PROBABILISTIC EXCITATION OF L/H TRANSITION IN TOROIDAL PLASMAS

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1. Introduction

Transition phenomena in plasmas (e.g., L/H transition) are widely observed in various toroidal confinement devices. The hysteresis characteristic has been observed in the L/H transition [1]. A transition (bifurcation) is modelled by the equation which includes a cusp-type bifurcation. In the L/H transition study, the model equation is tested [2] to explain the experimentally-observed dithering ELMs [3]. This study clearly demonstrates that the high temperature plasma has a transition nature. To understand the nature of sudden onsets of L/H transitions, consideration of the basic physics mechanism is needed.

Dynamics of transition in confined plasmas has been discussed by use of the critical condition. However, there exist statistical variances in relevant variables, and a concept of discrete critical condition is not necessarily valid. The turbulence level itself has a distribution and the transition occurs from a turbulent state to another. We here explore a new aspect of transition, that is, a probabilistic nature of occurrence. The statistical variance affects the transition nature and introduces a change from the deterministic phenomenon to the probabilistic one. In this paper, we shall show the probabilistic nature of the transition by using the simple (zero-dimensional) dynamical model.

2. Zero-dimensional dynamical model equations

We here adopt two basic equations: One is the temporal evolution of pressure gradient. The other is the dynamics of the loss rate that produces the hysteresis of the flux-gradient relation. We choose two representative variables, i.e., the pressure gradient α and the loss rate γ. The loss rate is directly related to the turbulence level and the particle diffusivity.

The reduction to the 0-D model from the transport equation has been discussed [4]. A layer with a finite width is considered, and the averaged value within this layer is treated as a scalar quantity. Though highly simplified, the model form of the hysteresis has successfully applied to the investigation of the dynamics of dithering ELMs. The model equation used here takes the forms

$$\frac{\partial}{\partial t}\alpha = S - \gamma\alpha \tag{1}$$

and

$$\xi\frac{\partial}{\partial t}\gamma = \alpha - 1 + a(\gamma - 1) - b(\gamma - 1)^3, \tag{2}$$

where S is the particle influx into the layer, $\xi(=O(B_p^2/B_t^2))$ denotes the dynamical time difference between α and γ, and the cubic equation $a(\gamma-1) - b(\gamma-1)^3$ describes the shape of the hysteresis in the gradient-flux relation.

The dynamical nature of the set of equations has been studied. If all the coefficients (S,a,b,ξ) are constant in time, Eqs.(1) and (2) predict the stable stationary solutions or the dynamical solution of a limit cycle. Stationary solutions are obtained for $S<S_1$ (lower flux branch), $S_2<S$ (higher flux branch) and a limit cycle appears for $S_1<S<S_2$ where $S_1 = \left(1-\sqrt{a/3b}\right)\left\{1+(2a/3)\sqrt{a/3b}\right\}$ and $S_2 = \left(1+\sqrt{a/3b}\right)\left\{1-(2a/3)\sqrt{a/3b}\right\}$.

As the physics origin of the transport, the turbulence level is often discussed in terms of the statistical averages. The relation between its averaged level and the gradient of the global plasma parameter has been a main subject of the plasma physics. The turbulence level is also associated with the statistical variance. For the confined plasmas, which is far from thermal equilibrium, the statistical variance is as important a quantity as the statistical average. The nonlinear simulation has shown a large temporal variation around the average [5]. The experimental observation has demonstrated that the statistical deviation from the mean value could be as large as the average itself [6]. Based on these considerations, we consider that parameters (S,a,b) are statistical variables and have fluctuation parts in time. For instance, the heat flux into the region of subject could fluctuate in time with some statistical distribution. The temporal variation of the relevant magnetic perturbation amplitude has a statistical nature. These processes give the statistical nature of (S,a,b). We set $S=S_0+\varepsilon_s$ and $a=a_0+\varepsilon_a$ consider ε_s and ε_a as statistical variables, e.g., $\langle\varepsilon_a\rangle=0$ and $\langle\varepsilon_a^2\rangle\neq 0$. The variance ε_s represents that of the fluctuations of the source from upstream. The variance ε_a comes from the deviation from the mean fluctuation level. The study about the effect of ε_s in influx was previously done [7]. Therefore, we here mainly focus the effect of the statistical variances ε_a on the transition property.

