

# TIME EVOLUTION OF THE ION DISTRIBUTION FUNCTION IN THE PERPENDICULAR COLLISIONLESS SHOCK WAVE

S.F. Garanin and S.D. Kuznetsov

*All-Russian Research Institute of Experimental Physics  
37 Mir Ave., Sarov, 607190, Russia*

## 1. Introduction

In precisely perpendicular collisionless shock wave (CSW) gyrating ions should move in a plane perpendicular to the ambient magnetic field. Such situation is realized in 1-D calculations of the CSW [1-3] where ion movement along the field are impossible due to the problem set up. However anisotropic ion distribution formed downstream the front turns out to be unstable respective to oscillation modes with wave number components along the magnetic field. These modes development and their influence on the ion distribution function can result to the anisotropy decrease.

These noise essential influence on the CSW structure was demonstrated in a direct 2-D and 3-D CSW modeling with large Mach number  $M_A \sim 13$  [4]. Paper [5] studied unstable modes development downstream the CSW front in the quasilinear approximation assuming transversal and longitudinal ion distribution functions being bimaxwellian and considering relaxation of corresponding temperatures. It was shown in the paper that this distribution function anisotropy is considerably relaxed. Nevertheless keeping in mind a real ion distribution there are questions: does isotropization take place for all regions of the distribution function, what is a rate of this process for small and large ion velocities and how real ion distribution results in wave spectrum?

The ion distribution function issue is also important practically for plasma facilities that realize plasma heating via CSW (see e. g. [6]), because in CSW large energy fraction is located in high velocity ions. An information about these ions was obtained in [7] using thermonuclear DT-neutrons spectrum measurements. It was found no anisotropy for main part of neutron spectrum and consequently high-energy ions. These results explanation in this paper propose that the ion distribution function enriched according to [3] of energetic ions is isotropized rather rapidly under action of arising noise due to instability development.

Here we consider the ion distribution function evolution due instability development of the Alfvén ion cyclotron mode with wave vector parallel to the ambient magnetic field.

## 2. Dispersion relation for longitudinal waves

The dispersion equation for waves with  $\vec{k} \parallel \vec{B}$  in the case  $c_A \ll c$ , for wave numbers

$k \sim c / \omega_{pi}$ ,  $\frac{\Omega_e}{k v_{Te}} \gg 1$  and frequencies  $\omega \sim \Omega_i$ ,  $\omega \ll \Omega_e$  can be formulated as

$$k^2 = \mp \omega + \frac{1}{2} \int d\vec{v} \frac{(\omega - k v_{\parallel}) \frac{\partial f}{\partial v_{\perp}} + k v_{\perp} \frac{\partial f}{\partial v_{\parallel}}}{\omega - k v_{\parallel} \mp 1}, \quad (1)$$

here dimensionless units are used  $k = k \frac{\omega_{pi}}{c}$ ,  $\omega = \frac{\omega}{\Omega_i}$ ,  $v = \frac{v}{c_A}$  and the velocity distribution function  $f(\vec{v})$  is normalized to 1.

As the ion movement in the ideal case of perpendicular CSW without perturbation takes place in the plain orthogonal to the magnetic field in the downstream vicinity of the shock front parallel to the field components of the ion velocities are small. If we completely neglect parallel ion velocity  $v_{\parallel}$  then relation (1) transforms to the cubic equation for  $\omega$

$$k^2 = \frac{\omega^2}{1 - \omega} - k^2 \frac{\overline{v_{\perp}^2}}{2(1 - \omega)^2}.$$

This equation has complex roots for sufficiently large  $k$ . As the coefficients of this equation are real the imaginary part of one of the conjugated roots is positive and it means that the ion distribution is unstable. The root of equation (1) is presented in Fig. 1 for  $\overline{v_{\perp}^2} = \beta = 0.5$ , that corresponds to the ion distribution downstream the CSW front obtained in 1-D calculation with the Mach number  $M_A \cong 4.4$  [8], the normalized ion transverse energy distribution function  $\varphi(v_{\perp})$  is also given in Fig. 1 ( $\varphi(v_{\perp}) dv_{\perp}$  gives transverse ion energy differential in the interval  $dv_{\perp}$ ), it is assumed small dispersion of longitudinal velocities. So the anisotropic ion distribution with small longitudinal  $v_{\parallel}$  is unstable, and characteristic wave numbers of the unstable mode are  $k \sim \frac{\omega_{pi}}{c}$  and for  $\beta \sim 1$  characteristic increments are  $\gamma \sim \Omega_i$ .

