BIREFRINGENCE EFFECTS IN MAGNETIZED PLASMAS

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Abstract

The effects of elliptical (linear) birefringence on radiation linearly (elliptically) polarized perpendicularly to the magnetic field are investigated on the basis of the far-field approach to the transport of polarized radiation in anisotropic media. Cyclic conversion of linear into elliptical polarization and, on referring to synchrotron emission, circular depolarization are described.

1. Introduction

Birefringence effects on the transport of polarized radiation in a magnetized plasma are usually dealt with in the limiting cases of circular and linear birefringence, namely, Faraday rotation and Cotton-Mouton effect, respectively [1]. Here the effects of elliptical birefringence on radiation linearly polarized perpendicularly to the magnetic field as well as the effect of depolarization of elliptically polarized synchrotron emission due to linear birefringence are discussed on the basis of the far-field approach to the transport of polarized radiation in anisotropic media [1,2]. More specifically, with reference to a homogeneous and uniformly magnetized medium, the radiation intensity tensor [1] normalized to the (total) radiation intensity, to be referred to as the **radiation polarization tensor** $p_{\alpha\beta}$, is given by [2]

$$p_{\alpha\beta}(\mathbf{r}) = U_{\alpha\sigma}\left\{ e^{\mathrm{i}(\omega/c)\left(n^{\sigma}-n^{\sigma'}\right)\left|\mathbf{r}-\boldsymbol{\rho}\right|} U^{*}_{\gamma\sigma}\, p_{\gamma\delta}(\boldsymbol{\rho}) U_{\delta\sigma'} \right\} U^{*}_{\beta\sigma'} \;,\quad U_{\alpha\sigma} \equiv \mathbf{e}_{\alpha}\cdot\mathbf{e}^{\sigma} , \qquad \text{(1a,b)}$$

with $\alpha,\beta = 1{,}2$ and $\sigma,\sigma' = \mathrm{X},\mathrm{O}$, summation over dummy indices being implied. In (1a), $\mathbf{r}$ and $\boldsymbol{\rho}$ are, respectively, the observation point, at which the radiation is collected, and the reference point, at which the radiation is emitted or injected; n^{σ} is the refractive index of either the extraordinary ($\sigma = \mathrm{X}$) or ordinary ($\sigma = \mathrm{O}$) mode. The unitary matrix (1b) represents the matrix of basis change from the basis of the mode polarization vectors $\mathbf{e}^{\mathrm{X}},\mathbf{e}^{\mathrm{O}}$ to the reference basis $\mathbf{e}_1,\mathbf{e}_2$, with respect to which the polarization measurements are performed. Result (1) is valid in the limit of optically thin media for which absorption is negligible and applies to the case for which the radiation source is sufficiently localized around point $\boldsymbol{\rho}$. The birefringence effects are accounted for in (1a) through the exponential (oscillating) factor the exponent of which is the relative phase picked up by the two (X and O) modes during propagation.

2. Elliptical birefringence of linearly polarized radiation

With reference to a magnetized plasma, the polarization vectors of the two natural (transverse) modes can be expressed as [1]

$$\mathbf{e}^{\mathrm{X}} = \frac{\mathbf{e}_2 + \mathrm{i}T\mathbf{e}_1}{\left(1+T^2\right)^{1/2}} \quad , \quad \mathbf{e}^{\mathrm{O}} = \frac{\mathbf{e}_1 + \mathrm{i}T\mathbf{e}_2}{\left(1+T^2\right)^{1/2}} \quad , \quad \mathbf{e}^{\mathrm{X}} \cdot \left(\mathbf{e}^{\mathrm{O}}\right)^* = 0 , \qquad \text{(2a,b,c)}$$

with T the mode ellipticity shown in Fig. 1 ($|T| \le 1$ yields the axial ratio of the polarization ellipse), the basis vectors $\mathbf{e}_1$ and $\mathbf{e}_2$, perpendicular to the wave vector, are such that $\mathbf{e}_2$ is perpendicular to the ambient magnetic field $\mathbf{B}_0$.

