# THE ELECTROMAGNETIC SPECTRUM AND THE ENERGY OF A PLASMA IN THERMAL EQUILIBRIUM

**Merav Opher** and Reuven Opher

*Instituto Astronômico e Geofísico - IAG/USP, Av. Miguel Stéfano, 4200*
*CEP 04301-904 São Paulo, S.P., Brazil*

**Abstract**

We derive the electromagnetic spectrum based on the *fluctuation-dissipation theorem* that describes the electromagnetic fluctuations in a plasma. Our description includes thermal and collisional effects in a plasma. The electromagnetic spectrum obtained differs from the blackbody spectrum in vacuum. We also evaluate the energy density of the plasma and find it appreciably less than the standard calculation.

## 1. Introduction

It is well known that a plasma, even non-magnetized and in thermal equilibrium, has electromagnetic fluctuations. The electromagnetic fluctuations in a plasma were studied in several investigations [1] using the description of the test-particle and the formalism based on the *fluctuation-dissipation theorem*. However, in all these studies, little attention was given to the question of what is the electromagnetic spectrum in a plasma. Usually, the electromagnetic spectrum in a plasma is deduced in a kind of "gedanken experiment" such as described by Dawson [2], where a slab of plasma at a temperature $T$ is placed between two blackbodies at the same temperature $T$ with vacuum regions between them. By assuming that the three bodies are in thermal equilibrium, and calculating the modes that enter the plasma, it is possible to deduce the electromagnetic spectrum. However, this kind of analysis only takes into account the modes that propagate. However, in a study of the electromagnetic fluctuations, it is possible to deduce the electromagnetic spectrum which takes into account all fluctuations in a plasma, including those that do not propagate.

Another interesting question is how much the energy of a plasma differs from the energy of an ideal gas. This correction is called the *correlation energy*. However, in the studies that have been made [3], the correlation energy was estimated assuming that the frequency of the fluctuations $\omega$ is much less than the temperature, $\omega \ll T$ ($\hbar = k_B = 1$). In a study of the electromagnetic fluctuations, it is possible to estimate the energy in a plasma without this assumption.

In this study (see also Opher & Opher [4]) we focus on these two questions, using the formalism based on the *fluctuation-dissipation theorem*. The study is made for the most simple case: a non-magnetized, isotropic, fully ionized plasma in thermal equilibrium. The intensities of the electromagnetic fluctuations are very dependent on how the plasma is described. Therefore, we used a kinetic description taking into account the thermal and collisional effects.

This study is a general study for plasmas, although we made the calculations for the plasma in the early universe. In the beginning of the epoch when the light elements (like $D$, $^4He$, $^3He$ and $^7Li$) were formed, at a time $1\ sec$ after the big bang, the universe was an electron-positron plasma. After $1\ sec$, electron-positron annihilation occurs and the universe becomes an electron-proton plasma. The ranges of densities and temperatures of this epoch are, respectively, $10^{24} - 10^{34}\ cm^{-3}$ and $10^{10} - 10^{7}\ K$.

This study can have consequences for cosmology since the calculations of the primordial universe assume that the plasma was in thermal equilibrium and that the electromagnetic spectrum was a blackbody spectrum in vacuum. These assumptions enter, for example, in the calculation of the energy density, where the energy density of the photons is taken as the energy density of a blackbody.

## 2. Electromagnetic Fluctuations

The *fluctuation-dissipation theorem* connects the electromagnetic fluctuations with the dissipative properties of the system [5]. The fluctuations of the electric field and the magnetic field are given by

$$\frac{\langle E^2\rangle_{\mathbf{k}\omega}}{8\pi} = \frac{\hbar}{e^{\hbar\omega/T}-1}\frac{Im\,\varepsilon_L}{\mid \varepsilon_L \mid^2} \;+\; 2\frac{\hbar}{e^{\hbar\omega/T}-1}\frac{Im\,\varepsilon_T}{\mid \varepsilon_T - \left(\frac{kc}{\omega}\right)^2 \mid^2}\,, \tag{1}$$

$$\frac{\langle B^2\rangle_{\mathbf{k}\omega}}{8\pi} = 2\frac{\hbar}{e^{\hbar\omega/T}-1}\left(\frac{kc}{\omega}\right)^2\frac{Im\,\varepsilon_T}{\mid \varepsilon_T - \left(\frac{kc}{\omega}\right)^2 \mid^2}\,, \tag{2}$$

where $\varepsilon_L$ and $\varepsilon_T$ are, respectively, the longitudinal and transverse dielectric permittivities of the plasma. The first and the second term of Eq. (1) are, respectively, the fluctuations of the longitudinal and transverse electric field. We observe that the fluctuations of a plasma are related to the dielectric permittivities, that are dependent on the treatment used to describe the plasma. Therefore, the electromagnetic fluctuations evaluated are very dependent on the description of the plasma.

