PHASE TRANSITIONS IN DUSTY PLASMA CRYSTALS: MOLECULAR DYNAMICS SIMULATION AND THEORY

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Dusty plasma crystal is modeled as a Yukawa system confined by a one-dimensional potential and low temperature structural transitions and melting are analyzed by simulation and theory. Relations of confining potential to experimental parameters are discussed and structures of Yukawa mixtures are obtained. Experiments with intentionally doped heavy impurities are proposed.

1. Dusty plasma as confined Yukawa system

Physics of dusty plasmas, assemblies of macroscopic particles immersed in plasmas, is one of important practical problems in plasma processes of semiconductor manufacturing and is also an interesting subject of basic statistical physics [1]. In our recent work [2, 3, 4, 5, 6, 7], we have regarded dust particles as interacting via the isotropic repulsive Yukawa potential

$$\frac{q^2}{r}\exp(-\kappa r), \tag{1}$$

where $-q$ is the (negative) charge on a dust particle, and trapped in a one-dimensional potential well of the form

$$v_{ext}(z) = \frac{1}{2}k(z-z_0)^2. \tag{2}$$

1.1. Low temperature structures

Defining the mean distance between dust particles by $a = (\pi N_S)^{-1/2}$ from the surface number density N_S, we describe our system at low temperatures by parameters

$$\xi = \kappa a \quad \text{and} \quad \eta = \frac{\pi^{1/2}}{2}\frac{(1/2)ka^2}{q^2/a}, \tag{3}$$

the latter being the strength of the confining potential relative to mutual repulsion. Distribution of particles in this system is determined by a competition between these two forces. At low temperatures, particles are organized into layers perpendicular to the confining force and the number of layers is expressed as a phase diagram Fig.1 in the (ξ, η)-plane [3]. In each layer, we have triangular or square lattice according to mutual distance. When $\eta \gg 1$, we have purely two-dimensional Yukawa system. With the decrease of η, the number of layers increases discretely. We have shown that this phase diagram is theoretically reproduced by a model which takes the correlation energy in layers into account [3].

1.2. Melting

Inter-layer vs. intra-layer orders

At low temperatures, the Yukawa system in the confining potential (2) has two kinds of orders, two-dimensional intra-layer order (in xy-plane) and one-dimensional inter-layer order (in z-direction).

At finite temperatures, these orders are destroyed by fluctuations. We introduce the parameter

$$\Gamma^* = q^2 \exp(-\xi)/k_B T a^*, \qquad (4)$$

where $a^* = (\pi N_S^*)^{-1/2}$ and N_S^* is the surface density of one layer. By molecular dynamics simulations, we have observed that the intra-layer order disappears at lower temperatures than the inter-layer order and there exists a domain of temperature where the system is composed of layers of two-dimensional liquids [7].

Two-dimensional melting

Let us now look into the melting of two-dimensional Yukawa lattice. We have the long-range orientational order and the quasi-long-range translational order: The long-range translational order cannot be maintained due to enhanced effect of long-wavelength fluctuations. The KTHNY theory predicts two-dimensional melting via two second order transitions: first to the hexatic phase with quasi-long-range orientational order and short-range translational order, and then to liquid where both orders are short-ranged [9].

We have analyzed the behavior of the orientational correlation function and the internal energy. An example of the latter is shown in Fig. 2. We see that the melting transition includes a first order transition. In Fig. 3, shown is the phase diagram of the two-dimensional Yukawa system which is realized for sufficiently large values of η. When η is not sufficiently large,we have melting of two-dimensional lattices under the influence of adjacent layers. The resultant phase diagram is given in Fig.4. It is clear that the effect of nearby layers stabilizes the two-dimensional lattice.

2. Experimental parameters and confinement

Let us consider the case where the dusty plasma is formed above a wide horizontal plane electrode as shown in Fig. 5 and discuss what determines the parameter η.

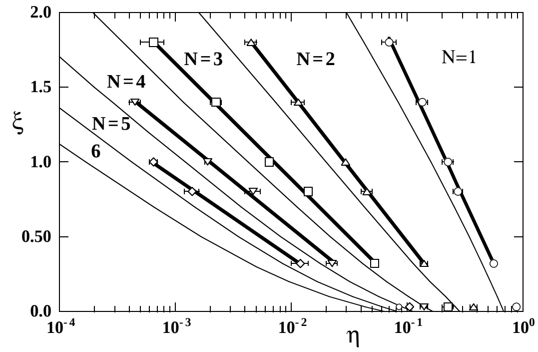

Figure 1. Number of layers. Simulation (thick lines with symbols) and theory (thin lines).