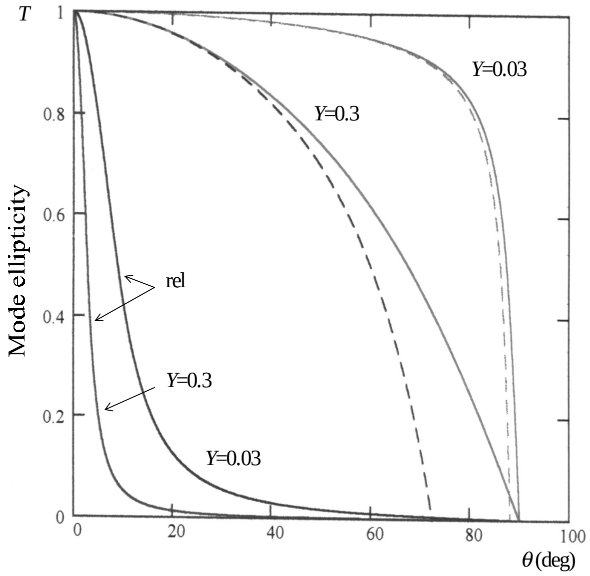

Figure 1. Mode ellipticity vs. $\theta \equiv \measuredangle(\mathbf{k},\mathbf{B}_0)$ for $X \equiv \left(\omega_{\mathrm{p}}/\omega\right)^2 = 0.5$ and various values of $Y \equiv \omega_{\mathrm{c}}/\omega$. Whereas the two "rel-curves" correspond to the relativistic ($T_{\mathrm{e}} = 5mc^2$) ellipticity [3], the other curves yield the ellipticity according to the cold plasma theory, the dashed curves refer to the approximation $T = 1 - Y\sin^2(\theta)/\left(2\left|(1-X)\cos(\theta)\right|\right)$ for propagation at angles such that $\left|(1-X)\cos(\theta)\right| >> (Y/2)\sin^2(\theta)$. Parallel, $\theta = 0$, and perpendicular, $\theta = 90°$, propagation correspond, respectively, to circular and linear mode polarization.

On the basis of (1) and (2) and assuming that the radiation at the reference point $\boldsymbol{\rho}$ is linearly polarized along $\mathbf{e}_2$, i.e., $p_{\gamma\delta}(\boldsymbol{\rho}) = \delta_{\gamma 2}\delta_{\delta 2}$ in (1a), one gets for the ellipticity T_{r} of the radiation at (the observation) point $\mathbf{r}$

$$T_{\mathrm{r}} = \frac{1-\sqrt{1-4A^2}}{2A} \quad , \quad A \equiv \frac{\left(1-T^2\right)T}{\left(1+T^2\right)^2}\left(1-\cos\Delta\varphi\right) , \qquad \text{(3a,b)}$$

where $\Delta\varphi \equiv (\omega/c)\left(n^{\mathrm{X}} - n^{\mathrm{O}}\right)\left|\mathbf{r}-\boldsymbol{\rho}\right|$. Note that result (3) remains valid if the polarization of the radiation at $\boldsymbol{\rho}$ is rotated by 90°, i.e., one is referring to input radiation linearly polarized along $\mathbf{e}_1$. The radiation ellipticity (3), shown in Fig. 2a for $\Delta\varphi = \pi$ and on the basis of the mode ellipticity T of cold plasma, *i)* exhibits a maximum $T_{\mathrm{r}} = 1$ (circular polarization) which occurs for propagation the nearer to perpendicular to $\mathbf{B}_0$ the smaller is Y ($\equiv \omega_{\mathrm{c}}/\omega$); *ii)* is zero, i.e., the radiation polarization remains unchanged through propagation in the medium, for either parallel or perpendicular propagation, for which, instead, Faraday rotation and Cotton-Mouton effect, respectively, can be effective. Thus, as a result of the elliptical-birefringence effect described by (3), a cyclic ($\approx \cos\Delta\varphi$) conversion of linear into elliptical polarization can occur. Such a process is expected to be effective for synchrotron radiation, for which $Y \approx \gamma^{-2} (<< 1)$, where γ is the Lorentz factor of the radiating particles, within a range of a few degree around the direction perpendicular to the magnetic field [4].

