CROSS-FIELD PLASMA TRANSPORT AND POTENTIAL FORMATION INDUCED BY ELECTROSTATIC WAVES

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Abstract. It has been proved theoretically that particle transport along and across a magnetic field and an electric field across a magnetic field can be induced by almost perpendicularly propagating electrostatic waves in a magnetized plasma.

I. INTRODUCTION

Particle transport along and across a magnetic field and an electric field across a magnetic field which are induced by almost perpendicularly propagating electrostatic waves in a magnetized plasma have been investigated theoretically based on quasilinear transport equations derived from Vlasov-Maxwell equations [1 - 3]. The electrostatic waves accelerate particles and the ratio of parallel and perpendicular drift velocities $v_{s\parallel}/v_d$ can be proved to be proportional to $k_\parallel/k_\perp$. As a result, the strong plasma transport across a magnetic field appears. Simultaneously the intense cross-field electric field $\mathbf{E}_0=\mathbf{B}_0\times v_d/c$ is genereated via the dynamo effect of cross-field particle drift to satisfy the generalized Ohm's law, that is, the electrostatic waves can produce the cross-field particle drift that is identical to $\mathbf{E}\times\mathbf{B}$ drift. This cross-field transport is able to explain the fluctuation-induced anomalous transport occuring in sawtooth crash [4] or edge plasmas in tokamaks and the perpendicular ion acceleration in tokamaks and space plasmas.

II. VELOCITY-SPACE DIFFUSION EQUATION

We consider the particle transport which arises from quasilinear velocity-space diffusion due to electrostatic waves propagating in the uniform magnetic field $\mathbf{B}_0 = (0, 0, B_0)$ and the uniform electric field $\mathbf{E}_0 = (0, E_0, 0)$. The dielectric constant $\varepsilon_{\mathbf{k}}=1+\sum_s \varepsilon_{\mathbf{k}}^{(s)} = \varepsilon'_{\mathbf{k}} + i\varepsilon''_{\mathbf{k}}$ $(\varepsilon_{\mathbf{k}}^{(s)} = \varepsilon_{\mathbf{k}}'^{(s)} + i\varepsilon_{\mathbf{k}}''^{(s)})$ is obtained as

$$\varepsilon_{\mathbf{k}}^{(s)} = -\frac{\omega_{ps}^2}{k^2}\sum_{r=-\infty}^{\infty}\int d v \frac{J_r^2(\mu_{\mathbf{k}})U_r(\mathbf{k})\, g_{s0}}{k_\parallel v_\parallel + k_\perp v_d - \omega_{\mathbf{k}} + r\omega_{cs}} \qquad (1)$$

$$g_s = a_s \sum_{m=0}^{\infty} \frac{1}{m!} v_x^m \left(-\frac{v_d}{v_\perp} \frac{\partial}{\partial v_\perp} \right)^m g_{s0} \, (v_\perp, v_\parallel, t)$$

$$= a_s g_{s0}((v_\perp^2 - 2 v_x v_d)^{1/2}, v_\parallel, t) \qquad , \tag{2}$$

where

$$U_r(\mathbb{k}) = k_\parallel \frac{\partial}{\partial v_\parallel} + \frac{r\omega_{cs}}{v_\perp} \frac{\partial}{\partial v_\perp} \qquad , \tag{3}$$

J_r is the Bessel function of rth order, $\mu_{\mathbb{k}}=k_\perp v_\perp/\omega_{cs}$, $\omega_{ps}^2=4\pi n_s e_s^2/m_s$, $\omega_{cs}=|e_s|B_0/m_s c$, $v_x=v_\perp\cos\theta$, $\mathbb{v}=(v_\perp, \theta, v_\parallel)$ is given by the cylindrical coordinate in velocity space, $\mathbb{v}_d=(v_d, 0, 0)=c\mathbb{E}_0 \times \mathbb{B}_0/B_0^2$ equals $\mathbb{E}\times\mathbb{B}$ drift velocity, $\mathbb{k}=(k_\perp, 0, k_\parallel)$, g_s is the background velocity distribution function containing the fluctuation-induced cross-field drift $v_d=\int d\mathbb{v} v_x g_s$, and a_s is the normalization constant. Equation (2) satisfies the unperturbed Vlasov equation

$$(\mathbb{E}_0 + (\mathbb{v}/c)\times\mathbb{B}_0)\cdot(\partial g_s/\partial\mathbb{v}) = 0 \qquad , \tag{4}$$

which leads to the generalized Ohm's law for a collisionless plasma

$$\mathbb{E}_0 + \frac{1}{c}\mathbb{v}_d \times \mathbb{B}_0 = 0 \qquad , \tag{5}$$

by integration of Eq. (4) multiplied by $\mathbb{v}$ in velocity space. This means that $\mathbb{E}_0$ is produced by the dynamo effect of cross-field particle drift arising from the acceleration due to electrostatic waves. When $\mathbb{v}_d=0$, g_s is reduced to g_{s0} being symmetric with respect to the magnetic field.

