SIMULATION STUDY OF NONLINEAR PLASMA MASER

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Plasma maser is a new nonlinear process in plasma turbulence which coexists with the quasilinear process between electrons and the resonant mode. Since the prediction of the process [1], it attracted much attention because multi-modes turbulence which contains enhanced high frequency fluctuations in addition to low frequency turbulence are quite often observed in laboratory and astrophysical plasmas [2]. Here, we report the first study on the comparison between theory and numerical simulation of the nonlinear plasma maser driven by electron beam instability.

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\begin{align*}
\text{(a)} \quad & E_x^2, \\
\text{(b)} \quad & E_y^2, \\
\text{(c)} \quad & E_z^2
\end{align*}
\]

**Fig. 1.** Time history of electric field energies (a) \(E_x^2\), (b) \(E_y^2\), and (c) \(E_z^2\). Solid lines show a case with \(\omega_{pe}/\Omega_c = 10\), \(v_b = 0.7c\), and \(n_b/n_o = 1.0/30.0\). Dotted lines correspond to a case without the electron beam.

According to the recent numerical simulations using TRISTAN code [3], R-mode electromagnetic waves are effectively generated from Langmuir mode turbulence driven by electron beam \((T/T_b = 2)\), where \(T\) is the background temperature and \(T_b\) is the beam temperature. It is well known that an electron beam along a magnetic field in the z-direction becomes unstable to excite Langmuir waves. Solid lines in Figs. 1(a) and 1(c) show the time history of electric field energies \(E_x^2\) and \(E_z^2\) for a case of \(\omega_{pe}/\Omega_c = 10\) with the beam of \(v_o = 0.7c\), \(n_b/n_o = 1/30\), while the dotted lines show the time history for a case without the electron beam, here \(n_b\) and \(n_o\) are number densities defined for beam and background electrons, \(v_0\) and \(\Omega_e\) are the electron beam velocity, and the electron cyclotron frequency, respectively. As seen in Fig. 1(a) and Fig. 1(c),

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both components of the electric fields are strongly excited due to the beam instability. This means that the Langmuir waves are obliquely propagating to the external magnetic field. The solid line in Fig. 1(b) shows the time history of the electric field energy $E_y^2$, while the dotted line shows the case without the beam. As seen in Fig. 1(b), the electric field $E_y$ grows after the electric field energy associated with the beam instability becomes large.

![Graph](image)

**Fig. 2.** (a) The dispersion relation ($\omega$ vs $k$) for $E_y$, cutting the low frequency region of $\omega < 1.63\omega_{pe}$. (b) Time history obtained by the inverse-Fourier transformation from (a).

To investigate the behavior of this electric field $E_y$, we perform two-dimensional Fourier transformations in parallel to a magnetic field. The results show there appear two electromagnetic wave branches. The first one is broad spectrum with maximum amplitude of the frequency around $2\omega_{pe}$ and $\frac{cK}{\omega_{pe}} \approx 1.7$. The second branch observed is seen around $\omega \approx \omega_{pe}$ and $k \approx K$. This second excited branch can be explained by the emission mechanism through Cherenkov emission by interaction with the electron beam [4]. But the first branch could not be explained by the usual emission mechanisms because $\frac{\Omega}{K} \approx 1.2c$ and $\frac{\Omega - \omega_{pe}}{K} \approx 1.1c$, here $c$ is light velocity. We cut the low frequency region of $\omega < 1.63\omega_{pe}$, as seen in Fig. 2(a), then we perform the inverse-Fourier transformation to obtain the time histories for the excited high-frequency waves. As seen in Fig. 2(b), the high-frequency electromagnetic waves with $\Omega/\omega_{pe} \approx 2.0$ along a magnetic field can be excited only after about $35\omega_{pe}t$. We may conclude from the above observations that electromagnetic waves propagating parallel to a magnetic field can be excited by the trigger of the Langmuir waves due to the electron beam instability.
Fig. 3. (a) Growth rates vs beam density for cases with $E_y$ parallel to a magnetic field:
(1) $n_b/n_o = 1/50$, (2) $n_b/n_o = 1/40$, (3) $n_b/n_o = 1/35$, (4) $n_b/n_o = 1/30$, (5) $n_b/n_o = 1/25$, (6) $n_b/n_o = 1/20$.
(b) Time histries of $E_y^2$ for different magnetic field strength: (1) no magnetic field, (2) $\omega_{pe}/\omega_e = 10$, (3) $\omega_{pe}/\omega_e = 3$ with $n_b/n_o = 1/30$ and $v_b = 0.7c$.

