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NON-LINEAR THEORY OF CORRELATION REFLECTOMETRY

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1. Introduction

Correlation reflectometry is widely used technique providing the valuable information on plasma low frequency turbulence. Particularly it permits to observe density fluctuations behaviour in a tokamak discharge for indication of the transition into improved confinement regimes. The plasma probing by ordinary or extraordinary microwave is used in this diagnostics and the reflected wave spectral broadening is usually measured. In order to improve the fluctuation reflectometry wave number selectivity a more sophisticated correlative technique, using simultaneously different frequencies for probing was proposed [1,2]. The data interpretation in this technique is based on the hypothesis, that the signal backscattered off long wavelength fluctuations dominating in the turbulence spectra, comes predominately from the cut-off layer. Therefore, the turbulence correlation length is often determined there from the shift between two cut-offs at which the coherence of two reflectometry signals at different frequencies vanishes [1,3].

In contradiction to the above approach, the numerical analysis performed in [4] in 1D model in the framework of Born approximation had shown slow decay of coherence with increasing frequency difference of reflectometer channels. According to [4], poor localised small angle scattering responsible for this effect plays significant role in producing the fluctuation reflectometry signal. This prediction was not taken seriously by community, probably because in oversimplified 1D model the small angle scattering effect is caused by fluctuations possessing wave numbers, smaller or comparable to the inverse distance between the plasma boundary and the cut off.

The rigorous theoretical analysis of fluctuation reflectometry carried out in [5] in the frame of linear 2D model, applicable in the case of low turbulence level, when the probing line is observable in the spectrum of reflected wave, had shown that the scattering efficiency is maximal for poorly localised small angle scattering along the incident wave ray trajectory as well. However, unlike the 1D model, in 2D it is produced by fluctuations running in poloidal direction and possessing finite wave number. Thus it is not possible to rule this effect out by re-normalization of density profile. As in 1D model, small angle scattering results in slow decay of the correlation function of two signals and, consequently, makes questionable estimation of the turbulence correlation scale in linear regime of scattering [6].

In contradiction to these predictions a quick decay of coherence is often observed in experiment [1-3]. In some experiments the coherence is suppressed at cut off separation even smaller than the vacuum wavelength [1]. A linear theory is not able to explain these results; moreover one should not expect it to account for these observations made under conditions when no probing line was observable in the reflected spectrum [2,7].

1D model presented in this paper is based on the conclusions of the linear theory [4-6]. It treats small angle multi-scattering as the main non-linear effect and describes it using WKB approximation for the probing wave propagation. As a result the expression for the cross-correlation function for reflected signals at two probing frequencies describing the transition from linear to non-linear scattering regime and accounting for the fast loss of coherence observed in experiment is obtained. The expression for the average electric field describing the wave extinction in the non-linear regime is also given.

2. One-dimensional model

We consider O-mode reflectometry in 1D model. The propagation of incident and reflected waves is governed by the equation

$$\left\{\frac{d^2}{dx^2}+\frac{\omega^2}{c^2}-\frac{4\pi e^2\left[n(x)+\delta n(x,t)\right]}{m_e c^2}\right\}E_z(x,\omega,t)=0\,, \qquad (1)$$

where the plasma density consists of the background stationary part, $n(x)$, and turbulent perturbations, $\delta n(x,t)$, with the time-scale being slow compared to the inverse probing wave frequency and its transit time in plasma. Further we will restrict ourselves to the case of statistically homogeneous turbulence and linear density profile $n(x)=n_c\cdot x/x_c$, where x_c indicates the cut off position. Neglecting the possibility of numerous reflections of the probing wave from the plasma boundary, metallic walls and the cut-off, which is a reasonable supposition for the case of large fusion machines, and assuming density fluctuations to be small $\left|\delta n/n_c\right|\ll 1$ and long wavelength, so that $l_c \gg c/\omega$, we obtain the amplitude of the wave reflected from the cut off in the framework of WKB approximation as

$$E_{zs}(\omega,t)=E_{zi}\exp\left(i\phi_r\right). \qquad (2)$$

Here E_{zi} is the incident wave amplitude and the phase consists of two parts $\phi_r=\phi_0+\delta\phi$, regular and random, respectively. In the case of $l_c \ll x_c$, the random value

$$\delta\phi\left(t,\omega\right)=-k_0^2\cdot\int_0^{x_c}\frac{\delta n(x,t)}{n_c}\frac{dx}{k(x,\omega)}$$

with $k_0=\omega/c, k(x,\omega)=\left[k_0^2-4\pi e^2 n(x)/m_e c^2\right]^{1/2}$ can be considered as one resulting of many independent random phase contributions. Thus one can conclude that it should be a normal random process for which the average reflected field, $\left\langle E_{zs}(\omega_k,t_k)\right\rangle$, and the cross-correlation function of reflected signals at two frequencies channels $j\neq k$,

$$K_{jk}=\left\langle\left(E_{zs}(\omega_j,t_j)-\left\langle E_{zs}(\omega_j,t_j)\right\rangle\right)\left(E_{zs}(\omega_k,t_k)-\left\langle E_{zs}(\omega_k,t_k)\right\rangle\right)^*\right\rangle,$$

can be obtained in form firstly proposed in [2]

$$\left\langle E_{zs}(\omega_k,t_k)\right\rangle=E_{zi}\exp\left[i\phi_0\left(\omega_k\right)-\frac{\sigma_{kk}}{2}\right],$$

$$K_{jk}=E_{zi}\left(\omega_j\right)E_{zi}^*\left(\omega_k\right)\exp\left\{i\left[\phi_0(\omega_j)-\phi_0(\omega_k)\right]-(\sigma_{kk}+\sigma_{jj})/2\right\}\left\{\exp\left(\sigma_{jk}\right)-1\right\}.$$

