THE JEANS-BUNEMAN INSTABILITY IN THE PRESENCE OF AN ION BEAM IN A DUSTY PLASMA

S.R. Pillay[1], **R. Bharuthram**[1] and F. Verheest[2]

[1] *Physics Department, University of Durban-Westville, Durban 4000, South Africa*

[2] *Sterrenkundig Observatorium, Universiteit Gent, Krijgslaan 281, B-9000 Gent, Belgium*

1. Introduction

A dusty plasma consists of fairly massive (about $10^6 - 10^{12}$ times the ion mass) charged dust grains coupled to the ambient plasma. They feature significantly in the studies of space environments such as planetary rings and cometary tails, as well as in industrial and laboratory devices. Much attention has been given to linear and nonlinear wave phenomena in such plasmas. The consideration of gravitational effects on wave behaviour has been more recently studied. A self-gravitating system being subjected to a Jeans instability has been fairly well known [1,2]. The Jeans instability in a dusty plasma has also been studied fairly extensively [e.g. 3,4,5,]. It has been found that in the presence of particle streaming, both the Jeans instability and the usual Buneman instability can overlap [6]. Further studies examined the influence of dust size distribution on the Jeans-Buneman instability [7]. This paper focuses on numerical solutions of the dispersion relation with particular emphasis on the transition from the Jeans to the Buneman instability as a function of the ion beam speed, ion to electron temperature ratio and the wave number.

2. Theory

Our model is that of a plasma consisting of warm electrons and ions, and cold dust grains. The electrons are assumed to be in electrostatic equilibrium, merely providing Debye shielding. The massive, negatively charged dust grain species is treated as a stationary cold fluid, ignoring thermal effects. On the other hand, the ions are taken as warm and provide the energy to drive the instability through their streaming motion.

Following the usual fluid analysis for a self-gravitating plasma [4,7] we obtain the linear dispersion relation:

$$
\begin{aligned}
0 \;=\; & \omega^4\left[1+\frac{1}{K^2}\frac{n_{eo}}{n_{io}}\right]+\omega^3\left[-2KV_o\left(1+\frac{1}{K^2}\frac{n_{eo}}{n_{io}}\right)\right] \\
& +\omega^2\left[k^2\left(V_o^2-\frac{T_i}{T_e}\right)\left(1+\frac{1}{K^2}\frac{n_{eo}}{n_{io}}\right)+\beta^2 Z_d^2\frac{m_i^2}{m_d^2}\left(1+\frac{1}{K^2}\frac{n_{eo}}{n_{io}}\right)-1\right. \\
& \left.+\beta^2 Z_d^2\frac{n_{do}m_i}{n_{io}m_d}\left(1+\frac{1}{K^2}\frac{n_{eo}}{n_{io}}\right)-Z_d^2\frac{n_{do}m_i}{n_{io}m_d}\right] \\
& +\omega\left[2KV_o\left\{Z_d^2\frac{n_{do}m_i}{n_{io}m_d}-\beta^2 Z_d^2\frac{n_{do}m_i}{n_{io}m_d}\left(1+\frac{1}{K^2}\frac{n_{eo}}{n_{io}}\right)\right\}\right] \\
& +\left[K^2\left(V_o^2-\frac{T_i}{T_e}\right)\left\{\beta^2 Z_d^2\frac{n_{do}m_i}{n_{io}m_d}\left(1+\frac{1}{K^2}\frac{n_{eo}}{n_{io}}\right)-Z_d^2\frac{n_{do}m_i}{n_{io}m_d}\right\}\right. \\
& \left.-\left\{(1+\alpha_i)R\beta^2 Z_d^4\frac{n_{do}}{n_{io}}\left(\frac{m_i}{m_d}\right)^3+(1+\alpha_d)\beta^2 Z_d^2\frac{n_{do}m_i}{n_{io}m_d}\right\}\right]
\end{aligned}
\tag{1}
$$

where $\alpha_i = m_d/Z_d m_i$ and $\alpha_d = Z_d m_i/m_d$. The quantity $R = G_i/G_d$, is defined such that the dispersion relation can be analysed to compare the cases including and excluding ion gravitational effects. The following normalisations are used : distance by $\lambda_d = (T_e/4\pi n_{io}e^2)^{1/2}$, time by ${\omega_{pd}}^{-1} = (4\pi n_{do}Z_d e^2/m_d)^{-1/2}$ and speed by $c_s = (T_e/m_i)^{1/2}$. The plasma frequencies for the different species are defined by $\omega_{p\alpha}^2 = 4\pi n_{o\alpha}q_\alpha^2/m_\alpha$, whilst the Jeans frequencies by $\omega_{J\alpha}^2 = 4\pi G n_{o\alpha}m_\alpha$. The quantities $n_{o\alpha}$ is the equilibrium density of species α and V_o is the ion drift speed. The factor β is the ratio of the dust Jeans frequency to the dust plasma frequency, i.e. $\beta = \omega_{jd}/\omega_{pd}$.