3. Probabilistic Transition

Temporal evolutions are examined by solving Eqs. (1) and (2). The variation in a hysteresis curve is investigated. The variance of a is taken into account with $\bar{\varepsilon}_a = \sqrt{\langle a^2\rangle - \langle a\rangle^2}$. The parameters are chosen as $a_0=0.5$ and $b=1$. The critical fluxes are $S_1=0.67$ and $S_2=1.21$. We set the smallness parameter ξ is to be zero to obtain the maximum probability of critical parameters clearly. We introduce the statistics for ε_a, the frequency spectrum of which obeys the power law, i.e., $I(f)\propto f^{-k}$. At first, the case with k=1 is studied. In this example, f^{-1}-noise generator is used to obtain fluctuation quantity ε in the domain $-0.05\leq\varepsilon\leq 10^{-5}$. We set $\varepsilon_a=\varepsilon+\varepsilon_1$ ($\varepsilon_1=0.014$) so that $\langle\varepsilon_a\rangle=0$ and $\bar{\varepsilon}_a=0.015$ are satisfied. Probability function of ε_a, $P(\varepsilon_a)$ shows a sharp peak near $\varepsilon_a\cong\varepsilon_1$ and is associated with a long tail. This tail is approximated by a power law like $(C+\varepsilon_a)^{-1}$ (C being some constant) in a wide region of the tail. We assume $P(\varepsilon_a)=P(\varepsilon_1-10^{-5})$(=const.) ($\varepsilon_1-10^{-5}<\varepsilon_a\leq\varepsilon_1$)and $P(\varepsilon_a)=0$ ($\varepsilon_a<-\varepsilon_2$, $\varepsilon_a>\varepsilon_1$)where $\varepsilon_2=0.036$. We set S=1.0.

The oscillations like limit cycles with irregular bursts are obtained. To show the probabilistic excitation of the transition, the observed critical pressure gradient is examined where the transition from H-mode to L-mode occurs. The distribution function of α_c at the onset of the transition, $P(\alpha_c)$, is obtained in Fig. 1. This distribution clearly demonstrates the onset has the probabilistic nature. A peak of the probability for the transition is observed in lower values of α_c, compared with the case in the absence of the noise. A long tail distribution of transition probability with respect to α_c is obtained. In the case of the deterministic picture, the critical pressure at the onset becomes one value α_{co}. For the parameters a=0.5 and b=1.0, the critical parameter is $\alpha_{co}=1.136$ without the noise. The smallest value of α_c for the occurrence of the transition, $\alpha_{c,min}$, is restricted with the cut-off of the power law statistics $P(\varepsilon_a)$. If $(\varepsilon_1+\varepsilon_2)$ becomes large, the value of $\alpha_{c,min}$ decreases. If the variation of the quantity a , $a \to a+\delta a$, is static, the change of critical α_c, $\alpha_{co} \to \alpha_{co}+\delta\alpha$ satisfies $\delta\alpha/\delta a=\sqrt{a/3}$. In the statistical fluctuations, we observe $\delta\alpha/\delta a=\sqrt{a/3}$. The distribution function $P(\alpha_c)$ is found to obey the same power law of the frequency with the variance in a range $\alpha_c \ll \alpha_{co}$ as $P(\alpha_c) \propto (\alpha_{co}-\alpha_c)^{-1}$. The fitting is done with correlation coefficient R=0.90. Furthermore, we examine the flux values where the transition from H-mode to L-mode ends and the periods of bursts. The probabilistic distributions of critical values of flux and periods of bursts are shown in Fig. 2 and Fig. 3, respectively. The deterministic picture predicts that the critical value of flux is $\Gamma_{c0}=2.064$ and the period is $T_{c0}=0.890$. In the presence of ε_a, distribution functions of $P(\Gamma_c)$ and $P(T_c)$ are also found to obey the same power law with the variance in a range $\Gamma_c \ll \Gamma_{co}$ and $T_c \ll T_{co}$ as $P(\Gamma_c) \propto (\Gamma_{co}-\Gamma_c)^{-1}$ and $P(T_c) \propto (T_{co}-T_c)^{-1}$, respectively. In the case of $P(\Gamma_c)$, the correlation coefficient is R=0.92. On the other hand, in the case of $P(T_c)$, the correlation coefficient is R=0.85.

Next, we study noises with other power dependences (f^{-k}) up to k=4. It is found that the probability distributions in terms of the critical parameter have the same power law, e.g., $(\alpha_{co}-\alpha_c)^{-k}$ in an asymptotic behavior.

Finally, we compare the effects of the variance ε_s in the source and those of the variance ε_a in the mean fluctuation level on the critical parameters. Parameters are chosen as $S_0=1.0$, $a_0=0.5$, b=1.0 and $\xi=0.0$. The noise is set $I(f) \propto f^{-1}$. In the presence of only the noise in the source, i.e., $\varepsilon_s \neq 0$, $\varepsilon_a=0$ and $\varepsilon_s/S_0=0.3$, the probabilistic functions of α_c and Γ_c have sharp distributions around α_{c0} and Γ_{c0}, respectively. In this case, the probabilistic function of T_c has the distribution around T_{c0} to some extent. Therefore, the plot on Γ_c -T_c plane has the sharp distribution of Γ_c and the distribution of T_c to some extent. Next, we study the case of only the noise in the mean fluctuation level, i.e., $\varepsilon_s=0$, $\varepsilon_a \neq 0$. To compare the previous case: $\varepsilon_s \neq 0$, $\varepsilon_a=0$, we set $\bar{\varepsilon}_a/a_0=0.3$. In this case, all critical parameters have the much wider distributions than the case of $\varepsilon_s \neq 0$, $\varepsilon_a=0$. The correlation plot on Γ_c -T_c plane, Fig. 4, has the

