PLASMA HEATING BY REGULAR ELECTROMAGNETIC WAVE DUE TO DYNAMICAL CHAOS ARISING UNDER WAVE-PARTICLE TYPE OF INTERACTION

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This report is devoted to theoretical research of electrons heating in the low density unbounded plasma on account of dynamical chaos arising under interaction of charged particles with high frequency electromagnetic fields in the presence of uniform external magnetic field. In this case stochastic instability of particle motion is developed due to overlapping of nonlinear cyclotron resonance's. We have demonstrated a capability of fast heating of electrons with low initial energies on account of dynamical chaos arising.

We consider the motion of a charged particle in a constant, externally applied magnetic field $\vec{H} = \{0,0,H_o\}$ and in the field of an electromagnetic plane wave. In term of the dimensionless variables ($t \to \omega t, \vec{r} \to \vec{r}\omega / c, \vec{p} \to \vec{p} / mc, \vec{k} \to \vec{k}c / \omega$) the equations of particle motion can be reduced to the form:

$$\dot{\vec{p}} = (1 - \vec{k}\vec{p} / \gamma)\operatorname{Re}(\vec{E}e^{i\psi}) + (\omega_h / \gamma)[\vec{p}\vec{h}] + \vec{k} / \gamma \operatorname{Re}\{(\vec{p}\vec{E})e^{i\psi}\}, \ \dot{\vec{r}} = \dot{\vec{p}} / \gamma, \ \dot{\psi} = \vec{k}\vec{p} / \gamma - 1, \quad (1)$$

where $\vec{h} = \vec{H} / H_o, \omega_h = eH_o / mc\omega, \vec{E} = eE_o\vec{\alpha} / mc\omega$, $\psi = \vec{k}\vec{r} - t$, $\gamma = (1 + \vec{p}^2)^{1/2}$ - the particle energy and $\vec{p}$ - its momentum. The set of Eqs.(1) has the integral of motion:

$$\vec{p} - \operatorname{Re}(i\vec{E}e^{i\psi}) + \omega_h[\vec{r},\vec{h}] - \vec{k}\gamma = const. \quad (2)$$

For subsequent analysis, it is convenient to transform to new variables $p_\perp, p_z, \theta, \xi, \eta$ defined by $p_x = p_\perp \cos\theta, p_y = p_\perp \sin\theta, p_z = p_z, x = \xi - p_\perp / \omega_h \sin\theta, y = \eta + p_\perp / \omega_h \cos\theta$. Suppose that the amplitude of the electromagnetic field is sufficiently small and taken into account that the particle will interact efficiently with the wave if it fulfills one of the resonance conditions:

$$k_z p_z + s\omega_h - \gamma = 0, \qquad s = \ldots,-2,-1,0,1,2,\ldots \quad (3)$$

after averaging over the fast time scale one can find following equations, which describe particle motion in the case of an isolated resonance:

$$\dot{p}_\perp = \frac{1 - k_z v_z}{p_\perp} W_s \varepsilon_o \cos(\theta_s) \qquad \dot{\gamma} = \frac{1}{\gamma} W_s \varepsilon_o \cos(\theta_s), \qquad \dot{\theta}_s = k_z v_z - 1 + s\frac{\omega_h}{\gamma}, \quad (4)$$

where $W_s = \dfrac{\alpha_x p_\perp s}{\mu} J_s(\mu) - \alpha_y p_\perp J_s'(\mu) + \alpha_z p_z J_s(\mu)$, $\mu = k_x p_\perp / \omega_h$, $J_s(\mu)$ - the Bessel function, $J_s'(\mu)$ - its derivative on the argument, $\vec{\alpha} = \{\alpha_x, \alpha_y, \alpha_z\}$ - polarization vector. Let`s suppose, that during the particle interaction with the wave its energy varies a little, i.e. $\gamma = \gamma_{0s} + \tilde{\gamma}_s, \tilde{\gamma}_s << \gamma_{0s}$, where γ_{0s} is met the resonant condition (3). In view of the approximating integral of movement:

$$p_z - k_z \gamma = a = const, \tag{5}$$

which can be obtained from (2), one can get a closed set of equations for $\dot{\theta}_s$ and $\tilde{\gamma}_s$ which are the equations of mathematical pendulum, find the width of the nonlinear isolated resonance and the separation between adjacent resonances. The generalized Chirikov's criterion for development of the local instability of particles motion in this case is [1],[2]:

$$\varepsilon_o > \frac{\omega_h^2}{16 W_s (1 - k_z^2)}, \tag{6}$$

In the energy-momentum space $\{\gamma, p_\perp > 0, p_z\}$ the charged particle can move only on the surface:

$$\gamma^2 = 1 + p_\perp^2 + p_z^2, \tag{7}$$

which is a hyperboloid of rotation. The effective particle interaction with waves determined by resonant conditions (3), which represents a section of hyperboloid (7) by the plane (3). The integral (5), along which the particle moves, also is a section of hyperboloid (7) by the plane (5). As it is known, the section of hyperboloid by plane can be only a hyperbola, parabola or ellipse. If the particle moves in vacuum, then $k_z = \cos(\phi) \le 1$, where ϕ - angle between external magnetic field and waves vector $\vec{k}$. In this case the resonant conditions are closed curves (in common case ellipses), except for the case when $k_z^2 = 1$ (autoresonance). When the particle moves in a dielectric, k_z can be more than 1. In this case the resonant conditions are non-closed lines (in common case hyperbolas). The integral of the motion (5), when $k_z^2 \le 1$ is a non-closed line (in common case hyperbolas). This means that it cannot limited the energy gain by the particle under its stochastic heating. When $k_z^2 > 1$, the integral (5) became a closed line (in common case ellipses). It leads to the limiting the maximum energy, which the particle can obtained. From the other side, the closed integral of the motion can allow us to control the distribution function of the charged particles on the energy. Indeed, in the conditions of the well-developed stochastic instability the diffusion on the energy of charged particles takes place. Particles are distributed uniformly in borders of the available energies, which are determined by the integral (5) and hiperboloid (7). Changing k_z , we can obtain the various average energy of particles and various width of the functions of their distribution on energies. Computer simulation confirm this statement.