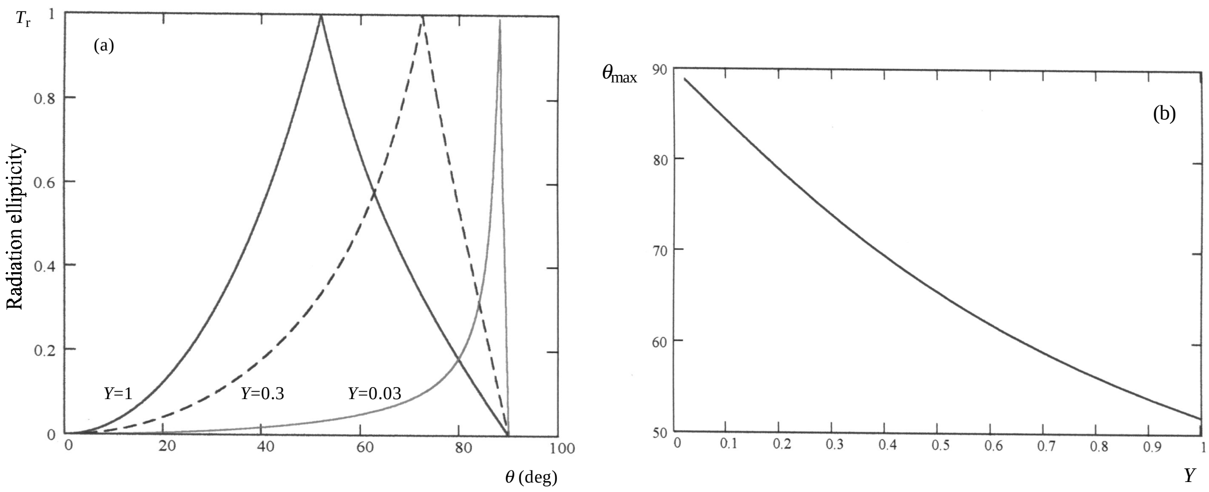

Figure 2. a) The ellipticity (3) vs. θ in a cold plasma. The maximum occurs for $T = \sqrt{2} - 1 \approx 0.41$. b) The angle $\theta_{\max}$, for which $T_\mathrm{r} = 1$, vs. $Y\,(\equiv \omega_\mathrm{c}/\omega)$.

3. Circular depolarization of synchrotron emission

Let us consider now the case of synchrotron emission (SE) by an isotropic distribution of electrons with a power-law energy spectrum, for which the (intrinsic) polarization tensor is

$$p_{\alpha\beta}^{(\mathrm{SE})} = \begin{pmatrix} \dfrac{2}{3a+7} & -\mathrm{i}\dfrac{r_\mathrm{c}}{2} \\ \mathrm{i}\dfrac{r_\mathrm{c}}{2} & \dfrac{3a+5}{3a+7} \end{pmatrix}, \tag{4a}$$

the corresponding degrees of circular and linear polarization being, respectively,

$$r_\mathrm{c} = \left[4\cos\theta \sqrt{\frac{Y}{3|\sin\theta|}} \frac{2+a}{a} f(a) \right] r_\mathrm{l} \quad , \quad r_\mathrm{l} = \frac{a+1}{a+7/3} \tag{4b,c}$$

with

$$f(a) \equiv \Gamma\left(\frac{3a+8}{12}\right)\Gamma\left(\frac{3a+4}{12}\right)\left[\Gamma\left(\frac{3a+7}{12}\right)\Gamma\left(\frac{3a-1}{12}\right)\right]^{-1} \quad , \quad a > \frac{1}{3}, \tag{4d}$$

a being the spectral index of the power-law distribution [5] (note that the degree of circular polarization r_c given by Eq. (4.106) of Ref. [5] is a factor 4 smaller than (4b)). With reference to (4), one should note that *i)* r_l depends only on *a*, with $r_\mathrm{l} > 0.5$ for $a > 1/3$, i.e., SE tends to be linearly polarized (e.g., $r_\mathrm{l} = 0.724$ for $a = 2.5$); *ii)* $f(a)$ is an increasing function of *a*, namely, $0.1 \lesssim f(a) \lesssim 1$ for $1/3 < a < 5$; *iii)* $r_\mathrm{c}/r_\mathrm{l}$ is a slowly increasing function of *a*, e.g., $r_\mathrm{c}/r_\mathrm{l} \approx 20\%$ for $2 \lesssim a \lesssim 5$ ($\theta = 70°$ and $Y = 0.04$).

