THEORY AND SIMULATION OF TURBULENCE IN TOROIDAL MAGNETIZED PLASMAS I

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A number of toroidal plasma devices with a purely toroidal external magnetic field has been constructed during the last two decades to investigate basic plasma phenomena. The profiles of these discharge plasmas are determined by the location of the hot cathode. While this is situated close to the center of the cross-section in the Blamann device at Tromsø, Norway [1], resulting in nearly circular equi-potential and density contours, it has often been placed on the inner side of the cross-section in the BETA device at Gandhinagar, India [2], giving slab-like equilibrium profiles.

In this paper we present a model describing low-frequency, low-β, electrostatic purely toroidal magnetized plasmas. The fully nonlinear model is reduced in the local approximation using the drift ordering. The resulting equations are solved numerically with periodic boundaries, and the results are compared with experiments, showing agreement with the spectra measured in slab-like equilibria in the BETA device.

Starting from the two-fluid equations, assuming quasi-neutrality, two-dimensional cold ion motion, and neglecting electron mass in the **B**-perpendicular dynamics but retaining electron-ion collisions along the magnetic field, we get the continuity equation for isothermal electrons

$$\frac{d\ln n}{dt} + \frac{2T_e}{eBR}\widehat{\mathbf{z}}\cdot\nabla\left(\ln n - \frac{e\phi}{T_e}\right) = \frac{T_e m_i}{e^2\eta}{\nabla_\parallel}^2\left(\ln n - \frac{e\phi}{T_e}\right), \qquad (1)$$

where last term on the left hand side is due to the compressible $\mathbf{E}\times\mathbf{B}$ and pressure drifts in the toroidal magnetic field, and z is the major torus axis. The quasi-neutrality condition can be written as

$$(\nabla\ln n + \nabla)\cdot\left(\frac{m_i}{eB^2}\frac{d\nabla_\perp\phi}{dt}\right) + \frac{2T_e}{eBR}\widehat{\mathbf{z}}\cdot\nabla\ln n = \frac{T_e m_i}{e^2\eta}{\nabla_\parallel}^2\left(\ln n - \frac{e\phi}{T_e}\right), \qquad (2)$$

where the last term on the left hand side comes from the pressure driven current and the rest is the divergence of the ion inertia current. The right hand side of the above model describes the compressional **B**-parallel electron motion. We note that this model is scale invariant in density. These fully nonlinear equations are solved numerically in an accompanying paper (see J.-V. Paulsen *et al.*) giving a self-consistent formation of equilibrium profiles. Here we will be concerned with the local features of the fluctuations. Thus, applying the local approximation by dividing the density into an inhomogeneous equilibrium part n_0 and one fluctuating part $\widetilde{n}$, and using the drift ordering, we obtain a generalization of the Hasegawa-Wakatani equations for a toroidal magnetic field including ion-neutral collisions,

$$\frac{\partial n}{\partial t} + \{\varphi, n\} + (1-\varepsilon)\frac{\partial \varphi}{\partial y} + \varepsilon\frac{\partial n}{\partial y} = \nabla_{\parallel}{}^2 \, (n - \varphi) \, , \tag{3}$$

$$\frac{\partial \Omega}{\partial t} + \{\varphi, \Omega\} + \varepsilon\frac{\partial n}{\partial y} + \nu\Omega = \nabla_{\parallel}{}^2 \, (n - \varphi) \, , \tag{4}$$

where $\Omega = \nabla_{\perp}{}^2\varphi$ is the vorticity and we have used the gyro-Bohm scaling. The magnetic field is along the z axis, y is the poloidal direction and x is the radial direction in this local system. Here $\varepsilon = 2L_{\perp}/R$, where $L_{\perp}$ is the background density scale length and R is the magnetic field of curvature radius.