We can derive the quasilinear velocity-space diffusion equation by the same manner as the previous works[1-3] as follows:

$$\frac{\partial g_s}{\partial t} = \sum_{\mathbb{k}\neq 0} |E_{\mathbb{k}}|^2 Q_{\mathbb{k}} g_{s0} \qquad , \tag{6}$$

with the θ-dependent velocity-space diffusion coefficient

$$Q_{\mathbb{k}} = \mathrm{Im}\left[\frac{e_s^2}{k^2} \sum_{n,\, r=-\infty}^{\infty} Y_{n,r}(\mathbb{k}) \frac{e^{-i(r-n)\theta} J_r(\mu_{\mathbb{k}}) U_r(\mathbb{k})}{k_\parallel v_\parallel + k_\perp v_d - \omega_{\mathbb{k}} + r\omega_{cs}} \right] \qquad , \tag{7}$$

$$Y_{n,r}(\mathbb{k}) = U_n(\mathbb{k}) J_n(\mu_{\mathbb{k}}) + \frac{k_\perp}{v_\perp}(r-n) J_n'(\mu_{\mathbb{k}}) \qquad , \tag{8}$$

where $Q_{\mathbb{k}}$ is expressed in the displaced cylindrical coordinate in velocity-space ($v_x = v_\perp\cos\theta + v_d$, $v_y = v_\perp\sin\theta$, $v_z = v_\parallel$), thereby g_{s0} appears in the right-hand of Eq. (6), and J_n' is the first derivative of J_n with respect to the argument. The azimuthal dependence of $Q_{\mathbb{k}}$ reperesents the anisotropy of the velocity-space diffusion around the magnetic field and the resulting particle transport in the x-direction.

III. TRANSPORT EQUATIONS

The velocity-space integration of the velocity-space diffusion equation multiplied by the energy and monemtum of particle yields the quasilinear transport equations as described in Refs. 1-3. Thus the transport equations indicating the time development of the energy and momentum densities of magnetized particles of s-species are given as follows:

$$\frac{\partial U_s}{\partial t} = -2\gamma_{\mathbf{k}}^{(s)} U_{\mathbf{k}} \quad , \tag{9}$$

$$\frac{\partial \mathbf{P}_s}{\partial t} = -\frac{2\gamma_{\mathbf{k}}^{(s)}\mathbf{k}}{\omega_{\mathbf{k}}} U_{\mathbf{k}} \quad , \tag{10}$$

where $U_{\mathbf{k}} = (1/8\pi)\,[\partial(\varepsilon'_{\mathbf{k}}\,\omega_{\mathbf{k}})/\partial\omega_{\mathbf{k}}]\,|E_{\mathbf{k}}|^2$ is the wave energy density, $\mathbf{k}U_{\mathbf{k}}/\omega_{\mathbf{k}}$ the wave momentum density, $U_s = \int d\mathbf{v}\, \frac{1}{2} n_s m_s v^2 g_s$ and $\mathbf{P}_s = \int d\mathbf{v} n_s m_s \mathbf{v} g_s$ are the energy and momentum densities of particles of s-species, $\gamma_{\mathbf{k}}^{(s)} = -\varepsilon''^{(s)}_{\mathbf{k}}/(\partial\varepsilon'_{\mathbf{k}}/\omega_{\mathbf{k}})$ $(\gamma_{\mathbf{k}} = \Sigma_s \gamma_{\mathbf{k}}^{(s)})$ is the linear damping rate ascribed to particles of s-species, and $\mathbf{P}_s = (P_{s\perp}\ ,\ 0,\ P_{s\parallel})$, $P_{s\perp} = m_s n_s v_d$, $P_{s\parallel} = m_s n_s v_\parallel$. Transport equations (1) and (2) predict clearly that the electrostatic waves generate anomalous transport or strong particle acceleration along and across the magnetic field. The relation of $P_{\parallel s}/P_{\perp s} = k_\parallel/k_\perp$ $(\mathbf{P}_s /\!/ \mathbf{k})$ found from Eq. (10) with $\mathbf{P}_s(0)=0$ shows that the small parallel and large perpendicular particle transports appear simultaneously. Consequently, it is verified that there exists the strong coupling between the transport along and across the magnetic field and that the parallel transport also becomes anomalous[5].

In the absence of nonlinear wave-wave and wave-particle interaction the kinetic wave equation is expressed by

$$\frac{\partial U_{\mathbf{k}}}{\partial t} = 2\gamma_{\mathbf{k}} U_{\mathbf{k}} \quad . \tag{11}$$

This equation and the transport equations (9) and (10) yield the conservation laws for total energy and momentum densities of waves and particles and they are represented as

$$\frac{\partial}{\partial t}\left(\sum_{\mathbf{k}} U_{\mathbf{k}} + \sum_{s} U_s\right) = 0 \quad , \tag{12}$$

$$\frac{\partial}{\partial t}\left(\sum_{\mathbf{k}} \frac{\mathbf{k}}{\omega_{\mathbf{k}}} U_{\mathbf{k}} + \sum_{s} \mathbf{P}_{s}\right) = 0 \quad . \tag{13}$$

It can be also proved by adding the nonlinear terms to Eqs. (9)-(11) that these conservation laws hold in the presence of nonlinear wave-wave and wave-particle interaction. Moreover, Eqs. (5) and (9)-(13) can be applicable to the particle transport in the relativistic magnetized plasma by replacing Eqs. (1)-(4) and (6)-(8) by the corresponding relativistic expressions[2, 3]. On the other hand, it was proved that the same transport equations can be also derived from the single-particle theory which is easy to understand the physical mechanism of the cross-field particle acceleration and anomalous transport.

In conclusion, it is verified by means of the quasilinear transport equations that the anomalous particle transport along and across the magnetic field and the electric field across the magnetic field can be generated simultaneously by the almost perpendicularly propagating electrostatic waves. The cross-field electric field is produced by the dynamo effect of cross-field transport to satisfy the generalized Ohm's law. The fluctuation-induced transport can explain successfully anomalous transport and perpendicular ion acceleration in tokamaks, and an electron beam-plasma system[6] as well as space plasmas.

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