PRIVILEGED NEGATIVE MAGNETIC SHEAR EQUILIBRIA IN AUXILIARLY HEATED TOKAMAKS

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1. Introduction

The privileged equilibria are associated, in our definition, with a stationary magnetic entropy, a quantity that measures, in the framework of information theory, the probability of coarse - grained current density configurations in a suitably constrained possibility space of the macroscopic plasma equilibria. The constraints involve: 1) a fixed (but unspecified) value of the current density dispersion generated by the underlying particle structure and 2) a fixed value of the magnetic energy plus the possible interaction energy of the magnetic system with the external world [1]. In an isolated system (e.g. a plasma completely surrounded by a perfectly conductive shell) the interaction energy is zero and the magnetic entropy is not only stationary but maximum. In an open system, as is the tokamak in interaction with the ohmic transformer and with the auxiliary heating, the magnetic entropy may be stationary, at the most.

In this work we study the dependence of the tokamak states with stationary entropy on the intensity and on the deposition profile of the auxiliary heating. These factors are found to have considerable bearings on the magnetic equilibrium, in particular on the generation of states with negative magnetic shear and on the form of the pressure and of the thermal diffusivity.

2. Tokamak Equilibria with Stationary Magnetic Entropy

The magnetic entropy S is stationary in a tokamak when the toroidal current density $\vec{j} = j_\phi \vec{e}_\phi$ satisfies the following relation (see [1] and the references quoted therein)

$$\frac{dS}{dt} = \int_{\Delta V} \frac{\vec{E}}{\mu^2}(\Delta\vec{j} + \mu^2\vec{j})dV + \int_{\Delta V} p_A dV = 0 \qquad (1)$$

Here Δ is the Laplacian, $\vec{E} = E\vec{e}_\phi$ with $E = E_0 R_0 / R$ is the externally induced electric field, p_A is the auxiliary power density and μ^2 is a parameter which arises as a Lagrangian multiplier related to the constraint 2) and which labels the family of states with constant entropy. We require that S is stationary locally so that ΔV is any sector of the plasma column delimited by two magnetic surfaces. It follows that j_ϕ must satisfy the equation

$$\Delta j_\phi + (\mu^2 - 1/R^2) j_\phi = -\mu^2 p_A / E \qquad (2)$$

This equation determines the poloidal magnetic configuration in terms of the auxiliary power density. Hollow current density profiles and negative magnetic shear are generated depending on $p_A(r)$ when μ^2 is taken as negative in a region adjacent to the minor axis. Since μ^2 must be positive near the border in order to satisfy the boundary conditions there, a point $r = \xi$

must exist where μ^2 changes its sign. The point ξ is fixed by matching continuously the solution and its derivative.

The pressure and the profile of the toroidal field follow from the compatibility between the stationarity condition (1) and the Grad-Shafranov-Schlüter equation

$$4\pi j_\phi / cR = \nabla \bullet R^{-2}\nabla\psi = -\left[\left(F^2(\psi)\right)^{'} / 2R^2 + 4\pi p^{'}(\psi)\right] \tag{3}$$

Applying the first equality (3), Eq. (1) can be reduced to the following condition:

$$\int_{S(\psi)} (E\nabla j_\phi - j_\phi \nabla E) \bullet d\vec{S} + (\mu^2 E_0 R_0 c / 4\pi) \int_{S(\psi)} R^{-2}\nabla\psi \bullet d\vec{S} + \mu^2 \int_{V(\psi)} p_A dV = C \tag{4}$$

where $S(\psi)$ is any surface with constant poloidal flux ψ enclosing the volume $V(\psi)$ and C is a constant. Applying the second equality (3) one obtains from (4) the relation (in the cylindrical limit with circular magnetic surfaces)

$$8\pi R_0^2 p''(\psi) + \left(F^2(\psi)\right)^{''} = 2\mu^2 + (4\pi\mu^2 / E_0 I(\psi)) \int_0^{r(\psi)} p_A r dr - C / \pi E_0 R_0 I(\psi) \tag{5}$$

where $I(\psi)$ is the current flowing inside the surface $S(\psi)$, $I(\psi) = cr(d\psi / dr) / 2R_0$. To satisfy (5) we put

$$p''(\psi) = \varepsilon(\mu^2 / 2E_0 R_0^2 I(\psi)) \int_0^{r(\psi)} p_A r dr \tag{6}$$

$$(F^2(\psi))'' = 2\mu^2 + (1-\varepsilon)(4\pi\mu^2 / E_0 I(\psi)) \int_0^{r(\psi)} p_A r dr - C / \pi E_0 R_0 I(\psi)$$

Thus the indetermination of the toroidal equilibrium due to two arbitrary functions $p(\psi), F(\psi)$ has been reduced to that of the free parameters ε and μ which label a family of states with constant entropy and different paramagnetism and pressure gradient. Acceptable states with positive pressure profiles are obtained with positive or negative values of the parameter $|\varepsilon| << 1$.

