# MAPPING OF A STOCHASTIC MAGNETIC FIELD IN A GENERAL CONFIGURATION

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## 1. Introduction

The topologic study of a stochastic magnetic field is investigated by the tracing of magnetic field lines using a symplectic symmetric map. This method allows to study statistical properties of the dynamics of the system such as the number of iterations for open field lines or the local diffusion coefficient. We study the effect of the **D**ynamic **E**rgodic **D**ivertor (DED) [1] on a **TEXTOR** equilibrium computed with the VMEC code. The magnetic field of the DED is computed using the Biot-Savart law applied to the superposition of sets of current carrying filaments.

## 2. The Mapping Method

If we invoke the condition $\nabla \cdot \mathbf{B} \equiv 0$, we can write in a general way the magnetic field $\mathbf{B}$ as

$$\mathbf{B} = \nabla\Psi \times \nabla\theta^* + \nabla v \times \nabla\chi(\Psi, \theta^*, v)$$

where $\Psi$ and $\chi$ constitute a measure of the toroidal and poloidal flux, respectively, and $\theta^*$ is a poloidal coordinate [2]. In this case, the magnetic field lines become

$$\frac{d\theta^*}{dv} = \frac{\partial\chi}{\partial\Psi},\ \frac{d\Psi}{dv} = -\frac{\partial\chi}{\partial\theta^*} \tag{1}$$

from which we recognize the Hamiltonian equations with $\chi \leftrightarrow H,\ (\Psi, \theta^*) \leftrightarrow (p, q)$ and $v \leftrightarrow t$. The Hamiltonian $\chi$ can be written in the form

$$\chi(\Psi, \theta^*, v) = \chi_o(\Psi) + \chi_1(\Psi, \theta^*, v)$$

where $\chi_o$ is the contribution from the unperturbed equilibrium such that $\frac{d\chi_o}{d\Psi} = \iota(\Psi)$, and $\iota$ is the rotational transform. The calculation of the Hamiltonian and the canonical coordinates are performed numerically [2].

If we examine now the intersections of the trajectory with the plane $(\Psi, \theta^*)$ at a cross section at $v = constant$, the equations of motion can be described by a map. This map should satisfy the same properties as the Hamiltonian equations (1), namely it must be symplectic and symmetric with respect to the toroidal angle [3, 4]

$$\hat{T}_+(J_n, \theta_n) = \begin{cases} J_{n+o} &= J_n + \frac{\epsilon}{2} f(J_n, \theta_{n+o}), \\ \theta_{n+o} &= \theta_n + \frac{\epsilon}{2} g(J_n, \theta_{n+o})\ (mod\ 2\pi) \end{cases} \tag{2a}$$

$$\hat{T}_{\iota}(J_{n+o},\theta_{n+o}) = \begin{cases} J_{n+1-o} &= J_{n+o}, \\ \theta_{n+1-o} &= \theta_{n+o} + \frac{2\pi}{L}\iota(J_{n+1-o})\ (mod\ 2\pi) \end{cases} \tag{2b}$$

$$\hat{T}_{-}(J_{n+1-o},\theta_{n+1-o}) = \begin{cases} J_{n+1} &= J_{n+1-o} + \frac{\epsilon}{2}f(J_{n+1},\theta_{n+1-o}), \\ \theta_{n+1} &= \theta_{n+1-o} + \frac{\epsilon}{2}g(J_{n+1},\theta_{n+1-o})\ (mod\ 2\pi) \end{cases} \tag{2c}$$

where $J = \Psi$ can be associated with a radial coordinate and $\theta = \theta^*$. The functions $f$ and $g$ are defined by

$$f(J,\theta) = -\int_0^{2\pi/L} dv \frac{\partial \chi_1}{\partial \theta^*}(\Psi,\theta^*,v)\Big|_{\Psi=J,\,\theta^*=\theta+\iota(\Psi)v},\ g(J,\theta) = -\int_0^{\theta} d\theta' \frac{\partial f}{\partial J}(J,\theta'). \tag{3}$$

The equations (2a), (2b) and (2c) define a symplectic symmetric map and equation (3) guarantees the preservation of area.

### 3. Numerical Results

The calculation for the TEXTOR equilibrium has been based on the following data: the plasma current $I_p = 356\ kA$, the major radius $R = 1.75\ m$, the minor radius $a = 0.46\ m$, the resonant $\iota = 1/3$ is located at $r = 0.43\,m$. The DED coils are aligned to approximately coincide with the pitch of the field lines on the $\iota = 1/3$ rational surface [1]. The coil arrangement consists of 16 helical conductors carrying a current $I_j,\ j = 1,...16$. We model the current $I_j$ as a simple trigonometric function,$I_j = cc * I_0 sin((j-1)\varphi + \pi/4t)$ with $I_0 = 7.5\,kA, \varphi = \pi/4$ for the case $m/n = 6/2$ and $I_0 = 3.75\ kA, \varphi = \pi/8$ for the case $m/n = 3/1$. Consequently, the perturbed fields generated by the DED coils have a structure dominated by the $m/n = 6/2$ or $m/n = 3/1$ component where $m$ ($n$) is the poloidal (toroidal) mode number.