The linear dispersion relation for flute perturbations in a fully ionized plasma is given by

$$\frac{\omega}{\omega_{\rm fm}}\left(\frac{\omega}{\omega_{\rm fm}} - 1\right) + \frac{1}{\alpha} = 0, \tag{5}$$

where $\alpha = \varepsilon k_{\perp}^2/(1 - \varepsilon)$ and $\omega_{\rm fm} = \varepsilon k_y$. The dispersion relation and linear phase difference $\theta_{\mathbf{k}}$ between density and potential fluctuations, defined by $n_{\mathbf{k}} = |n_{\mathbf{k}}/\varphi_{\mathbf{k}}|\varphi_{\mathbf{k}} \exp \theta_{\mathbf{k}}$, are shown if Fig. 1. The positive branch is unstable for $0 < \alpha < 4$, with a phase shift up to $3\pi/2$, indicating large anomalous transport. Ion-neutral collisions destabilize all waves with $\alpha > 4$, while it can be shown that finite ion temperature effects stabilize all waves α larger than some critical value always less than 4.

The flute model is solved numerically using a spectral collocation method, implying periodic boundary conditions. In the absence of ion-neutral collisions we reproduce the simulation of Das *et al.* [3] where the model nonlinearly saturates with a shear flow in the poloidal direction. Hence, the spectrum saturates at $k_y = 0$ and the turbulent flux due to the background density gradient vanishes. For finite ion-neutral collision frequency, however, we get a turbulent state. Fig. 2 shows the evolution of the total energy $E = (1/2)\int d\mathbf{x}\,[n^2 + (\nabla_{\perp}\varphi)^2]$. We see that this has exponential growth until the nonlinear terms stops the growth at $t \sim 200$. The plasma then becomes turbulent, while the energy stays constant when averaged over the turbulent time scale. The anomalous flux $\Gamma = -\int d\mathbf{x}\, n\partial_y\varphi$ also shown in Fig. 2 is large in the turbulent phase, and balanced by collisional dissipation.

In Fig. 3 we show the electric field and density power spectra for the same simulation as shown in Fig. 2, averaged from $t = 200$ to $t = 600$. The inertial

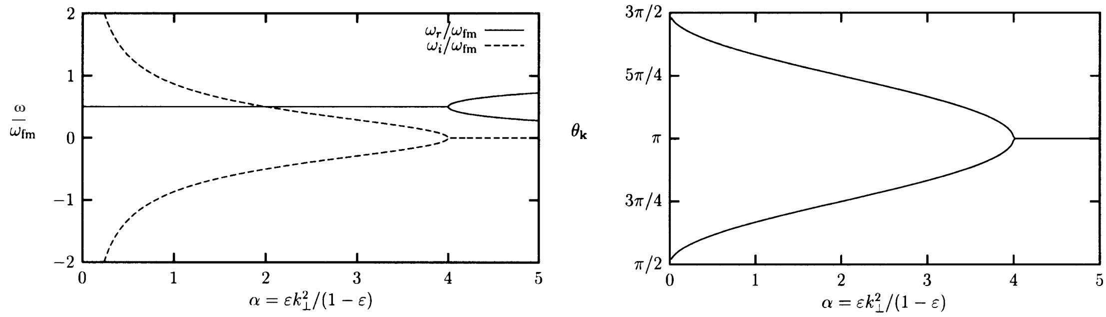

Figure 1: The linear dispersion relation (left) and phase difference between density and potential (right) for flute perturbations in a curved magnetic field. For the multiple valued curves the upper part corresponds to the positive branch which is unstable wave, and the lower part corresponds to the negative branch which is damped.