3. Condition on Thermal Diffusivity

The realization of magnetic states described by Eq. (2) implies a condition on the thermal diffusivity in the one-fluid approximation of the power balance. In a cylinder the balance is described by the equation $\frac{1}{r}\frac{d}{dr}(rn\chi\frac{dT}{dr}) = Ej_\phi + p_A$. In the same approximation Eq. (2) takes the form $-\frac{E}{\mu^2}\frac{1}{r}\frac{d}{dr}(r\frac{dj_\phi}{dr}) = Ej_\phi + p_A$. It follows by comparison that

$$\chi = -\frac{E}{\mu^2}\frac{dj_\phi / dr}{dp / dr} - \frac{C_0}{r dp / dr} \tag{7}$$

where $p = nT$ is the pressure and n has been taken as uniform for simplicity. The constant $C_0 > 0$ is determined by continuity across $r = \xi$. Thus the stationary entropy current density profile which *must* guarantee on inertial time scales the establishment of mechanical force balance (Grad-Shafranov-Schlüter equilibrium) in auxiliarly heated discharges has a definite relation with the energy transport equation, thereby constraining the *profile* of the thermal diffusivity coefficient χ to a self-consistent radial dependence whatever may be its scaling,

$\chi = \frac{T}{B} F(\rho^*, \nu^*, \beta)$ with the typical transport dimensionless parameters ρ^*, ν^*, β . This agrees with the profile consistency concept proposed by Coppi in 1979 [2] for ohmic discharges, which is now seen from the point of view of equilibria with stationary magnetic entropy.

The expression above for χ implies that the zero shear regions have a minimum heat diffusivity. Reverse shear configurations with an off-axis maximum of j_ϕ have therefore a localized minimum of χ , akin to so called "thermal barriers" but related to the position of the maximum of j_ϕ or of the minimum of q, rather than to the rational q surfaces as suggested recently [3]. In the case of sharply localized power deposition such as ECR the "barrier" appears therefore essentially related to the position of the heating power. Consequently, in such conditions there is no need to invoke sudden special modifications of the microscopic properties of the medium at the rational q surfaces, to explain apparent heat flow barriers. In conclusion it has been shown that negative shear equilibria may exist as steady state optimal profiles for an open system with stationary entropy conditions, which constitute a self-organization criterion: the self consistency of transport and equilibrium imply profile modifications of the heat conductivity similar to "transport barriers".

References

[1] E. Minardi: J. Plasma Phys. **48** (1992) 281.

[2] B. Coppi: "Plasma thermal energy transport: theory and experiment" in *Proceedings of Symposium on Physics of Plasmas close to Thermonuclear Conditions*, Sept. 1979, Varenna, Italy p.479, EUR FU BRU/XII/476/80.

[3] M. de Baar et al.: *Proc. 24th EPS Conf. on Contr. Fusion Plasma Phys.*, Berchtesgaden 9th-13th June 1997, Part I, p.585

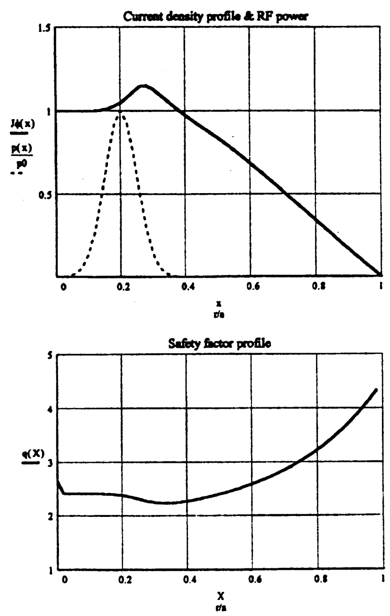
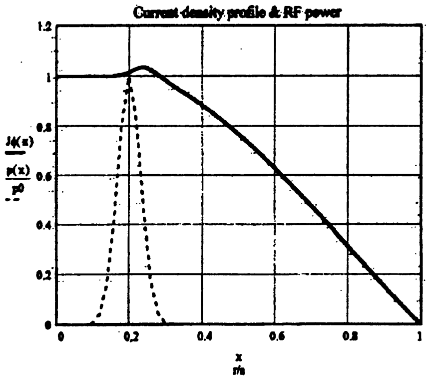


