BALLOONING MODES INSTABILITY IN THE SPACE PLASMAS

O.S. Burdo*, O.K. Cheremnykh* and O.P. Verkhoglyadova**

* *Space Research Institute NASU & NSAU, Kiev, Ukraine*
** *Taras Shevchenko Kiev University, Kiev, Ukraine*

Introduction

Ballooning and flute modes have been considered in a number of papers in association with the geomagnetic pulsations and motions of the magnetic surfaces in the plasmasphere of the Earth (see, for example, [1], [2], [3]). Their stability examination received much attention in connection with the substorm activity and its consequences. The papers differ significantly in approach and theoretical methods applied. We think that use of the well-known set of ballooning mode equations obtained naturally for finite beta plasmas in general magnetic field geometries [4], [5] gives a possibility to solve the problem and to study the stability regions in a general way. The proposed study is an attempt to apply the set of equations to the Earth magnetic field.

Model

The study is made for dipole magnetic field (see Fig. 1). Magnetic shell displacements in the meridian plane are considered.

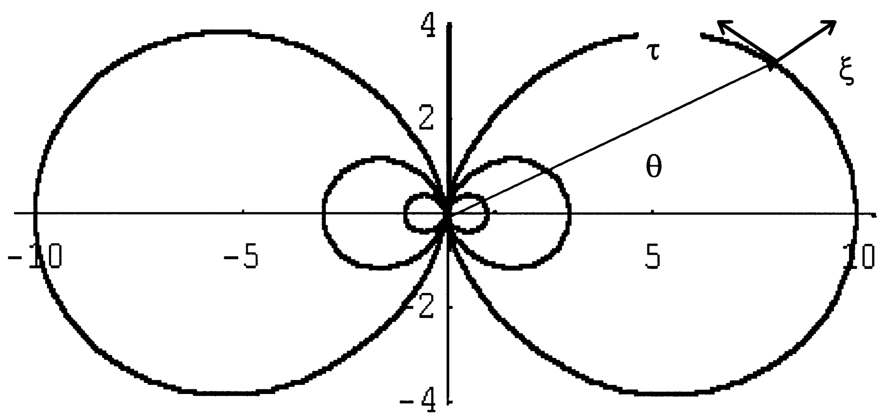

Figure 1. Sketch of the field geometry.

Here, τ and ξ are field line displacements along the undisturbed field and transversely to the fieldline whose equation is $r = L\cos^2\theta$, where r is the radius (in the Earth's radii) and θ

is polar angle in right-handed spherical co-ordinate system, L is the equatorial magnetic shell distance. The respective undisturbed field components are

$$\begin{cases} B_\theta = -\dfrac{M\cos\theta}{r^3} \\ B_r = \dfrac{2M\sin\theta}{r^3} \end{cases}$$

and the absolute value is $B = \dfrac{M\sqrt{a}}{r^3}$, $a = 1+3\sin^2\theta$, where M - is the Earth dipole moment.

Main equations

We start from the basic set of equations for ballooning-type displacements in the well-known form [5]:

$$\rho\omega^2\frac{k_\perp^2}{B^2k_{\perp\beta}^2}\hat{\xi} = -\vec{B}\nabla\left(\frac{k_\perp^2}{B^2k_{\perp\beta}^2}\vec{B}\nabla\hat{\xi}\right)+$$

$$+2\frac{\left[\vec{B}\times\vec{\kappa}\right]\vec{k}_\perp}{B^2k_{\perp\beta}}\left\{-p'\hat{\xi}+\frac{\gamma pB^2}{\gamma p+B^2}\left(2\hat{\xi}\frac{\left[\vec{B}\times\vec{\kappa}\right]\vec{k}_\perp}{B^2k_{\perp\beta}}-div\,\hat{\tau}\frac{\vec{B}}{B^2}\right)\right\},$$

$$\rho\omega^2\hat{\tau} = \vec{B}\nabla\left\{\frac{\gamma pB^2}{\gamma p+B^2}\left(2\hat{\xi}\frac{\left[\vec{B}\times\vec{\kappa}\right]\vec{k}_\perp}{B^2k_{\perp\beta}}-div\frac{\vec{B}}{B^2}\right)\right\}$$

Here we use the following notations: $\tau, \xi = \hat{\tau}, \hat{\xi}\exp\left\{-i\omega t+\frac{i}{\varepsilon}\hat{S}\right\}$, $\varepsilon \ll 1$, where $\vec{k}_\perp = \nabla\hat{S}$, $\vec{k}_\perp\vec{B} = 0$, $p' = \dfrac{\partial p}{\partial\psi}$.

