, as shown in the Fig. 2. So, it is difficult to determine values \( a_{\text{max}} = a(i_{\text{cr}}) \) and \( I_{\text{max}} = I(i_{\text{cr}}) \) numerically in this critical point with better accuracy. If \( a \) is large enough (\( a \geq 3d \)), \( b \) does not depend on \( a \) and achieve a value about \( 0.75d \). Curvature radius of the end part of such an elongated sheet is also almost a constant, \( r_e = 0.5d \). The most elongated configuration for \( l/d = 5 \) is shown in Fig. 3.

![Fig. 2. Normalized total pinch current \( I/I_{PB} \) versus parameter \( r_{PB}/d \) for the different values of \( l/d \).](image)

![Fig. 3. Plasma density profile for \( l/d = 5 \) at the critical value of \( r_{PB}/d = 1.038 \).](image)

In order to obtain sheet Z-pinches with negative curvature of central part firstly we proposed a pair of hyperbolically curved conductors (\( r_1(x) = d(l + (x/h)^2)^{1/2}, \ r_2(x) = -r_1(x), -l \leq x \leq l \)). It appears that increasing of \( h \) results only in decreasing of \( a_{\text{max}} \), but plasma boundary remains flat, so the result has no essential difference with a pair of plane conductors. We found that elongated sheet Z-pinches with some negative curvature of central part can be produced in the chambers with hyperbolic central part and elliptic ends (Fig. 4).

![Fig. 4. Density profile of elongated plasma sheet with negative curvature of boundary in the central part that can be produced in special chamber.](image)
While the central part of sheet pinch is neutral (in the case of zero plasma boundary curvature) or stable (in the case of negative plasma boundary curvature) against the interchange mode instabilities, it is objected to the tearing mode instability. But growth rate of tearing mode is essentially smaller than that of interchange mode. Although the end parts of the sheet pinch are strongly unstable for interchange mode in the same way, as a fiber pinch of the radius $r_c$, it requires significant time for these perturbations to reach central part of sheet. Thus, the central part of sheet Z-pinch will be much more stable than fiber pinch.

References