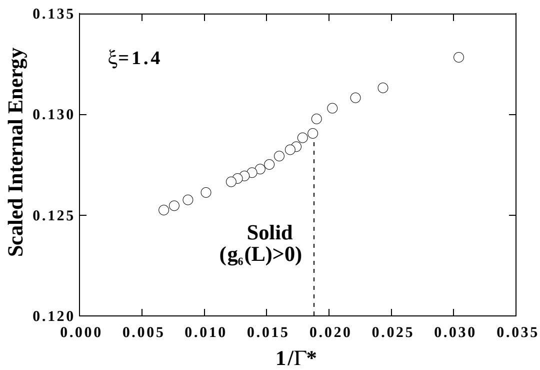

Figure 2. Internal energy of two-dimensional Yukawa system vs. $1/\Gamma^*$ for $\xi = 1.4$.

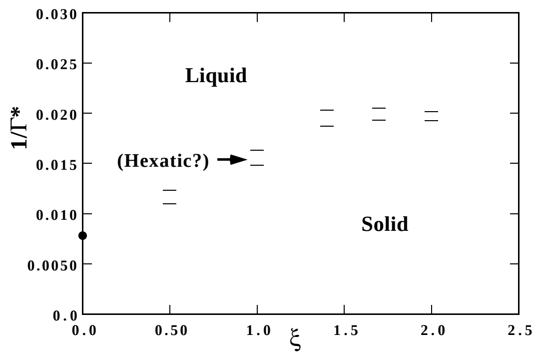

Figure 3. Phase diagram of two-dimensional Yukawa system.

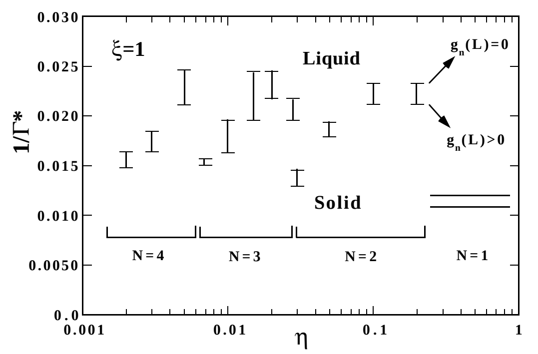

Figure 4. Phase diagram of confined Yukawa system.

When we assume that the density of charges in the sheath (except for those of dust particles) is given by en_{sh}, e being the elementary charge, and is nearly constant in the domain of dust crystals, the sum of gravitational and the electrostatic potentials for a dust particle of mass m and charge $-q$ is written as

$$\phi_{ext}(z < 0) \tag{5}$$

$$z_0 = -\frac{mg}{4\pi q e n_{sh}} = -\frac{g}{4\pi e n_{sh}}\frac{m}{q} < 0. \tag{6}$$

The confining potential is thus given by (2)with$k = 4\pi q e n_{sh}$andη is calculated as

$$\eta = \left(\frac{e}{q}\right)\left(\frac{n_{sh}}{N_S^{3/2}}\right). \tag{7}$$

Note that η is the ratio of the charge densities due to the space charge in the sheath $en_{sh}$and the charge due to dust particles, the latter being extended three-dimensionally.

3. Crystal of mixtures

When we have only one species of dust particles, the structure at low temperatures is completely determined by parameters ξ and η[3]. In the case where there are two or more species of dusts, we have to also take the dependence of z_0 on species into account. We thus define a parameter δ by

$$\delta = -\frac{z_0}{a} = \frac{1}{2}\frac{mga}{2\pi q e n_{sh} a^2} = \frac{g}{4\pi e n_{sh} a}\frac{m}{q} \tag{8}$$

to represent the separation in z-direction: The equilibrium position $z_0$which is proportional to the charge-to-mass ratio $q/m$is compared with the mean distance a. Examples of structures of dusty plasma mixture composed of species 1 and 2 with

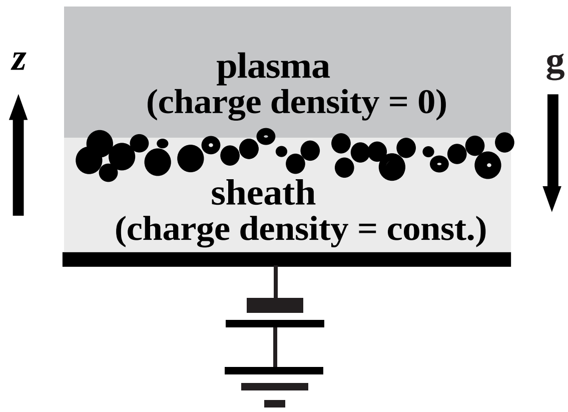

Figure 5. Plasma, sheath, and dusts near plane electrode.