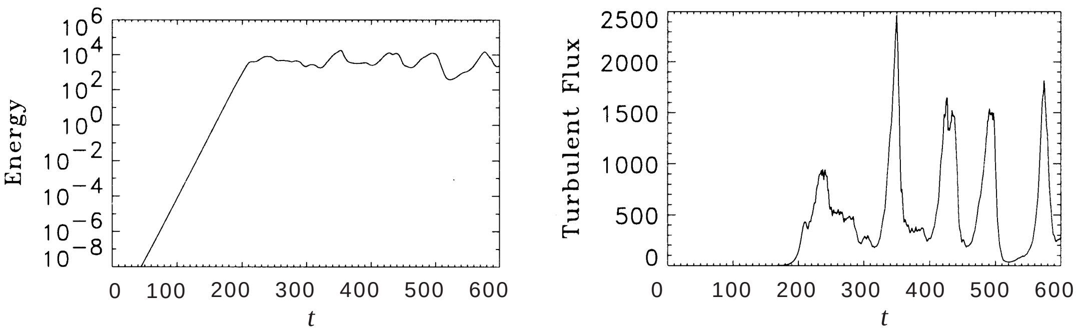

Figure 2: Time history for the energy E to the left and the turbulent flux Γ to the right for a simulation with ε = 0.1, ν =1 and spatial resolution of 256 x 256. The initial exponential growth is saturated after t ~200, where the turbulent flux is balanced by dissipation.

range spectral index for the electric field is close to 2.7, and 2.3 for the density which is very close to the spectra observed in the BETA device with slab like equilibria [2], as well as spectra observed in equatorial spread F [4]. The much higher power in the density fluctuations is also consistent with experiments on BETA. However, the spectra observed in the BETA device and equatorial spread F both show a break in the density spectrum at $k_\perp$ = 1.5, which is not seen to appear in the simulations. The common belief is that this is due to a secondary drift wave instability growing on the density irregularities caused by the interchange instability. Another possibility is that the break is due to finite Larmor radius effects. This is supported by linear theory, where finite ion temperature effects have been included consistently with the drift approximation. Simulation results including these effects are now in progress, and will be presented in a forthcoming paper, accompanied by with a detailed parameter study.

Despite the success of the local simulations in reproducing the experimen-

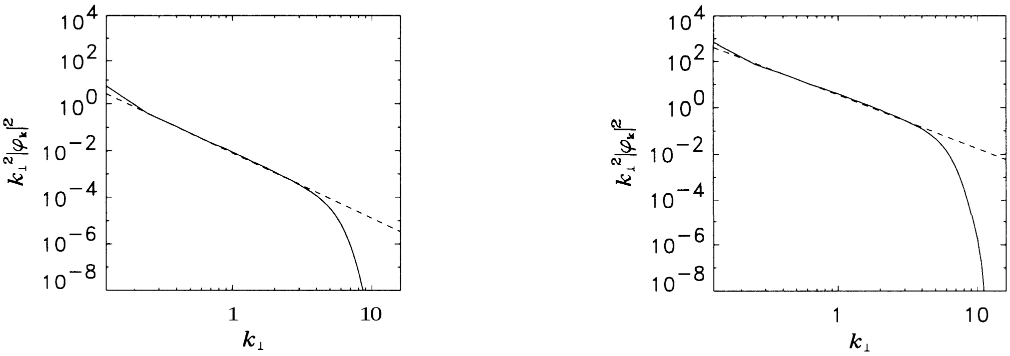

Figure 3: The electric field (left) and density (right) power spectrum for the same simulation as in Fig 2. In the inertial range the spectral index for the electric field is close to 2.7 (broken line), while for the density spectrum this is close to 2.3.

tally observed power spectra, there are several factors that motivate simulation of the fully nonlinear problem. First of all, the fluctuation amplitudes in the local simulations are very high, up to 200 in the gyro Bohm scaling. Thus, the drift ordering is strictly not valid and all nonlinearities must be retained. Also, the adoption of a locally rectangular coordinate system is heuristic. Of more fundamental interest is the possibility to implement realistic sources and sinks in the model. This calls for a global simulation with finite boundaries. Such a code has been constructed [5] and some results are presented in an accompanying paper (see Paulsen et *al.*). Finally, it should be noted that these global simulations show that potential fluctuations are larger than the density fluctuations, which is in agreement with experiments where the cathode is positioned at the center of the poloidal cross-section [2, 6].

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