At $k_z^2 < 1$ the approximated integral (5) does not limited the energizing of the particle in conditions of the well-developed dynamical chaos. But with the growth of the particle energy and its diffusion into the region of higher energies, the condition (6) can be broken. Let's find such angles ϕ between external magnetic field and wave vector $\vec{k}$, under which the width of the nonlinear resonance, with the growth of the particle energy, stay more, than distance between neighbour resonances.

Assuming that the resonance number s, in which the particle is situated well over than the resonance number n from which this particle began to move stochastically, from (3), (5) we can find:

$$p_{zs} = (a(n) + k_z s\omega_h)/(1-k_z^2), \quad p_{\perp s}^2 = (s^2\omega_h^2 - a^2(n))/(1-k_z^2) - 1, \tag{8}$$

where the constant $a(n)$ is defined by the resonance conditions (3) and the integral (5) for resonance with number n. Using (8), and that $s >> n$, it is possible to give the asymptotic evaluation of the non-linear resonance width :

$$\Delta\tilde{\gamma}_s = 4\sqrt{\frac{\varepsilon_o\omega_h s^{1/3}}{k_x(1-k_z^2)}\{(\alpha_x + a_z k_z / k_x)c_o s^{1/3} - a_y c_1\}}, \; c_0 \approx 0.447, \; c_1 \approx 0.411. \tag{9}$$

From the criterion (6), for wave, with linear ($\vec{\alpha} = \{0, i, 0\}$, $\vec{k} = \{\sin(\phi), 0, \cos(\phi)\}$) or circular ($\vec{\alpha} = \{\cos(\phi), 1, -\sin(\phi)\}$, $\vec{k} = \{\sin(\phi), 0, \cos(\phi)\}$) polarization we have the following condition for angle ϕ , under which the width of the nonlinear resonances, stay more, than distance between neighbour resonances:

$$16\varepsilon_o c_1 s^{1/3}\sin(\phi)/\omega_h > 1, \tag{10}.$$

It is visible from (10), that under the propagation of electromagnetic wave perpendicularly to the external magnetic field, with the growth of particle energy the condition of resonance overlapping (6) is fulfilled better and better ($\varepsilon_o > s^{-1/3}$). So let's consider the movement of particle in a field of the plane wave with polarization $\vec{\alpha} = \{0, i, 0\}$, which is propagated perpendicularly to an external magnetic field. From (1), for movement in isolated resonance, one can obtain:

$$\dot{p}_\perp = -\varepsilon_o J_s' \cos(\theta_s), \; p_z = const, \dot{\theta}_s = s\frac{\omega_h}{\gamma} - 1 + \frac{\varepsilon_o}{\omega_h}\{\frac{s^2\omega_h^2}{p_\perp^2} - 1\}J_s\sin(\theta_s), \tag{11}$$

The set of equations (11) has an integral:

$$H = \frac{s\gamma}{\omega_h} - \frac{\mu^2}{2} + \frac{\varepsilon_o}{\omega_h}\mu J_s' \sin(\theta_s)\,, \tag{12}$$

The changing of wave amplitude or gyrofrequency leads to the qualitative changing of the phase portrait of the system (11) for low energy particles. This connected with the right part of equation for phase θ_1 (11), which includes addend $\propto \varepsilon_o / p_\perp$. This leads to changing the area of energies, into which particles move (we assume that the initial energy is $\gamma(t=0) \approx 1$, $\omega_h \approx 1$). So, if at $\varepsilon_0 = 0.05$ particles with $\gamma(t=0) \approx 1$ can lie in area $\gamma < 1.05$, at $\varepsilon_0 = 0.093$, particles, moving in conditions of the isolated resonance can achieve energies $\gamma_{\max} \propto 1.5$. The value $\gamma_{\max} \propto 1.5$ is easy to find from (12). At $\varepsilon_o > 0.15$ the dynamics of particles is chaotic. In conditions of the developed dynamical chaos there is a diffusion of particles to the area of high energies. The diffusion coefficient was found in [1]. For our case it can be represented in a form:

$$D \approx \left(\frac{\varepsilon_o^{4/3}}{2 + \varepsilon_o^{2/3}}\right)^2 . \tag{13}$$

For the demonstration of the diffusion character of particles movement in conditions of devloped stochastic instability of particle motion and a possibility of heating thermal electrons of low density plasma by regular electromagnetic fields, for 1000 particles, which are evenly distributed on the wave length, it was solved the equation (2). The initial energy of all particles are $\gamma = 1.00001$ It was found that the diffusion coefficient (13) in a good agreement with results of numerical simulation and that at $\varepsilon_0 = 0.2$ at 100 periods of HF wave, the average energy of particles $<\gamma>$ grows on 3 times from $<\gamma>\approx 1$ to $<\gamma>\approx 3$, at $\varepsilon_0 = 0.3$ on 5 times, and at $\varepsilon_0 = 0.5$ on 7 times.

Acknowledgements

The authors would like to thank Prof. K.N.Stepanov for his interest in this work and for useful discussions. This work was supported by STCU, grant No 253.

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