In the Fig. 1a, we see the Poincaré section for the case $m/n = 6/2$ with $cc = 0.2$ and the formation of magnetic islands (six on the $\iota = 1/3$ surface), stochastic regions and KAM barriers. The value $J/J_{max} = 1$ corresponds to the boundary of the plasma. Fig. 1b is the same but with the configuration $m/n = 3/1$ for the perturbation.

We can study the angular dependence of the number of iterations $N_\phi$ for $J/J_{max} = constant$ in the mapping for open field lines while they reach the boundary of the plasma. Fig. 2a represents $N_\phi$ for the case $m/n = 6/2$ with $cc = 0.4$ and $J/J_{max} = 0.93$ and Fig. 2b represents the case $m/n = 3/1$, $cc = 0.4$ and $J/J_{max} = 0.95$. We show clearly a fractal structure of this quantity. It is due to the fact that the dynamics of the system is self-similar.

We expect that the radial diffusion is normal for few iterations and so the mean square radial displacement of field lines should be linear with the number of iterations. In this case, we can introduce a local diffusion coefficient $D_{FL}$. The profile of $D_{FL}$ as a function of $J/J_{max}$ is shown in Fig. 3a for the case $m/n = 6/2$ for different values of $cc$. We see clearly that $D_{FL}$

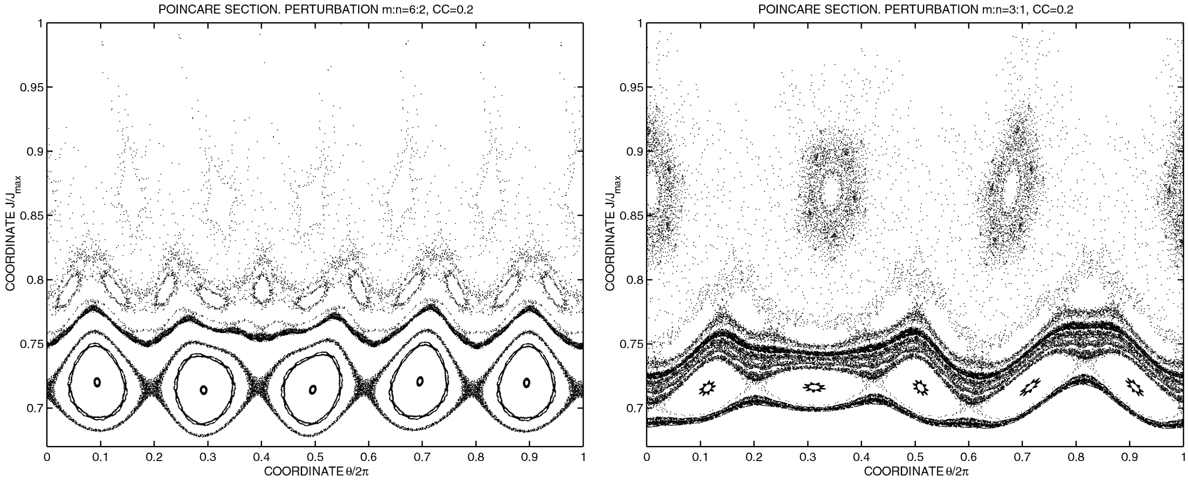


**Fig. 1:** a) Poincaré section in $(\theta, J/J_{max})$ coordinates for the perturbation $m/n = 6/2$ with $cc = 0.2$ (left) and b) for the case $m/n = 3/1$ with $cc = 0.2$ (right).