3. Numerical Results and Discussion

The dispersion relation is solved numerically for the following fixed parameters: $m_i/m_e = 1836$, $Z_d = 7 \times 10^2$, $k\lambda_d = 0.1$ and $n_{eo}/n_{io} = 0.5$. Figure 1 compares the growth rate at $\beta = 0.1$ (for which one finds $m_d/m_i = 7.77 \times 10^{19}$) as a function of the ion drift speed, V_o, for different T_i/T_e ratios. For $V_o = 0$,$\gamma > 0$ for $T_i/T_e \leq 0.1$. This represents the pure Jeans Instability (JI). As T_i/T_e increases, the JI dies away and the Buneman Instability (BI) is excited at some onset value V_o, which increases with T_i/T_e. Ion gravitational effects are found to be negligible, as expected [2,8]. In each case the the growth rate for the BI rises sharply, after which it drops sharply to zero and does not reappear, even at much higher ion drift velocities.

For $\beta = 0.5$ (i.e. $m_d/m_i = 3,89 \times 10^{20}$) (Figure 2), corresponding to stronger dust gravitational influence, the behaviour of the curves is the same as in figure 1. However, an interesting feature is that the JI grows again and levels off, after remaining damped over a certain range of intermediate ion drift speeds. This seems to indicate that there is a range of ion drift speeds over which neither the Jeans nor the Buneman instabilities exist.

Figures 3 and 4 examines the growth rates as a function of V_o at $T_i/T_e = 0$ and 1.0. As before, it is noted that at extremely low V_o-values the growth rates correspond to the JI, which is in turn overtaken by the BI for higher V_o. The re-appearance of the JI after the BI damps is clearer here. An interesting feature is that the band over which there is no growth in the instability becomes smaller as $\beta \to 1$, corresponding to the state when the gravitational and electrostatic effects on the dust are equal.

An investigation into the k dependance of the growth rate was also carried out. It is observed that for $V_o^2 = T_i/T_e$, the growth rate is independant of k, and is greater for larger β values. This suggests a Jeans mode which is k independant for the condition mentioned. Figures 5 and 6 examine the growth rate as a function of $k\lambda_d$ for $V_o = 0$ and 1.0, $T_i/T_e = 0.5$, at varying β values. For the case with $V_o = 0$, as β increases the range of k values over which the instability exists increases. With $V_o = 0$, it is clear that this is a pure Jeans mode which has a maximum $k\lambda_d$ value for each β value, beyond which the instability does not exist. For $V_o = 1.0$ (figure 6) the instability changes from the JI to the BI as $k\lambda_d$ increases. We note that for the JI it is the high β curve that has the largest γ (as expected), whilst the opposite holds for the BI. Also the upper k-limit for $\gamma > 0$ is independant of β for the BI. Comparing figures 5 and 6, one notes that there is some critical ion drift velocity, below which the Jeans instability dominates, and above which the Buneman take over.

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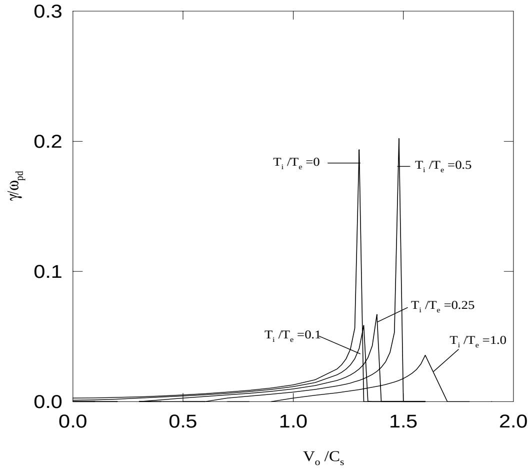

Figure 1. γ vs V_o at $\beta = 0.1$ for $T_i/T_e = 0; 0.1; 0.25; 0.5$ and 1.0.