## 3. Description of a Plasma

To describe the plasma, we used the Vlasov equation in first order with a suitable collision term in order to obtain the best expression for the dielectric permittivities. We are dealing with a fully ionized plasma, therefore the collision term that has to be used is the Fokker-Planck collision term. In this kind of plasma the charged particles interact simultaneously with a large number of neighboring particles, suffering multiple Coulomb collisions. To be more precise, even in a fully ionized plasma, there are two regions to treat the collisions. At large distances the effect of many particles and screening is important and the Fokker-Planck collision term has to be used. But at small distances, the individual effects of the particles are important since they strongly interact in binary collisions and the Boltzmann collision term has to be used. Therefore, a unified treatment is necessary. This has been done by Thompson & Hubbard and Hubbard [6] for the Fokker-Planck collision term. However, it is extremely difficult to solve the kinetic equations with the Fokker-Planck collision term since the equations turn out to be integro-differential. For this reason, we used the BGK (Bhatnagar-Gross-Krook) collision term that is an approximation to the Boltzmann collision term, that describes the collisions as binary. This is an approximation to the inclusion of collisions. In fact, for the temperatures and densities that we are dealing with, the collision frequency is much less than the plasma frequency so this will not affect appreciably the results. However, when integrating over wave number, we have a divergence at high wave number. This is due to the fact that this description is not complete.

With the BGK collision term [5], the dielectric permittivities are,

$$\varepsilon_L(\omega,\mathbf{k}) = 1 + \sum_\alpha \frac{{\omega_{p\alpha}}^2}{k^2{v_\alpha}^2}\frac{1+\frac{(\omega+i\eta)}{\sqrt{2}kv_\alpha}Z\left(\frac{\omega+i\eta_\alpha}{\sqrt{2}kv_\alpha}\right)}{1+\frac{i\eta}{\sqrt{2}kv_\alpha}Z\left(\frac{\omega+i\eta_\alpha}{\sqrt{2}kv_\alpha}\right)}\,, \tag{3}$$

$$\varepsilon_T(\omega, \mathbf{k}) = 1 + \sum_\alpha \frac{{\omega_{p\alpha}}^2}{\omega^2} \left( \frac{\omega}{\sqrt{2} k v_\alpha} \right) Z \left( \frac{\omega + i\eta_\alpha}{\sqrt{2} k v_\alpha} \right) , \tag{4}$$

where $\alpha$ is the label for each species of the plasma, $v_\alpha$ is the thermal velocity for each species and $Z(z)$ is the Fried $\&$ Conte function.

## 4. Electromagnetic Spectrum

The electromagnetic spectrum is given by the sum of the spectrum of the magnetic field and the transverse electric field,

$$S(\omega) = \frac{\langle B^2 \rangle_\omega}{8\pi} + \frac{\langle {E_T}^2 \rangle_\omega}{8\pi} . \tag{5}$$

With $\varepsilon_T(\omega, \mathbf{k})$, we obtain $S(\omega)$, using Eqs.(1)-(2) and integrating the spectra over wave number. When integrating over wave numbers, as we commented above, a divergence occurs. When treating binary collisions, a superior limit has to be taken since for small distances the Coulomb energies of the particles exceed their kinetic energies. Therefore, we take as a superior limit of integration one over the closest distance between a particle and an electron in a plasma (when the kinetic energy equals the Coulomb energy), $k_{max} = Mmv^2/(m + M) \mid eq \mid$. The derived $S(\omega)$ is shown in Fig. 1 and is compared with the blackbody spectrum in vacuum.