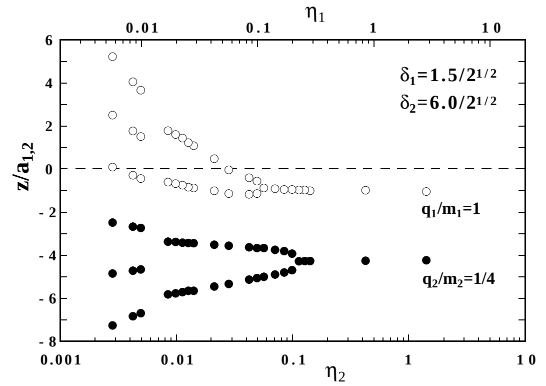

Figure 6. Crystal structure of dusty plasma mixture.

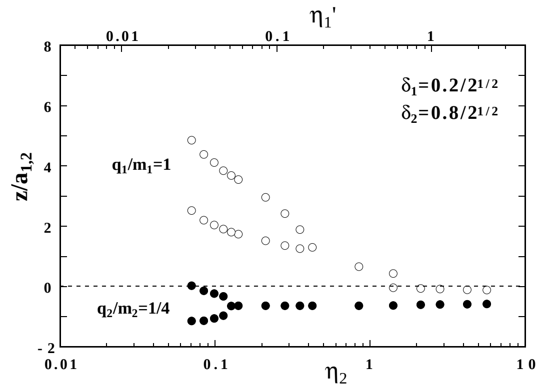

Figure 7. Crystal structure of dusty plasma mixture.

$$\frac{q_1}{q_2} = \frac{1}{2}, \quad \frac{m_1}{m_2} = \frac{1}{8}, \text{ and } \frac{N_1}{N_2} = 1 \tag{9}$$

are shown in Figs. 6 and 7 [8]. Here $-q_i$, m_i, and N_i are the charge, the mass, and the surface number density of the dust of species i. These conditions for charges and masses correspond to the case where both kinds of dust particles are of the same material, the ratio of radii is 2, and the electrostatic potentials are the same. As for the parameters ξ, we assume$\xi = 1$ evaluating the mean distance a by the total surface density $N_S = N_1 + N_2$.

We define the parameters for species i by

$$\eta_i = \left(\frac{e}{q_i}\right)\left(\frac{n_{sh}}{N_i^{3/2}}\right), \tag{10}$$

$$\delta_i = -\frac{z_i}{a_i} = \frac{g}{4\pi e n_{sh} a_i}\frac{m_i}{q_i}, \tag{11}$$

where$a_i = (\pi N_i)^{-1/2}$ and z_i is the equilibrium position of species i determined by $\phi_{ext}(z)$,

$$z_i = -\frac{g}{4\pi e n_{sh}}\frac{m_i}{q_i}. \tag{12}$$

In the above case,

$$\frac{\eta_1}{\eta_2} = \frac{q_2}{q_1}\left(\frac{N_2}{N_1}\right)^{3/2} = 2, \quad \frac{\delta_1}{\delta_2} = \frac{1}{4}. \tag{13}$$

When δ_i's are large, two species form separate systems, the one with smaller q/m lying below the other. With the decrease of η, the number of layers increases in each system and the critical values of transitions are consistent with the case of one component. When δ_i are sufficiently small andη is sufficiently large, both species of particles are almost in the same plane.

4. A proposal of experiment

We note that, for smaller values of η, we have structures where particles with larger q/m are levitated into the bulk plasma. Since there is no electric field in the latter domain, this levitation is due solely to repulsion from underlying layers. It is known that dust particles formed in reacting plasmas have rather small dispersion in their size [10]. By introducing external particles (impurities) with smaller charge to mass ratio, we may have a system of Yukawa particles in the domain of bulk plasma. In the sheath region, the interaction between dust particles has an anisotropic part induced by the ion flow in the domain of sheath. In the bulk plasma where the ion flow is small, the potential between particles may be well modeled by the isotropic Yukawa potential. We may thus have dusty plasma crystals which can be directly compared with results in statistical physics [1].

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