In the dipole field we introduce the magnetic flux function $\psi = -\dfrac{M\cos^2\theta}{r}$, $k_{\perp\beta} = \dfrac{\left[\vec{B}\times\nabla\psi\right]}{B^2}\vec{k}_\perp = k_\varphi r\cos\theta$, the azimuthal wave number k_φ, and the magnetic field curvature $\vec{\kappa}$, where $\left[\vec{B}\times\vec{\kappa}\right] = -\vec{e}_\varphi\dfrac{2M\cos\theta}{r^4a}$.

Introducing the dimensionless variables according to [6]:

$B = \dfrac{B_0\sqrt{a}}{\cos^6\theta}$, $\beta = \dfrac{\gamma p}{B_0^2}$, $B_0 = \dfrac{M}{L^3}$, $\Omega = \dfrac{\omega L\sqrt{\rho}}{B_0}$, where B_0 - is the dipole magnetic field intensity at the equator, we obtain:

$$\Omega^2\hat{\tau} + +\frac{1}{r^3}\hat{M}F = 0,$$

$$\Omega^2 \hat{\xi} \frac{k_\perp^2}{k_\varphi^2} + \frac{a\cos^2\theta}{r^7} \hat{M}\left(\frac{k_\perp^2 r}{a k_\varphi^2 \cos^2\theta} \hat{M}\hat{\xi} \right) - \frac{4\cos^2\theta}{a r^3}\left(p'\hat{\xi} + F\right) = 0,$$

$$F = \frac{\beta B^2}{\beta + B^2}\left(\frac{r^3}{a}\hat{M}\hat{\tau} + \frac{2r^2\sin\theta}{a^2}\left[4+5a\right]\hat{\tau} + \frac{4\hat{\xi}r}{a^2} \right)$$

Using the expression for fieldline we rewrite the derivative along the fixed magnetic shell:

$$\hat{M} \equiv r^3\left(\vec{B}\nabla\right) = r^3\frac{\partial B}{\partial s} = \frac{1}{\cos\theta}\frac{\partial}{\partial\theta},$$

where the length element along the fieldline is $ds = \cos\theta\sqrt{a}\,d\theta$.

Finally, we obtain the following set of equations which describe the ballooning-type displacements of a fieldline with specified L:

$$\Omega^2\hat{\tau} + \frac{1}{\cos^7\theta}\frac{d}{d\theta}F = 0,$$

$$\Omega^2\hat{\xi}(1+qa) + \frac{a}{\cos^{13}\theta}\left(\frac{1+qa}{a\cos\theta}\frac{d^2}{d\theta^2}\hat{\xi} + \frac{\sin\theta}{a^2\cos^2\theta}\left(3a+qa^2-8\right)\frac{d}{d\theta}\hat{\xi} \right) -$$

$$- \frac{4}{a\cos^4\theta}\left(\frac{\beta}{\gamma}\alpha\hat{\xi} + F \right) = 0,$$

$$F = \frac{\beta\alpha}{\beta\cos^{12}\theta + a}\left(\frac{\cos^5\theta}{a}\frac{d}{d\theta}\hat{\tau} + \frac{2\sin\theta\cos^4\theta}{a^2}\left[4+5a\right]\hat{\tau} + \frac{4\hat{\xi}\cos^2\theta}{a^2} \right),$$

$$\frac{k_\perp^2}{k_\varphi^2} = 1 + qa, \quad \alpha = \frac{L}{p}\frac{dp}{dL},$$

where q - is square of a ratio of the radial wave number to the azimuthal wave number and $\gamma = 5/3$ is the adiabatic index. The set of equations contains functions of L, namely α, β, Ω, and parameter q.

Flute stability criterion of the equatorial plasmapause

From the equations derived we can easily obtain the flute stability criterion. Let us consider region near the equator plane $\theta \approx 0$, and introduce the following form of displacements : $\hat{\tau}, \hat{\xi} \sim \exp(in\theta)$, $n \approx 0$. Given the pressure $p = p_0 L^{-m}$ and $\beta < 1$, we can conclude that the plasmapause is unstable for $m > 4\gamma/(\beta+1) \approx 20/3$ [1, 2, 3]. For incompressible plasma from the Suydam criterion we obtain just $m>0$.