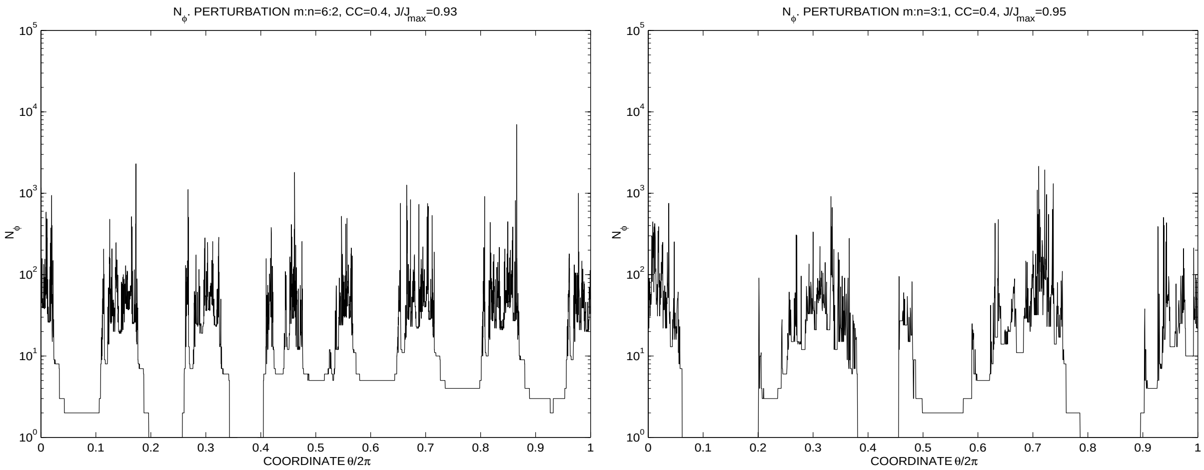


**Fig. 2:** a) Number of iterations $N_\phi$ for open field lines for the perturbation $m/n = 6/2$ and $cc = 0.4$ for $J/J_{max} = 0.93$ (left) and b) for the case $m/n = 3/1$ with $cc = 0.4$ and $J/J_{max} = 0.95$ (right).

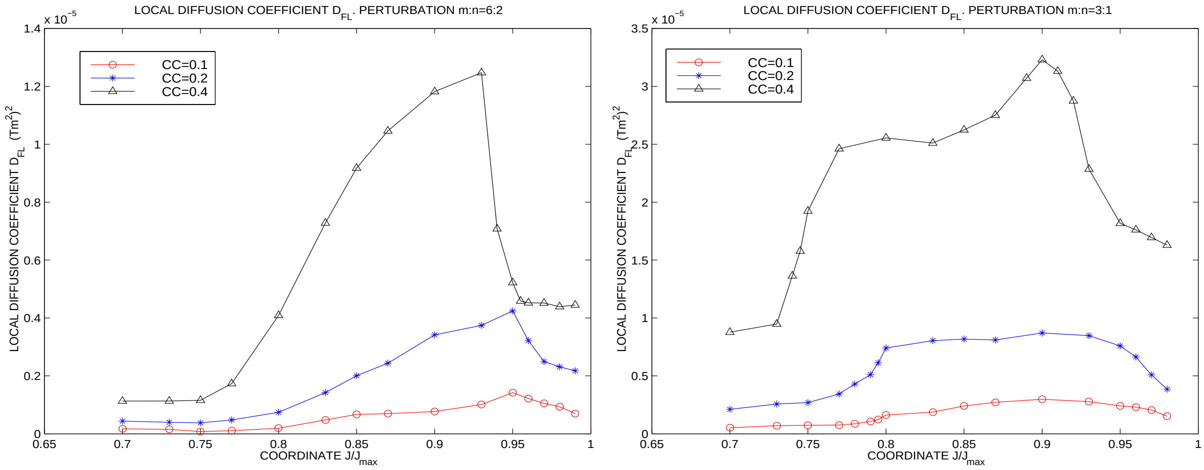


**Fig. 3:** a) Profile of the local diffusion coefficient $D_{FL}$ as a function of $J/J_{max}$ for the perturbation $m/n = 6/2$ for different values of $cc$ (left) and b) for the case $m/n = 3/1$ (right) (Circles: $cc = 0.1$. Stars: $cc = 0.2$. Triangles: $cc = 0.4$) .

increases until the value $J/J_{max} \sim 0.95$ and $\sim 0.90$ for the cases $m/n = 6/2$ and $m/n = 3/1$ (Fig. 3b), respectively. After this value, the laminar process dominates on the diffusion and the coefficient $D_{FL}$ decreases quickly.

## 4. Conclusion

We have studied the magnetic topology of the DED on a TEXTOR equilibrium by using a symplectic symmetric map. This map is general and the only restriction imposed is that the unperturbed equilibrium state should have nested magnetic flux surfaces. We can use this method to study more complicated system such as Stellarators. Concerning the numerical results, we have seen that the DED creates stochastic regions and the Poincaré section constitutes a very complicated KAM system: formation of magnetic islands, stochastic regions, KAM barriers, etc.. Concerning the statistical properties of the system, we have shown there exists a fractal structure for the number of iterations for open field lines. This is due to the self-similar properties. Finally, from the study of the profile of the local diffusion, we have seen that the laminar region is pushed inwards into the plasma as the structure of the perturbation is changed.

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