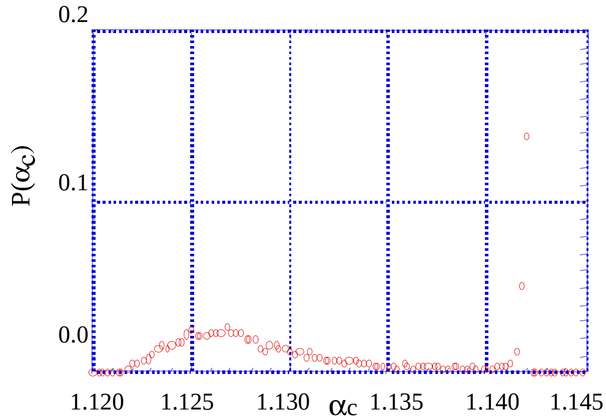

Fig. 1. The distribution of the critical gradient at the onset of the transition for the noise with power law.

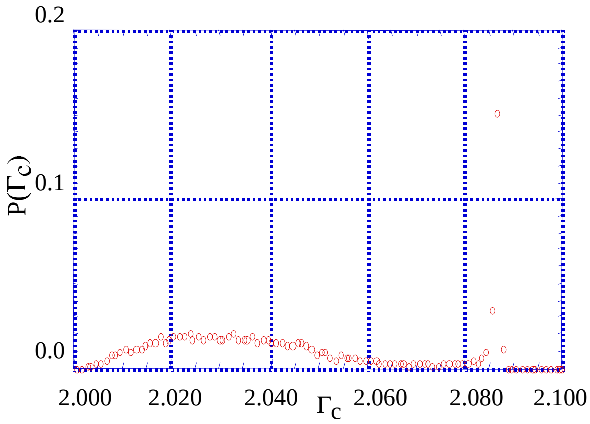

Fig. 2. The distribution function of the critical flux for the noise with power law.

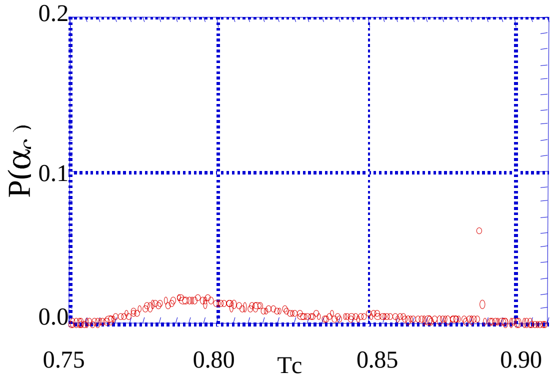

Fig. 3. The distribution of the period of burst for the noise with power law.

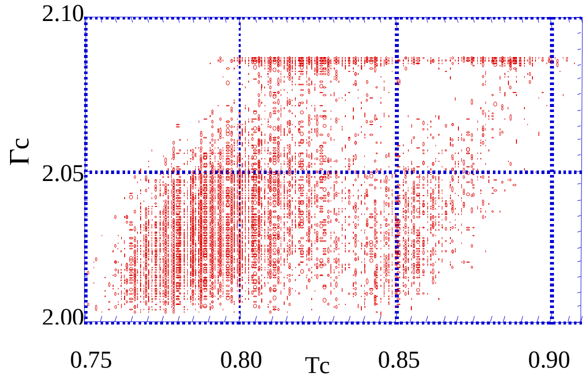

Fig. 4. Correlation plot with critical parameters of periods and particle flux in the case of $\varepsilon_a \neq 0$.

wide distribution of both T_c and Γ_c axes. The significant difference of the distributions from the origin of the fluctuations is examined. The qualitatively same results are obtained for other cases, k=2,3,4.

4. Summary and Discussions

In this article, we study the probabilistic nature of the transition by use of 0-D model equation. The statistical variance for relevant parameters expresses the probability of the transition based on the probabilistic view. It is found that the transition takes place with a finite probability around the threshold condition without the noise. The obtained critical parameters also show the large deviation. The distribution of critical parameters is found to reflect the statistical property of the background turbulent field. The statistical study on the experiments of transition will provide a unique information to understand the physics of the transition as well as the nature of the turbulence in plasmas. Especially, it can be tested whether the fluctuations are originated from the source or the intrinsic turbulence by the experimental study about the correlation between the critical parameters (e.g., T_c and Γ_c) in the system of the model equations used here. It should be also noted that a concept of lead time is naturally introduced. This is left for future work.

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