STOCHASTIC INSTABILITY OF PARTICLE MOTION IN THE CIRCULAR WAVEGUIDE

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The particle motion in a magnetic field and in fields of electromagnetic waves is studied quite well at present time. The criteria of the chaotic motion are obtained [1]. Unfortunately, all these investigation were carried out for plane electromagnetic waves. In this report we study dynamics of charged particles in the fields of H-waves of a circular metal waveguide that is placed in a constant magnetic field H_0. Let's use cylindrical frame $(r,\ \varphi,\ z)$ with z-axis along waveguide. Magnetic fields H_0 is directed along an axis of a waveguide, the radius of waveguide is a. In the chosen coordinate frame the wave H_{mn} has components:

$$\begin{gathered} E_r = E_0 \frac{m}{k_\perp r} J_m(k_\perp r)\sin(m\varphi)\sin(k_z z - \omega t), \quad E_\varphi = E_0 J'_m(k_\perp r)\cos(m\varphi)\sin(k_z z - \omega t) \\ E_z = 0, \quad H_r = -\frac{k_z}{k}E_\varphi, \quad H_\varphi = \frac{k_z}{k}E_r, \quad H_z = \frac{k_\perp}{k}E_0 J_m(k_\perp r)\cos(m\varphi)\cos(k_z z - \omega t) \end{gathered} \tag{1}$$

where J_m is Bessel function of the order m; J'_m is the derivative of Bessel function; $k_\perp = \nu_{mn}/a$, ν_{mn} is the root of J'_m, the index n is the root number; $k_z = \sqrt{k^2 - k_\perp^2}$; $k = \omega/c$. Using the dimensionless variable $\omega t \to t$, $\vec r\omega/c \to \vec r$, $\vec p/Mc \to \vec p$, $k_\perp c/\omega \to k_\perp$, $k_z c/\omega \to k_z$, we obtain the equations for angle and energy of particle in cylindrical frame $(p_\perp, \Theta, p_z)$:

$$\begin{gathered} \frac{d}{dt}\gamma = \frac{\varepsilon_0 p_\perp}{2\gamma}\operatorname{Im}\sum_{s=-\infty}^{\infty} i^s J'_s(k_\perp\rho)J_{m-s}(k_\perp r_c)\exp\left[i(m-s)\varphi_c - i\Theta_s\right] \\ -\frac{\varepsilon_0 p_\perp}{2\gamma}\operatorname{Im}\sum_{s=-\infty}^{\infty} i^s J'_s(k_\perp\rho)J_{m+s}(k_\perp r_c)\exp\left[i(m+s)\varphi_c + i\Theta_s\right],\ p_z - k_z\gamma = \text{const}, \\ \frac{d}{dt}\Theta = -\frac{\omega_H}{\gamma} - \frac{k_\perp}{\gamma}\varepsilon_0\cos(k_z z - t)\operatorname{Re}\sum_{s=-\infty}^{\infty} i^s J_{m+s}(k_\perp r_c)J_s(k_\perp\rho)\exp\left[i(m+s)\varphi_c - is\Theta\right] \\ +\frac{\gamma - k_z p_z}{p_\perp\gamma}\varepsilon_0\sin(k_z z - t)\operatorname{Re}\sum_{s=-\infty}^{\infty} i^{s+1}J_{m+s}(k_\perp r_c)\frac{s}{k_\perp\rho}J_s(k_\perp\rho)\exp\left[i(m+s)\varphi_c - is\Theta\right] \end{gathered} \tag{2}$$

where $\Theta_s = k_z z - s\Theta - t$ is a phase of cyclotron resonance of number s, (r_c, φ_c, z) are coordinates of Larmor center of particle, $\rho = p_\perp/\omega_H$ is Larmor radius, $\varepsilon_0 = eE_0/Mc\omega$, $\omega_H = eH_0/Mc\omega$, M is the particle mass, $\gamma = \sqrt{1+\vec p^{\,2}}$ is the particle energy.

Let's consider a case of small amplitude of wave ($\varepsilon_0 << 1$), that practically is always satisfied. We shall study particle motion near resonance

$$(k_z p_z + s\omega_H)/\gamma - 1 \approx 0\,, \ \ s = 0, \pm 1, \pm 2, ... \qquad (3)$$

Then in the equation for energy in (2) we can leave only terms of a sum with a slow resonance phase Θ_s. Let's analyze equation for energy after averaging for wave H_{m1} ($m \neq 0$, $n = 1$). The efficiency of interaction of the particle with the wave depends from magnitude of the right side of the equation. And as the right side of the equation depends on functions $\left|J_{m-s}(k_\perp r_c)\right|$ and $\left|J_{m+s}(k_\perp r_c)\right|$, then for particles near an axis of waveguide we can to leave only the term with Bessel function of a smaller order. From the point of view of effectiveness of interaction with wave for such particles the most favorable condition is realization of one of conditions $m \pm s = 0$, i.e. axial mode number of the wave is equal to cyclotron resonance number.

