Extended dissipative systems far from equilibrium may exhibit complex spatio-temporal dynamics (STD) leading to the formation of regular patterns or turbulence, respectively [1]. The understanding of these dynamics is a challenging problem offering a key for applications of non-linear systems. In discharges, which represent outstanding paradigms for field affected reaction-diffusion systems, non-linear wave states may occur in the positive column (PC) if a control parameter, e.g. the discharge current \(i\), is increased above a threshold value leading to a Hopf bifurcation [2]. Here, these Hopf patterns represent ionization waves [3] yielding a modulation of the light emerging from the discharge. In this paper we present investigations of regular and irregular patterns of ionization waves which will be characterized by the method of the biorthogonal decomposition (BD). Additionally, the observed STD will be comprehended by the numerical simulation of a fluid model.

Experiments were performed in a cylindrical (\(r = 2.0\) cm; \(L = 48\) cm; \(I < 60\) mA) dc glow discharge (pure neon; \(p = 3.1\) Torr) which was sustained by an external voltage; the discharge current was limited by a load resistor (\(R = 50\) k\(\Omega\)). As the waves correspond to a modulation of the light a fast CCD camera (256 pixels, line frequency < 33 kHz) was employed for wave detection. For discharge currents above \(i_1 > 4.8\) mA coherent ionization waves can be observed [4] which represent discrete longitudinal spatial eigenmodes. Results of the spatio-temporal dynamics are shown in Fig. 1. The gray-scale indicates the wave amplitude showing that the phases travel to the cathode whereas the group velocity propagates opposite reflecting the peculiar dispersion of ionization waves (p-type). It can be seen from Figs. 1 (a)-(d) that a high frequency oscillation appears on the left of the space-time diagram corresponding to the head of the PC. This oscillation occurs for currents above \(i_2 > 9.7\) mA and is known as \(\text{Säulenkopfschwingung}\) [5]. Its frequency \(f_h\) was found to increase linearly with the discharge current whereas the dominant frequency \((f_c)\) in the PC does not vary for currents above 15 mA.
Fig. 1: Spatio-temporal dynamics of ionization waves for a neon dc glow discharge (r=2 cm, p=3.1 Torr). (a) shows a regular state (i = 58.7 mA), (b) an intermittent state (i = 57.2 mA), (c) amplitude turbulence (i = 56.1 mA) and (d) a phase turbulence (i = 52.3 mA) in the anode region (right). The observations cover the entire PC and its head (L=48 cm).

Figure 1 shows that the Säulenkopfschwingung injects a higher frequency ionization wave into the PC. In the course of space this wave is damped. From Fig. 2 it can be concluded that the regularity of the spatio-temporal pattern depends on the commensuration of the dominant eigenmode of the PC and the injected wave whose frequency depends in turn on the discharge current. As it can be seen from Fig. 2 regular spatio-temporal states occur when the ratio of \( f_c/f_h \) is a rational number. A Farey sequence [6] indicates a spatio-temporal mode-locking leading to an energy transfer to the dominant mode in the PC.

For further examination the experimental results \( u_{ij} \) were decomposed by the BD (see [7]). The main purpose of this decomposition is to derive a weighted set (weights \( A_k \)) of eigenfunctions in space [topos \( \varphi_k(x) \)] and time [chronos \( \psi_k(t) \)].

\[
(1) \quad u_{ij} = \sum_{k=1}^{K} \varphi_{k}(x_j) A_k \psi_k(t_i) \quad \text{with} \quad i = 1, \ldots, N; \quad j = 1, \ldots, M; \quad K = \min(N, M)
\]

The BD is optimal in the sense that it captures the most energy possible in a given number of linear wave modes [8]. It is superior to other multivariate methods as the spatial and the
temporal eigenfunctions are treated simultaneously. This makes the BD a powerful tool for the
detection of spatio-temporal coherences that are indicated by degenerated modes, i.e. modes \( \phi_k A_k \psi_k \) with the same weight. As an overall measure of spatio-temporal complexity an entropy
can be introduced:

\[
H_{BD} = -\frac{1}{\ln K} \sum_{k=1}^{K} p_k \ln p_k \quad \text{with} \quad p_k = \frac{A_k^2}{\sum_i A_i^2} \quad \text{and} \quad \sum_k p_k = 1
\]

Table 1 shows the different entropy values which are compared to correlation
dimensions \( D_2 \) [9] of time series taken near the anode indicating the complexity of the
signals:

<table>
<thead>
<tr>
<th>current dataset</th>
<th>58.7 mA (a)</th>
<th>57.2 mA (b)</th>
<th>56.1 mA (c)</th>
<th>52.3 mA (d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_{BD} )</td>
<td>0.35</td>
<td>0.41 ¹) 0.55 ²)</td>
<td>0.52</td>
<td>0.50</td>
</tr>
<tr>
<td>( D_2 )</td>
<td>1.0</td>
<td>2.0 ¹) 3.6 ²)</td>
<td>3.7</td>
<td>3.3</td>
</tr>
</tbody>
</table>

Tab. 1 : Comparison of measures for the spatio temporal complexity; BD entropy \( H_{BD} \) and correlation
dimension \( D_2 \). ¹) quasiperiodic section ²) amplitude turbulent section.

Additionally a detailed analysis of the BD modes gives insights into the mechanism of the
spatio-temporal dynamics even in the case of large complexity. This can be seen from Fig. 3.

![Fig. 3](image)

Fig. 3 : Reconstruction of the first four couples of degenerated modes of the amplitude turbulent state
[Fig. 1(c)]. The first four modes with \( 2A_k/\Sigma A_k = 16.6\%, 10.4\%, 8.0\% \) and \( 6.8\% \), respectively, are shown.

The first mode [Fig. 3(a)] can be clearly identified as the ionization wave corresponding to the
dominant mode of the PC. This mode was found to decrease when spatio-temporal dislocations appear. For these times the mode activity of the next three modes switches to
different regimes indicating a side-band activity during these periods. As these side-bands can
be identified as spatial eigenmodes, the occurrence of dislocations is due to a non-linear interaction of spatial wavemodes.

The occurrence of irregular spatio-temporal patterns can be comprehended by numerical simulations of a fluid model of the PC [10]. Here the boundaries were forced periodically in order to simulate the Säulenkopf-schwingung. Fig. 4 shows a result of the calculations for p= 1.5 Torr, r=1 cm, L=15 cm, i=14 mA and $f_c/f_h = 1.8$ as input parameters of the model that will be described in [11].

It can be concluded from our work that the route to turbulence in the discharge investigated is governed by a current dependent oscillation at the head of the PC and its commensuration to the most stable eigenmode of the PC.

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References