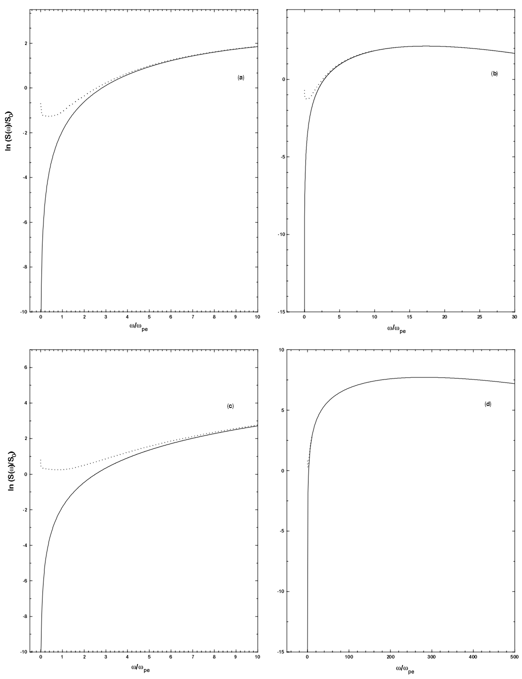

**Fig. 1:** The electromagnetic spectrum $ln[S(\omega)/S_0]$ vs $\omega/\omega_{pe}$ (where $S(\omega) = \langle B^2 \rangle_\omega/8\pi + \langle {E_T}^2 \rangle_\omega/8\pi$ and $S_0 = {\omega_{pe}}^2 k_B T/c^3$ is the normalization) for: (a) The electron-positron plasma at $T = 10^{10}\ K$ and $n_e = 1.4 \times 10^{31} cm^{-3}$ (the dotted curve is *our model* and the solid curve is the black-body spectrum in vacuum); (b) The same as case (a), extended to high frequencies; (c) The same as case (a) but for an electron-proton plasma at $T = 10^9 K$ and $n_e = 5.4 \times 10^{26}$; and (d) The same as case (c), extended to high frequencies.

We obtain that the electromagnetic spectrum in a plasma is not a blackbody spectrum in vacuum. It is a blackbody spectrum at high frequencies but at low frequencies it is a distorted spectrum. In particular, for the electron-positron plasma studied at $T = 10^{10}\ K$, there is appreciable distortion for $\omega \leq 3\omega_{pe}$ and for an electron proton plasma at $T = 10^9\ K$, an appreciable distortion for $\omega \leq 6\omega_{pe}$.

## 5. Energy in a Plasma

Using the dielectric permittivities $\varepsilon_L(\omega, \mathbf{k})$ and $\varepsilon_T(\omega, \mathbf{k})$, we can calculate the energy contained in the magnetic and the electric field, $\rho_B$, $\rho_{E_T}$ and $\rho_{E_L}$. As we saw above, for the transverse component, there is an additional energy compared to the blackbody spectrum in vacuum (see Fig. 1). We can thus write

$$\Delta\rho = \rho_B + \rho_{E_T} - \rho_\gamma \ , \tag{6}$$

where $\rho_\gamma = 8.418 T_9^4 gcm^{-3}$ is the blackbody energy density. To estimate the interaction energy for the longitudinal component, we have to subtract the "self-energy" of the particles, that is, their own fields acting on themselves. The "self-energy" is given by $E_{self} = \frac{n}{4\pi^2} \int dk k^2 \phi_k$, where $\phi_k$ is the static potential energy between particles. The total correction to the energy of the medium due to the fact that a plasma differs from an ideal gas is

$$\rho_{new} = \Delta\rho + \rho_L \ . \tag{7}$$

$\rho_{new}$, besides including the correlation energy, corrects for the fact that the electromagnetic spectrum in the plasma is not a blackbody spectrum in vacuum.

Calculating, in particular, for the electron positron plasma, for the range of temperatures and densities of the period of light element production in the early universe, we obtain $\frac{\rho_{new}}{\rho_\gamma} \approx \frac{\rho_L}{\rho_\gamma} = --10\%$. This result is appreciably different from the standard result which assumes that $\omega < T$: $-\frac{1}{3}\left(\frac{g}{4\pi}\right) \sim 0.003\%$, where $g$ is the plasma parameter (for this period, $g \sim 10^{-3}$).

Both results (spectrum distortion and energy deficit), may have important consequences for plasma and cosmological studies and show that collective effects can be very important. These effects, in principle, can be studied in the laboratory, for high density plasmas produced by lasers.

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