Let us assume now that SE (4) propagates in a relativistic (thermal) plasma whose (relativistic) eigenmodes are nearly linearly polarized, cf. the relativistic ellipticity shown in Fig. 1. Making use of (1), where the reference polarization tensor $p_{\gamma\delta}(\boldsymbol{\rho})$ is identified with

(4a), taking $U_{\alpha\sigma} = \delta_{\alpha\sigma}$, for linearly polarized modes, and integrating along the geometrical-optics ray yield the radiation polarization tensor accounting for the effects of linear birefringence. The corresponding degrees of polarization are

$$\tilde{r}_{\mathrm{c}} = \frac{\sin\Delta\varphi}{\Delta\varphi} r_{\mathrm{c}} \quad , \quad \tilde{r}_{\mathrm{l}} = \left[1 + \left(\frac{1-\cos\Delta\varphi}{\Delta\varphi}\right)^2 \left(\frac{r_{\mathrm{c}}}{r_{\mathrm{l}}}\right)^2\right]^{1/2} r_{\mathrm{l}} , \qquad (5a,b)$$

with $\Delta\varphi \equiv (\omega/c)(n^{\mathrm{X}} - n^{\mathrm{O}})s$, s being the relevant ray length; r_{c} and r_{l} are, respectively, the degrees of circular and linear polarization in the absence of birefringence, cf. (4b,c). From (5a) it appears that $\tilde{r}_{\mathrm{c}} < r_{\mathrm{c}}$, i.e., the degree of circular polarization tends to decrease with respect to its value in the absence of birefringence; such an effect, due to the extension of the radiating source, is referred to as **circular depolarization** and is shown in Fig. 3. The relative depolarization $|\tilde{r}_{\mathrm{c}} - r_{\mathrm{c}}|/r_{\mathrm{c}}$ is about 20% for $Y = 0.04$.

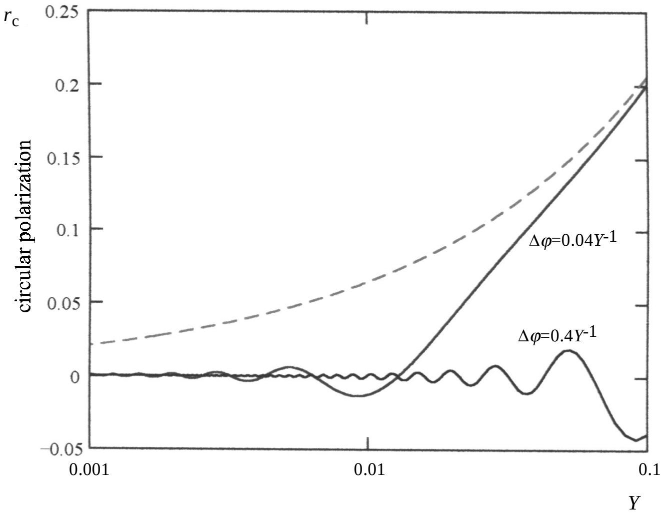

Fig. 3. The degree of circular polarization of synchrotron emission vs. $Y \equiv \omega_{\mathrm{c}}/\omega$, in the presence (full curves, Eq. (5a)) and in the absence (dashed curve, Eq. (4b)) of linear birefringence, for $a = 2.5$, $\theta = 70°$ and different values of $\Delta\varphi$.

As for the linear polarization, from (5b) it appears that $\tilde{r}_{\mathrm{l}} \gtrsim r_{\mathrm{l}}$, i.e., the plasma birefringence tends to enhance the degree of linear polarization of SE, such an effect being proportional to $(r_{\mathrm{c}}/r_{\mathrm{l}})^2 \approx 4\%$, which yields a relative effect $(\tilde{r}_{\mathrm{l}} - r_{\mathrm{l}})/r_{\mathrm{l}} \lesssim 4‰$, which is thus negligible with respect to the concomitant effect of circular depolarization.

References

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