IMPROVEMENT TO THE CALCULATION OF THE LOW FREQUENCY PART OF THE ELECTRICAL MICROFIELD DISTRIBUTION IN PLASMAS

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Abstract

A method is presented here to obtain the low frequency part of the electrical microfield distribution in plasma, by the quadrature of an analytical expression. This method considers all the interactions between ions inside a non-linear Debye sphere, using the new approximated non-linear potential recently published. This field is a very accurate solution of the non-linear Poisson equation for a point test charge in a plasma, described by a uniform charge density (ion charge z) at Boltzmann thermal equilibrium. The solution here presented is an improvement of previous procedures and more general as any other one, because the potential here used yields not only for single charged ions, but also for multicharged ones. For $Z = 1$ and $z = 1$ (for the surrounding charge density), we reproduce results of other authors.

1. Introduction

In a previous paper [1] a method was described through which the Holtsmark [2] distribution function can be corrected, using a new expression of the Debye screening fields given by P. Martin and R. Perez [4]. In this paper we present a similar procedure (using the solution of the non-linear Poisson equation) obtained by the quasi-fractional method [5], in order to get a highly accurate solution of them given by

$$\phi(r) = \frac{Z_1 e}{r} g(s) e^{-r/\lambda_D}, \tag{1}$$

where

$$g(s) = \frac{1 + p_1 s + p_2 s^2 + p_3 s^3}{(1+s)^3} \tag{2}$$

and the coefficients $p_1,\ p_2$ and p_3 are determined in [4]. Here it is dimmensionless variable s as $r/F\lambda_D$; where the coupling parameter F is defined as

$$F = \frac{2}{15}\sqrt{2\pi} Z \delta^3, \tag{3}$$

where δ is defined as

$$\delta = \left(\frac{15}{4}\right)^{1/3} \left(\frac{1}{\sqrt{2\pi}\, n^{1/3}}\right) \sqrt{\frac{4\pi n e^2}{kT}}.$$

The idea is to get exactly the Holtsmark distribution function in the high temperature limit, where non-linear corrections are not so important, and make such kind of adjustment when nonlinearity are present. In order to do that, we use Equation (1) instead of the Debye-Hückel, because these take in to account the non-linear part of the interactions, when all correlations are present in the nonlinear Debye sphere. On the other hand, the Debye-Hückel relation is of empirical nature while our potential, is an analytical and high precise approximation of the non-linear Poisson equation that yields in the whole range and reproduces their correct form for the asymptotical solution.

2. Theoretical treatment

In order to calculate the electric field $\vec{E}$ produced at a given charged point by particles located at $\vec{r_1}, \vec{r_2}, \ldots, \vec{r_n}$, we write

$$\vec{E} = \sum_{j=1}^{N} E_j(\vec{r}, \vec{r_j}). \tag{4}$$

Our primary interest is to obtain the probability distribution $W(\vec{E})$ of the electric field where in general we have:

$$W(\vec{E}) = \int \ldots \int \delta(\vec{E} - \vec{E}(\vec{r}))\rho \, d^{3N}r \, d^{3N}p \qquad (5),$$

where ρ is probability density in the $\Gamma - space, \vec{r_1} \ldots \vec{r}_N, \vec{p}_1 \ldots \vec{p}_N$. Now taking the Fourier transform of Eq. (5) we get

$$F(\vec{k}) = \int e^{i\vec{k}.\vec{E}} W(\vec{E}) dE \tag{6}$$

and through the inverse of Eq. (6) we obtain

$$W(\vec{E}) = \frac{1}{2\pi^3} \int e^{i\vec{k}.\vec{E}} F(\vec{k}) d^3k, \tag{7}$$

where $F(\vec{k}) = exp(nh(\vec{k})$, is called the *Spectral Function* and nh is the effective microfield distribution. After some mathematical manipulation and introducing the shielded field $E = |\vec{E}|$ at the origin defined by :

$$E = \frac{\partial \phi}{\partial r} = \frac{Zx}{y^2}\{-g + y\frac{\partial g}{\partial s}\frac{\delta}{F} - yg\delta\}e^{-\delta y}, \tag{8}$$

we can write the following expression for nh :

$$nh(x) = \frac{15}{2^{3/2}\sqrt{\pi}} \int_0^\infty y^2 dy \left(\frac{\sin(kE)}{kE} - 1 \right) \tag{9}$$

where $x = kE_0$, $E_0 = e/r_0^2$ and $y = r/r_0$.

We calculate the integrals (9) for each representative value of y. Fitting these data it is possible to obtain an analytical expression for $nh(x)$ in the form

$$nh(x) \cong -x^{3/2}(a + bx^c). \tag{10}$$

The fitting parameters a,b and c are calculated for each $\delta = r_0/\lambda_D$ and for several Z as shown in the following tables:

Table 1					Fitting parameters			Table 2
δ	Z	a	b	c	Z	a	b	c
0.0	1	1.0	0.0	0.0	1	1.2257	-0.37249	0.19864
0.2	1	1.23	-0.373	0.199	2	4.3328	-2.0905	0.1276
0.4	1	2.43	-2.21	0.0512	3	10.4821	-6.5895	0.083422
0.6	1	5.06	-4.47	0.0248	4	23.4450	-17.7281	0.05112

In order to calculate the electric field distribution, we consider spherical symmetry, thus

$$W(E) = 4\pi\, E^2\, W(\vec{E}), \tag{11}$$

because we are interested in the function $E = |\vec{E}|$ due to the isotropy of the field. Introducing the variable $\beta = E/E_0$ we find in a straight forward calculation:

$$H(\beta) = \frac{2\beta}{\pi} \int_0^\infty \sin(\beta x) \exp(nh(x)) x dx \tag{12}$$

