NEAR-THRESHOLD KINETIC INSTABILITIES AND FAST PARTICLE TRANSPORT: OBSERVATIONS & INTERPRETATION

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There are numerous fusion-related experiments in which collective modes are excited by a population of energetic particles [1-6] such as fusion-produced alpha particles, neutral beam produced ions or suprathermal rf heated ions. In some cases the modes grow to a level at which they cause enhanced transport and anomalous losses of the fast particles. In other cases, the losses are small but the modes exhibit an intricate nonlinear behavior: generation of sidebands, quasiperiodic bursts, change of the mode frequency in time, etc. This paper presents a first-principles physics basis for understanding these phenomena. The key idea that underlies our analysis and allows it to be rather general is that in most experiments the system cannot go far beyond the instability threshold because sources of energetic particles are relatively weak. The evolution of an unstable mode in this regime is governed by resonant wave-particle interaction and collision-like relaxation process for resonant particles that can be characterized by an effective collision frequency ν_{eff}. Near the instability threshold, the mode linear growth rate γ, which represents the difference between the energetic particle drive γ_L and the background damping rate γ_d, is small compared with both γ_L and γ_d. Even a relatively small nonlinear correction to γ_L can then be comparable with γ and therefore affect the mode dynamics. This feature allows us to treat the nonlinearity in a perturbative manner by expanding the energetic particle response in powers of the mode amplitude A and retaining only the lowest order nonlinear contribution. The resulting nonlinear equation for the mode amplitude A has the following form [7] :

$$\exp(-i\phi)\frac{dA}{dt} = \frac{\gamma}{\cos\phi}A - \frac{\gamma_L}{2}\int_0^{t/2}\tau^2 d\tau \int_0^{t-2\tau} d\tau_1 \exp[-\nu_{eff}^3\tau^2(2\tau/3+\tau_1)] \times A(t-\tau)A(t-\tau-\tau_1)A^*(t-2\tau-\tau_1) \qquad (1)$$

where the parameter ϕ characterizes the fast particle contribution to the real part of the mode frequency ω. This parameter, whose value is of the order of γ_L/ω, is responsible for asymmetry of nonlinear sidebands. Apart from a numerical factor, the absolute value of the amplitude A equals the square of the nonlinear bounce frequency, ω_b^2, for a typical resonant particle trapped in the wave. The neglect of higher order terms in Eq. (1) is validated by the fact that the mode saturates before the resonant particles complete a nonlinear bounce in the perturbed field. The structure of Eq. (1) readily indicates that the saturated amplitude must scale as $\gamma^{1/2}$. The corresponding value of ω_b in the saturated state turns out to be smaller than ν_{eff}, so that particles indeed decorrelate from the resonance faster than their motion becomes strongly nonlinear. It follows from the solution of Eq. (1) that the mode converges dynamically to a steady saturated state when γ is sufficiently small. However, this saturated state becomes unstable and bifurcates when γ exceeds a critical value γ_{cr} with $\gamma_{cr} = 0.486\,\nu_{eff}$ for ϕ << 1.

When $\gamma \geq \gamma_{cr}$, Eq. (1) has a limit cycle solution:

$$A = A_0 \exp(i\delta\omega t)[1 + \alpha_1 \exp(i\Delta\omega t) + \beta_1^* \exp(-i\Delta\omega t) + \alpha_2 \exp(2i\Delta\omega t) + \beta_2^* \exp(-2i\Delta\omega t) + \ldots],$$

where A_0 and $\delta\omega$ are the amplitude and the nonlinear frequency shift of the main spectral component, $\Delta\omega$ is the sideband frequency , and α_i and β_i are the relative amplitudes of the sidebands. Equation (1) leads to the following relationship between γ_{cr} , ν_{eff} , and $\Delta\omega$ at the bifurcation point : $\Delta\omega = 1.18\gamma_{cr} = 0.575\nu_{eff}$. As γ/ν_{eff} increases past the first bifurcation, the limit cycle solution exhibits further bifurcations leading to period doublings. This is illustrated in Fig. 1, which shows the power spectra of the function $A(t)$ for three values of γ/ν_{eff}.

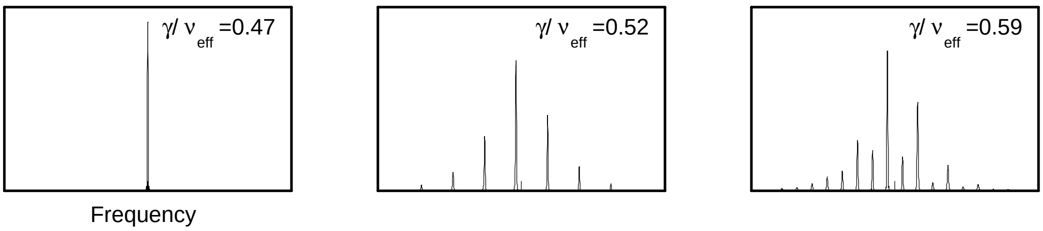

Fig. 1. Nonlinear splitting of the spectral line and the period doubling bifurcation: snapshots of the power spectrum of the saturated solution of Eq. (1) (with *ϕ=3π/64*).

Recent observations of toroidal Alfvén eigenmodes driven by energetic ions in JET give clear experimental evidence for the nonlinear scenario described by Eq. (1): the mode first appears as a single spectral line and then splits into several closely spaced spectral components.