Let's return to (2). Let $n = 1$ (H_{m1} – wave) and the particle is located near to an axes of waveguide. Then, after averaging on resonant phase $\tilde{\Theta}_s = k_z z - s\Theta - t + (m+s)\varphi_c$ we find a maximum perturbation of particle energy – the half-width of the cyclotron resonance

$$\Delta\gamma_s = \sqrt{2\left|\varepsilon_0 p_{\perp 0} J_{m+s}(k_\perp r_c) J_s'\left(k_\perp \rho_0\right)\right| / k_\perp^2} \qquad (4)$$

where $p_{\perp 0}$, ρ_0 satisfy the condition of a resonance (3). Deriving (4) we have neglected by terms of the order ε_0 in equation for angle Θ of system (2), as the perturbation of γ is order of $\sqrt{\varepsilon_0}$. However, it should be noted, that it is fair only in a case, when $p_\perp$ is far from zero.

The half-width of resonance (4) is similar to that for the case of particle motion in a field of plane electromagnetic wave, propagating perpendicularly to a constant magnetic field [8]. The difference is in the multiplier $\sqrt{0.5\, J_{m+s}(k_\perp r_c)}$,i.e. the half-width of resonance depends from particle position with respect to the axis of waveguide and from relation of axial mode number with cyclotron resonance number. So, for effective interaction of particle with the wave we need to choose the appropriate mode of wave in the waveguide.

Nonrelativistic particle

In this case the right side of equation for angle Θ of system (2) have singularity $p_\perp^{-1}$ which leads to the essential modification of particle dynamic. To illustrate this we have performed computer simulation for the case of electron motion in the field of H_{11}–wave and for the following values of parameters: $a = 8\ cm$, $\omega/2\pi = 2.8\ GHz$, $E_0 \approx 24\ KV/cm$, $H_0 = 1\ KG$, $k_z z_0 = \pi/4$, $p_z = 0$, $r_c = 0$,. The phase trajectories on a plane $\left(\Theta_s \equiv k_z z - s\Theta - t\ ,\ \gamma\right)$ for

$s=-1$ (first cyclotron resonance) are shown in Fig. 1. The points of trajectories are depicted in time $t_j = 2\pi j$, $j = 0, 1, \ldots$, that is equivalent to Poincare map. The maximal energy $\gamma \approx 1.6$ electron gains during the time $t \approx 30\pi$. Such behavior of nonrelativistic electrons can be used for plasma heating. It is necessary only to prevent regular phase oscillations of electrons, i.e. in the average, the electrons must acquire energy. This can be reached by transition into stochastic regime of the motion of electrons.

Stochastic motion.

Let's discuss now possibility of chaotic motion of the electron due to overlapping of cyclotron resonances. The neighboring cyclotron resonances are overlapped at $\Delta\gamma_s + \Delta\gamma_{s+1} \geq \omega_H / k_\perp^2$. Let the conditions of optimal resonance interaction are satisfied: $m+s=0$, $r_c \approx 0$. Then the width of the nearest resonance with number $s \pm 1$ is in $\sqrt{|J_1(k_\perp r_c)|}$ time less then the width of the resonance s, i.e. the width of adjacent resonances decreases rapidly. Besides for small $k_\perp$ the distance between resonances is increased faster ($k_\perp \leq 1$), than their width. Apparently, it is difficult to reach overlapping of cyclotron resonances for the purposes of stochastization of electron motion – the large amplitude of wave is required. Really, numerical simulation for H_{11}– wave and parameters: $a = 8\ cm$, $\omega/2\pi = 2.8\ GHz$, $H_0 = 1\ KG$ has confirmed, that the resonances are not overlapped up to values of $E_0 \approx 300\, KV/cm$. In order for the electron motion to be stochastic we have used standing waves H_{11} and H_{21} for heating of electrons of plasma in cylindrical resonator.