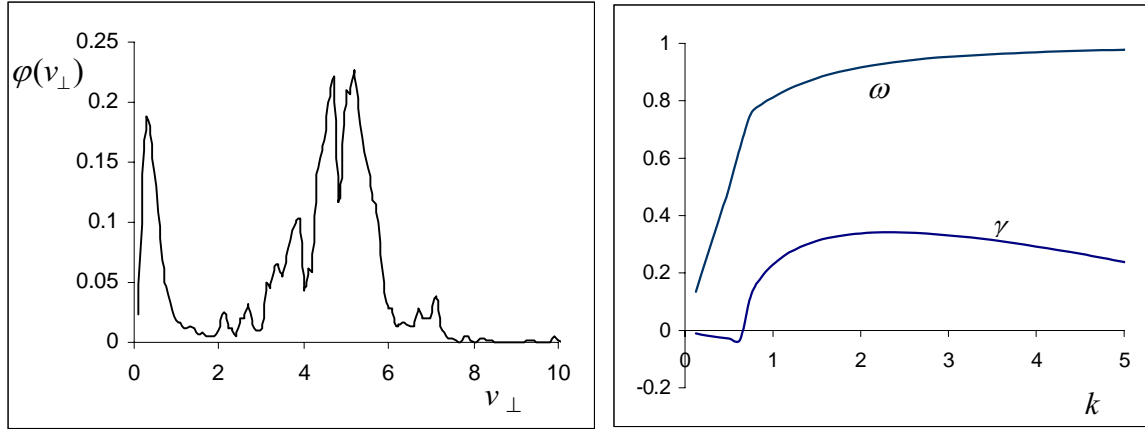


Figure 1.

### 3. Quasilinear diffusion of distribution function

The evolution of high-energy part of ion distribution is of greatest interest to us. Then the equation of quasilinear diffusion for  $\frac{k v}{\omega x} \gg 1$ , where  $x = \frac{v_{\parallel}}{v}$ , can be written in the form

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x} \frac{|\vec{B}_k^1|^2}{2} \frac{1-x^2}{|\frac{\partial \omega}{\partial k} - v x|} \frac{\partial f}{\partial x}, \quad (2)$$

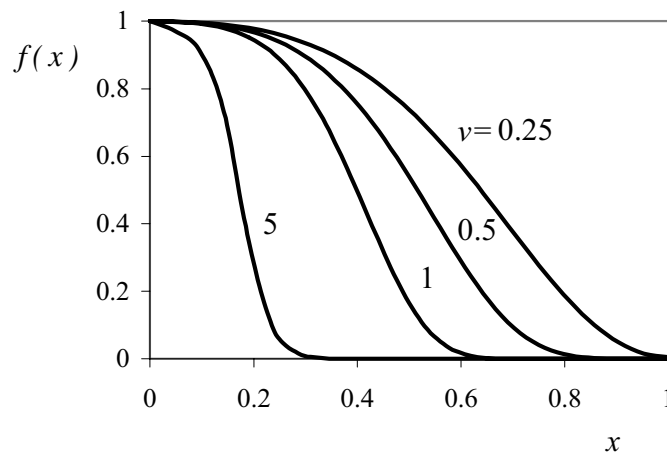
( $\vec{B}_k^1$  is an amplitude of perturbation of  $k$ -mode, corresponding to resonance condition  $\omega = 1 + k v x$ , imaginary part of the frequency  $\gamma \equiv \text{Im} \omega$  is assumed to be small), corresponding to angular diffusion for fixed module of velocity. The growth of magnetic field perturbations is described by the equation

$$\frac{\partial |\vec{B}_k^1|^2}{\partial t} = 2\gamma |\vec{B}_k^1|^2. \quad (3)$$

Evolution of distribution function and growth of magnetic field perturbation were computed according to the equations (1) – (3). Initial distribution function is given in Fig. 1, initial magnetic noise distribution was given by

$$|\vec{B}_k^1|^2 = \frac{\varepsilon}{1+k^2};$$

here,  $\varepsilon = 10^{-3}$  is small initial energy. Computation results are presented in Fig. 2 where the relative dependence of distribution function versus angular variable  $x$  is given for several values of velocity module at time  $t = 7$  when the energy of magnetic noise is 0.1. It can be seen that principal fraction of ions with the relatively small velocities reaches almost complete isotropy to that time. For high velocity ions carrying the main fraction of energy the isotropisation is more slow, it should be expected that isotropy could be reached in time of order of several tens of  $\Omega_i^{-1}$ .



**Figure 2.**

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