Fig. 1 shows the plot of Eq. (12) versus the parameter β. We recover exactly by $\delta = 0$ the Holtsmark profile. Comparing our data with the one given by BM (see Ref. [2]), we observe

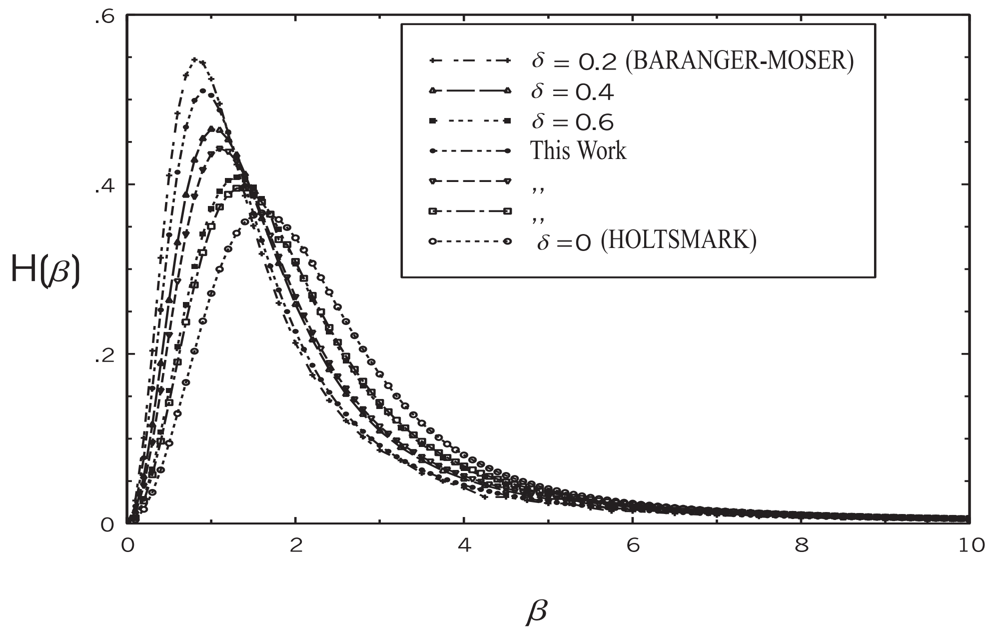

Figure 1. Distribution of the low frequency component at a positive charged point for several values of δ

some discrepancies in the region where an strongest field is expected. We think that it is due to the fact that our distribution takes into account the nonlinearity of the Poisson equation. We emphasize this point of view, because that is the correct form of the field for the ions in the proper Debye-sphere. High charged points are taken into account through the factor F defined by Eq. (2) with the variable s in the approximant. Fig. 2 shows this for $\delta = 0$ and several charge

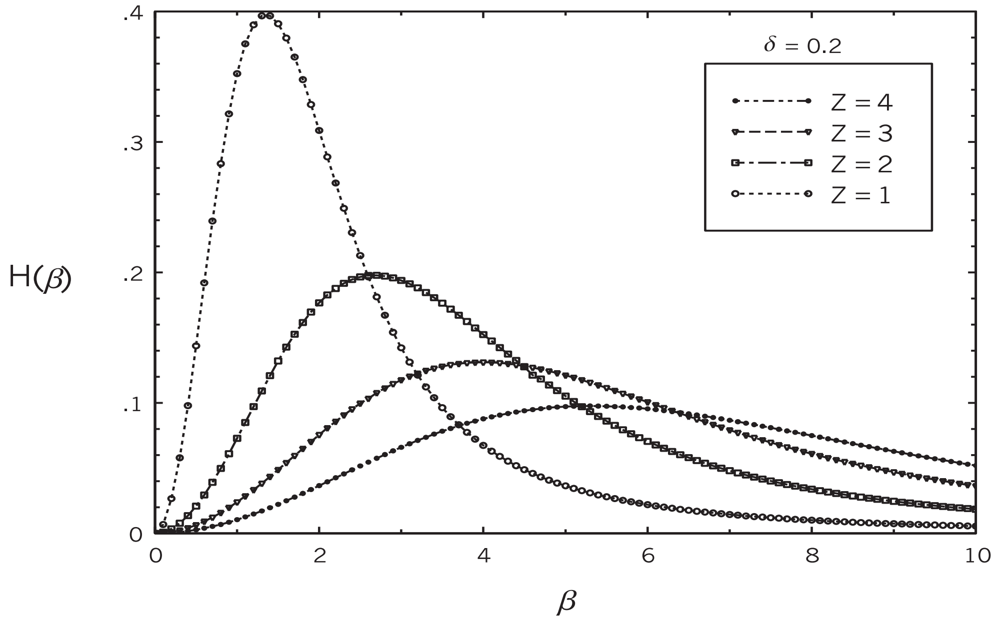

Figure 2. Distribution of the low frequency component at a positive charged point for several Z values

number Z. It is interesting to see that the region of strongest field decreases when Z increases. It is to be expected because by higher Z lower shielding is to be found.

3. Conclusion

Works on field distribution at a positive charged point have been analyzed for various authors. In our case, we compare our results with BM (see ref [2]), due to the accuracy of this work. We recover Holtsmark exactly. In all cases the peak of the distribution shifts toward smaller fields as expected, but in our case is lower, as given by BM due to the fact that non-linear interaction play a primary roll in our distribution. For the case of high charged points the peaks of the distributions are shifted toward much smaller fields. That means, lower shielding is expected.

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