Fig. 1. Demonstration case of RFS equilibrium *J* (upper) and *q* (lower) profiles with $P_A/P_\Omega=6$, *Ip=0.5MA*



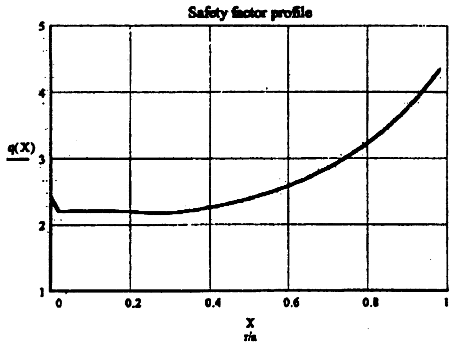
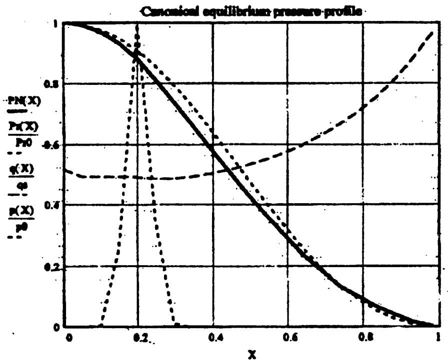


Fig. 2a. P_{Ω}*=700kW,* P_A*=2MW; Ip=0.5 MA* deposition width σ^2*=0.002; J(r)* and $P_A(r)$.

Fig. 2b. Safety factor *q* profile with reverse shear.



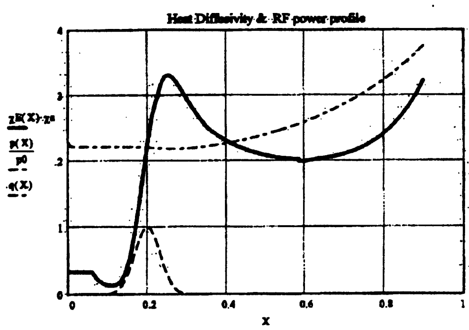


Fig. 2c. Canonical equilibrium pressure profile

Fig. 2d. Self-consistent thermal diffusivity profile.

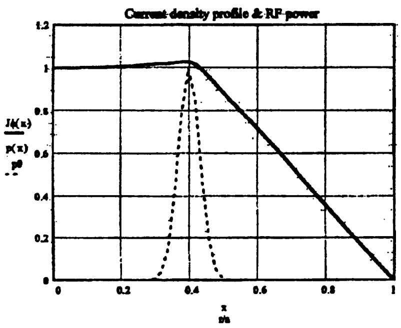
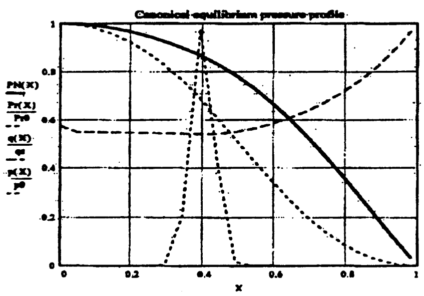


Fig. 3a. As Fig. 2a with P_A*=1.5 MW* peaked at *r/a=0.4* .



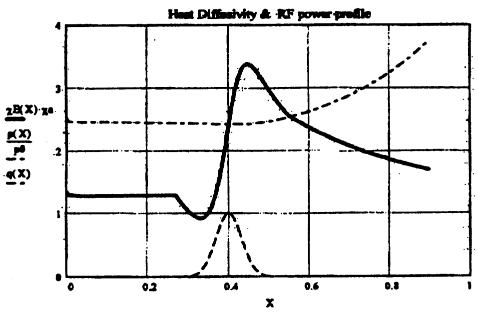


Fig. 3b. Canonical equilibrium pressure profile.

Fig. 3c. Self-consistent thermal diffusivity profile.