Stability of ballooning modes in the equatorial plane ($\theta \approx 0$)

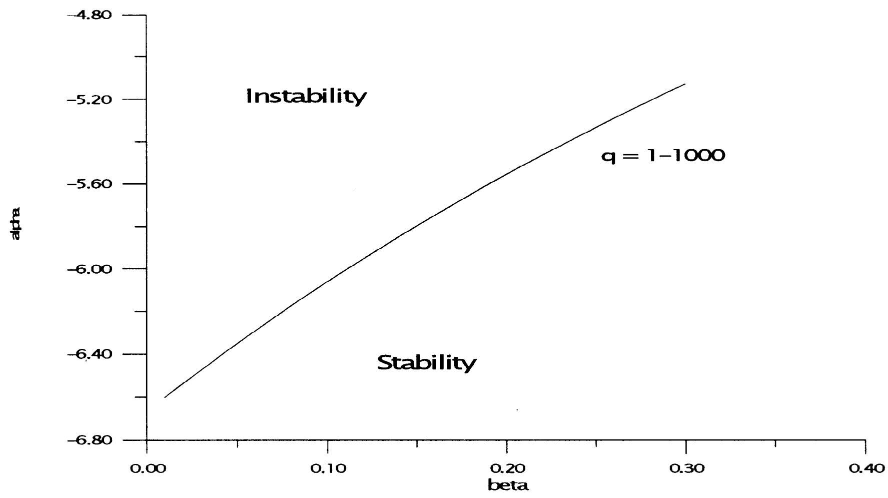

Figure 2. Stability boundary in the plane (β, α).

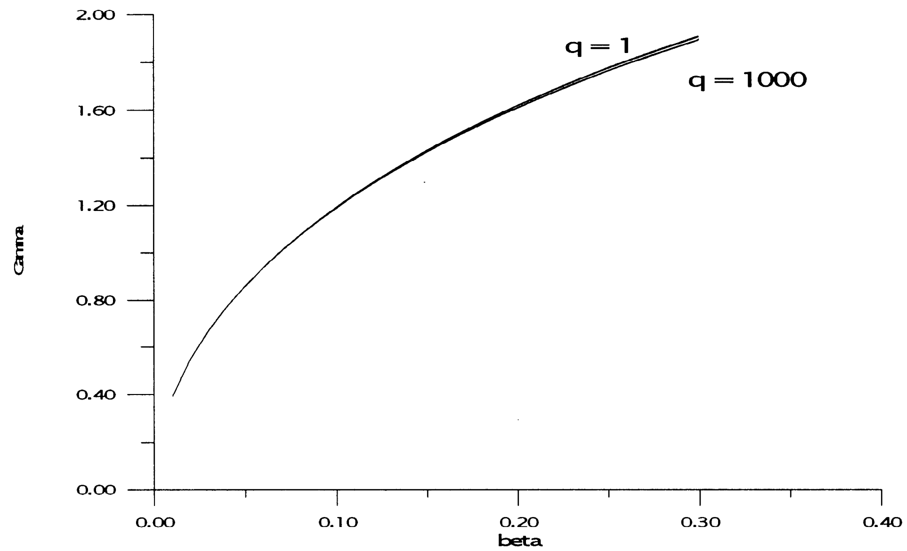

Figure 3. Growth rate $\Gamma = \sqrt{-\Omega^2}$ as function of β.

Conclusions

It is shown, that stability boundary and growth rate do not depend on wave numbers ratio (q). In the limit $\beta \to 0$ the results obtained are in a good accordance with the well-known flute stability criterion. These preliminary results are obtained for the vicinity of the equator plane.

Acknowledgements This work was supported, in part, by the National Academy of Sciences of Ukraine on the theme 0197U095786 and National Space Agency of Ukraine on the contract "Center-1".

References

[1]. Gold T., Journ. Geophys. Res., **64**, 1219 (1959). [2]. Ivanov V.N., Pokhotelov O.A., Sov. J. Plasma Phys., **13**, 1446 (1987). [3]. Lakhina G.S., Mond M., Hameiri E., Journ. Geophys. Res., **95**, 4007 (1990). [4]. Correa-Restrepo D., Z. Naturforschung, **37**, 848 (1982). [5]. Pustovitov V.D., Shafranov V.D., Reviews of Plasma Physics, **15**, 163 (1990). [6]. Cheng C.Z., Chang T.C., Lin C.A., Tsai W.H., Journ. Geophys. Res. , **98**, 11339 (1993).