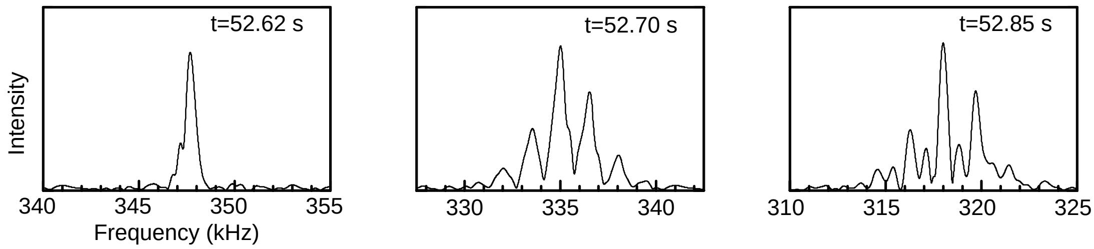

Fig. 2. Snapshots of the experimental spectra of magnetic activity for the toroidal Alfvén eigenmode with $n=7$ in JET discharge #40238.

The experimental spectra also show the half-frequency sidebands (period doubling) that arise somewhat later than the primary sidebands as the fast particle drive increases in time.

Our model relates the frequency shifts and the amplitudes of the sidebands to the values of the growth rate and the effective collision frequency. This simple model assumes that there is a prevailing resonance in the system and that ν_{eff} is essentially constant for that resonance. In general, Eq. (1) may require modifications to incorporate multiple resonances and to refine the descriptions of effective collisions. However, all modifications of Eq. (1) have an important common feature: they give nonlinear bifurcations and scaling laws which can be related to the experimental data with high precision. In the JET experiment being discussed the single resonance assumption appears appropriate. By matching the solution of Eq. (1) with the data presented in Fig. 2, we find that $\gamma_{cr} = 0.847\Delta\omega = 9\cdot 10^3 \sec^{-1}$ and $\nu_{eff} = 1.73\Delta\omega = 1.8\cdot 10^4 \sec^{-1}$. These values fall into tthe range expected from independent theoretical estimates. The model also explains the reversible character of line splitting and reproduces the entire time behavior of the experimental signal if one makes a natural assumption that the growth rate γ increases linearly with time from the instability threshold to the bifurcation point, passes through a maximum after that and eventually decreases to negative values.

The steady state and the limit cycle regimes require γ to be smaller than or comparable to ν_{eff}. When γ is larger than ν_{eff}, the solution of Eq. (1) does not saturate. Instead, it shows that the amplitude $A(t)$ grows to infinity in a finite time with simultaneous oscillations at increasing frequency. Equation (1) is applicable at the onset of this singularity but it fails to describe the evolution of the system beyond the explosive phase where the perturbative approach eventually breaks down. Note, however, that the onset of the oscillatory explosive singularity already has the signature of the frequency chirping effect observed for fishbone instability in PDX and JET experiments , for hot electron interchange modes in Terrella [8] and for Alfvén eigenmodes in TFTR, JT-60 and DIII-D. To verify the mechanism of the frequency chirping for fishbones, a next-level physical model has been developed which treats the energetic particles fully nonlinearly but uses a linear approximation for the background plasma response in the resonant layer near the q=1 surface. The code based on this model uses a new analytic expression for the MHD response [9]. Preliminary simulations [10] of the fishbone instability recover the nonlinear downshift in frequency, which accompanies mode saturation, and also the radial transport and possible losses of fast ions. This model is being generalized to incorporate the fluid nonlinearity of the resonant layer, which is shown to be competitive with the energetic particle nonlinearity.

It should be noted that the singular solution of Eq. (1) serves as a seed for spontaneous formation of phase-space holes and clumps at the post-explosive phase of many kinetic instabilities [11]. The clumps and holes represent nonlinear Bernstein-Greene-Kruskal modes with a frequency chirping effect that allows the mode to continuously extract energy from the distribution of energetic particles to balance the background dissipation. This mechanism may apply to fast chirping observed in toroidal Alfvén eigenmode experiments.

A single weakly unstable mode rarely causes strong losses of fast particles since it only affects a relatively small near-resonant area of phase space. Anomalous losses are usually associated with multiple modes that lead to global quasilinear diffusion. However, quasilinear diffusion requires the mode amplitudes to be sufficiently large to satisfy the resonance overlap condition. In presence of background damping, a weak source of energetic particles may not be able to continuously maintain this critical level of turbulence. On the other hand, given enough time without anomalous transport, the same weak source may create an inverted population of particles with free energy sufficient to trigger the overlap. This situation leads to an interesting regime of intermittent quasilinear diffusion when turbulent bursts and losses are followed by quiescent periods during which the system accumulates free energy needed for the next burst. It is essential that each burst releases much more energy than what would be released by a set of isolated resonances. Once triggered, the burst forces the mean gradient of the particle distribution to fall somewhat below the threshold of linear instability, which explains the decay of turbulence at the end of the burst. This picture has characteristic features of beam losses produced by toroidal Alfvén eigenmodes in TFTR and DIII-D. This avalanche-type scenario of intermittent global diffusion is not limited to the problem of fast particle transport but should be applicable as well to the more general problem of turbulent transport.

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