In this case there is some practical advantage: it is easier to receive large amplitudes of fields, it is possible to excite several types of oscillations by one generator. The results of a numerical simulation for 1000 electrons moving in waves are shown in Fig. 2. All electrons had the initial energy $\gamma = 1 + 10^{-4}$. Initially, electrons were allocated randomly inside the imaginary cylinder of a radius $a/8$ and length $2\pi/k_z$, which was located in a center of the waveguide. The direction of initial velocity of electrons was random too. The amplitudes of waves H_{11} and H_{21} were $E_0 \approx 24\, KV/cm$ (i.e. two running towards each other waves H_{11} had amplitude $12\, KV/cm$, the same for wave H_{21}), the other values were as follows: $a = 8\ cm$, $\omega/2\pi = 2.8\ GHz$, $H_0 = 1\ KG$. In Fig. 2 the solid lines show lines of resonances. The lines that intersect at point $(\gamma = 1, p_z = 0)$ are the lines of the first cyclotron resonance $(s=-1)$ of electron with forward and backward H_{11}–waves, the lines that intersect at point $(\gamma = 2, p_z = 0)$ are the lines of the second cyclotron resonance $(s=-2)$ of electron with H_{21}–waves. The dots show final positions of electrons in the end of calculation at $t = 200\pi$.

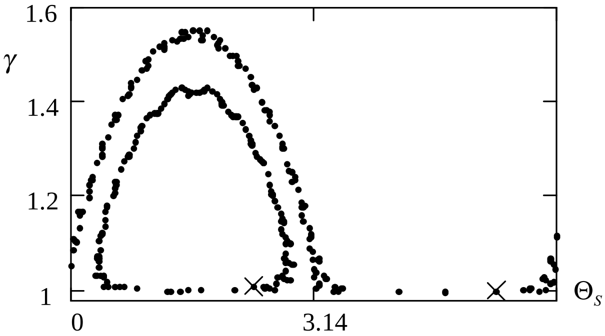

Fig. 1. Electron in first cyclotron resonance.

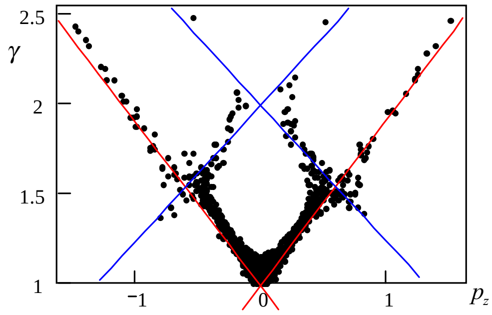

Fig. 2. Diffusion of electrons in the fields of two standing waves H_{11} and H_{21}.

From Fig. 2 we can see, that electrons initially located in the point $(p_z = 0\,, \gamma \approx 1)$ under the action of waves diffuse into area of large energy. At initial stage nonrelativistic electrons gain relativistic energy during time $t \approx 30\pi$. After that they start to interact with H_{21}–waves. The some electrons acquire energy up to $\gamma \approx 2.5$. The trajectory of single electron is shown on Fig. 3. The electron moves chaotically rambling over resonances. The energy distribution function of electrons for dimensionless time $t = 200\pi$ is presented in Fig. 4.

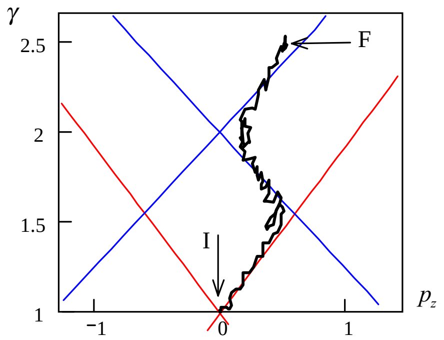

Fig. 3. Trajectory of single electron, I – initial position, F – final position.

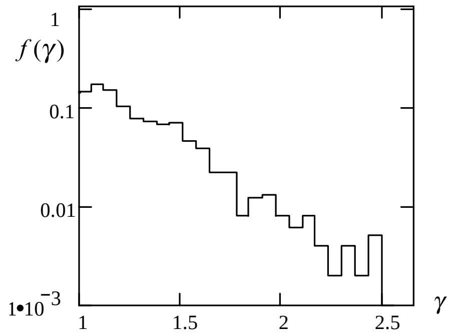

Fig. 4. Energy distribution function of electrons in the fields of waves H_{11} and H_{21} at $t = 200\pi$.

Stochastic plasma heating was investigated experimentally too. We have used cylindrical resonator with diameter 160mm and length 700mm. To excite H_{11} and H_{21} modes of the resonator we have used 10 cm pulse magnetron with the power of 900 kW. The duration of magnetron pulse was equal to 2µsec. The plasma was created by beam-plasma discharge. The magnetic field was varied from 1kG to 1.5kG. We have detected a lot of electrons with energies up to 1MeV when the fields amplitudes are large enough for chaos arising.

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References

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