To compare this simulation results with a theory, we study the growth rate of the instability with different parameters. Fig. 3(a) shows the growth rate measured from the simulations by changing the electron beam density: (1) $n_b/n_o = 1/50$, (2) $1/40$, (3) $1/35$, (4) $1/30$, (5) $1/25$, (6) $1/20$: parallel to a magnetic field. As seen in Fig. 3(a) we find the strong growth if the beam density increases.

We further investigate the emission process of electromagnetic waves by Langmuir waves. The most probable candidate for this emission process is the plasma maser mechanism. The plasma maser theory predicts that there appears no plasma maser mechanism when external magnetic field is absent [5]. Fig. 3(b) shows the time history of $E_y^2$: (1) noise case with electron beam with $n_b/n_o = 1/30$, $v_o = 0.7c$ and no magnetic field, (2) weak magnetic field with $\omega_{pe}/\Omega_e = 10$, $n_b/n_o = 1/30$ and $v_o = 0.7c$. (3) strong magnetic field with $\omega_{pe}/\Omega_e = 3$, $n_b/n_o = 1/30$ and $v_o = 0.7c$. As seen in Fig. 3(b), there is no enhancement of the electric field $E_y$ for unmagnetized plasma case (1), compared with magnetized plasma cases (2) and (3).

Thus, the plasma maser originates from polarization term for magnetized plasmas. After a little algebra, we find that the most dominant imaginary part contribution even with respect to $\Sigma_k$ comes from polarization term and the growth rate is given by

$$\gamma/\omega_{pe} \approx -\delta \frac{\sqrt{\pi} T}{2 T_b} \sum_k \frac{K}{|k|} \frac{|E_l(k, \omega)|^2}{4\pi n_0 T} \frac{1}{\Omega - \omega - \frac{\omega_{pe}^5}{(\Omega - \Omega_e)^2(\Omega - \omega - \Omega_e)^2}}\frac{k v_0 - \omega}{k v_e} \exp[-\left(\frac{k v_0 - \omega}{k v_e}\right)^2],$$

where $v_e$ is the thermal velocity of beam electrons. It should be mentioned that the polarization contribution vanishes for unmagnetized plasma [5].

Here, we compare the simulation result [case (4) of Fig. 3(a)] with Eq. (1). Inserting
simulation parameters $\delta = n_b/n_o = 1/30$, $T/T_b = 2$, $\omega \simeq \omega_{pe}$, $\omega_{pe}/\Omega_e = 10$, $\Omega \simeq 2.05\omega_{pe}$.

$K \sim k$, $|E_l(k, \omega)|^2/4\pi n_0 T = 1.57 \times 10^{-2}$ into Eq.(1), we find $\gamma/\omega_{pe} \simeq 3.83 \times 10^{-2}$, here we put $(kv_0 - \omega)/kv_e = 1$. The obtained value is smaller by factor three than that of simulation result.

The difference may be attributed to the difference in situations between theory and simulations. In simulations, Langmuir waves are propagating obliquely to the ambient magnetic field, whereas in the theoretical model they are propagating along the magnetic field.

The plasma maser effect is one of the lowest order mode coupling process in weak plasma turbulence. In contrast to the resonance broadening effect [6] which comes from the propagator correction, plasma maser originates from the vertex correction [7]. All of the standard textbooks on plasma physics assume that the lowest order mode coupling process in plasma is composed of two parts, i.e., the three wave resonance and nonlinear scattering. The third one, plasma maser, was predicted sometime ago for magnetized plasmas [1], and is relevant for many laboratory and space plasmas. However, the check by numerical simulation had been lacking. As is shown by Fig. 3(b), the presence of an external magnetic field is crucial for the amplification of electromagnetic radiation.

The fundamental significance of this paper lies in the fact that it reports for the first time the agreement between theoretical predictions and numerical simulation of plasma maser driven by electron beam instability. The numerical simulation verifies many important physical aspects of the process. Thus our paper confirms the fundamental physics of the plasma maser and clarified the existence of a new mode coupling process and opens a new direction in plasma physics.

References