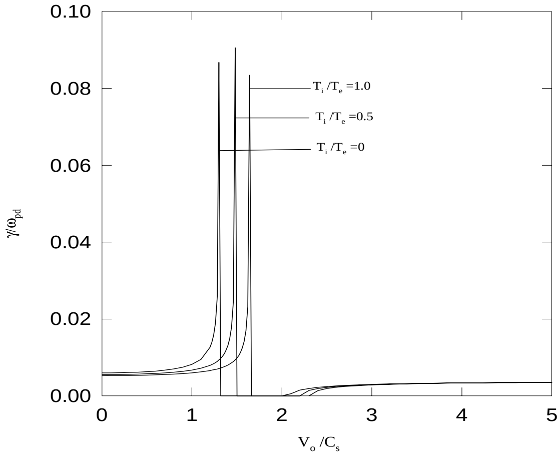

Figure 2. γ vs V_o at $\beta = 0.5$ for $T_i/T_e = 0; 0.5$ and 1.0.

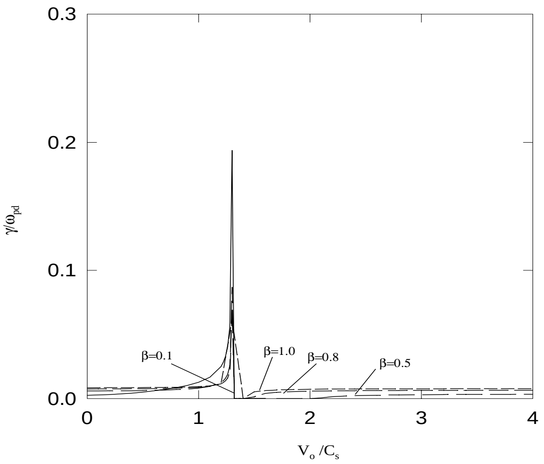

Figure 4. γ vs V_o at $T_i/T_e = 1.0$ for $\beta = 0.1; 0.5; 0.8$ and 1.0.

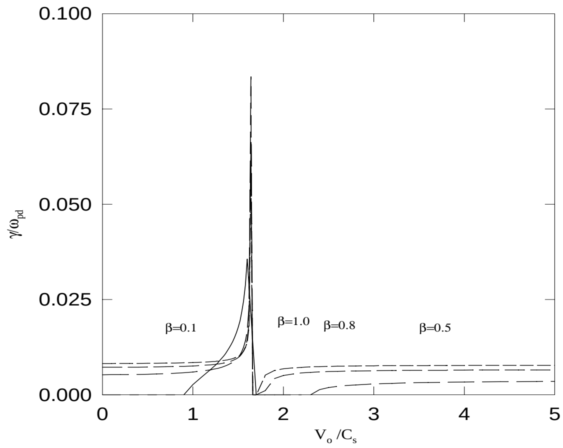

Figure 3. γ vs V_o at $T_i/T_e = 0$ for $\beta = 0.1; 0.5; 0.8$ and 1.0.

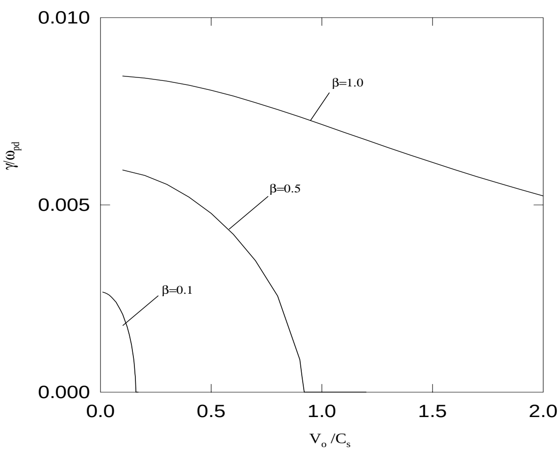

Figure 6. γ vs $k\lambda_d$ at $T_i/T_e = 0.5$ for $\beta = 0.1; 0.5$ and 1.0. Here $V_o = 1.0$.

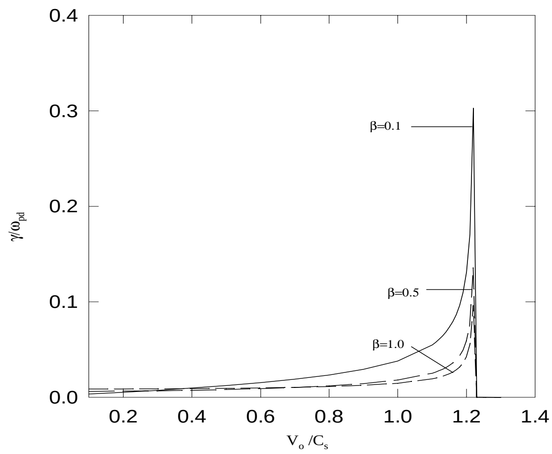

Figure 5. γ vs $k\lambda_d$ at $T_i/T_e = 0.5$ for $\beta = 0.1; 0.5$ and 1.0. Here $V_o = 0$.