In the above equation σ_{jk} is the correlation matrix, which can be expressed in terms of the fluctuation spectrum $\left\langle\delta n^2\left(x'-x'',t_j-t_k\right)\right\rangle=1/2\pi\cdot\int\tilde{n}_\kappa^2(\tau)\exp\left[i\kappa\left(x'-x''\right)\right]d\kappa$:

$$\sigma_{jk}(\Delta,\tau)=4k_0^2\sqrt{x_c(\omega_j)x_c(\omega_k)}\cdot\int\frac{d\kappa}{2\pi}\frac{\tilde{n}_\kappa^2(\tau)}{|\kappa|n_c^2}e^{i\kappa\Delta}F\left[\sqrt{\kappa x_c\left(\omega_j\right)}\right]F^*\left[\sqrt{\kappa x_c\left(\omega_k\right)}\right] \qquad (3)$$

where $\Delta = x_c(\omega_j) - x_c(\omega_k)$,$\tau = t_j - t_k$ and $F(s) = \int_0^s \exp\left(i\varsigma^2\right) d\varsigma$ is a Fresnel's integral.

3. Linear and nonlinear regimes.

It should be underlined, that poor localized small angle scattering, which in 1D model is produced by small wave numbers, make substantial contribution to σ_{jk} because of $1/|\kappa|$ singularity saturating only at $1/|\kappa| \sim x_c$. In [7] this effect was artificially suppressed by choosing a specific spectrum of density fluctuations, $\tilde{n}_\kappa^2 \sim \kappa$.

For large plasma, where $x_c \gg l_c$, σ_{kk} value can be estimated from Eq.(3) with logarithmic accuracy as

$$\sigma_{kk} \simeq k_0^2 x_c(\omega_k) \cdot \frac{\tilde{n}_{\kappa=0}^2(0)}{n_c^2} \ln \frac{x_c(\omega_k)}{l_c}. \tag{4}$$

For small density perturbation level, satisfying the condition

$$k_0^2 x_c \ln(x_c / l_c) \cdot \tilde{n}_{\kappa=0}^2(0) / n_c^2 \ll 1, \tag{5}$$

the probing line is dominant in the reflected spectrum and the Born approximation approach is valid. It results in the following simplified formula for the cross-correlation function:

$$K_{jk} = E_{zi}(\omega_j) E_{zi}^*(\omega_k) \sigma_{jk} \exp\left\{ i\left[\phi_0(\omega_j) - \phi_0(\omega_k) \right] \right\}, j \neq k.$$

Estimating σ_{jk} with logarithmic accuracy we obtain for $\Delta \gg l_c$

$$\sigma_{jk} \simeq k_0^2 x_c \ln(\Delta / l_c) \cdot \tilde{n}_{\kappa=0}^2(\tau) / n_c^2.$$

As it is seen, the coherence only very slowly decays with growing separation of cut offs. Such a logarithmic decay was observed in 1D numerical linear full-wave consideration [4] and obtained in 2D analytical theory, based upon full-wave Born approximation [6]. Thus we can conclude that it represents a meaningful physical effect caused by small angle scattering and not related to the WKB approximation used in the analysis. It is evident that under condition (5) the fluctuation reflectometry is providing data on the frequency spectrum of large-scale component of turbulence and is not capable of its coherence length estimation.

According to (4) transition to the non-linear regime where the probing line is no longer observable in the reflected spectrum occurs when the following criteria is fulfilled

$$k_0^2 x_c \cdot \tilde{n}_{\kappa=0}^2(0) / n_c^2 \ln(x_c / l_c) \geq 1. \tag{6}$$

When (6) holds, the situation can be considered in detail when the small angle scattering is already deep in the non-linear regime, so that an inequality $k_0^2 x_c \cdot \tilde{n}_{\kappa=0}^2(0) / n_c^2 > 1$ is valid. Under this condition the correlation in two channels is not negligible only for close enough cut offs $\Delta \ll l_c$ and for time difference τ shorter than the turbulence correlation time $\tau \ll t_c$. Using these inequalities and assuming the turbulence spectrum to be symmetric we can represent σ_{jk} in the form of Taylor series expansion in Δ / l_c and τ / t_c and finally obtain

$$K_{12}(\Delta, \tau) \simeq E_{zi}(\omega_1) E_{zi}^*(\omega_2) \exp\left\{ i\left[\phi_0(\omega_1) - \phi_0(\omega_2) \right] - (\kappa_r \Delta)^2 / 2 - (\Omega_r \tau)^2 / 2) \right\}$$

with

$$\Omega_r^2 = \frac{2}{\pi} k_0^2 x_c(\omega_1) \int \frac{d\kappa}{|\kappa| n_c^2} \left. \frac{\partial^2 \left[\tilde{n}_\kappa^2(\tau) \right]}{\partial \tau^2} \right|_{\tau=0} \left| F\left[\sqrt{\kappa x_c(\omega_1)} \right] \right|^2 \tag{7}$$

$$\kappa_r^2 = k_0^2 x_c(\omega_1) \int d\kappa \frac{|\kappa| \tilde{n}_\kappa^2(0)}{n_c^2} \tag{8}$$

According to (7) parameter Ω_r is dominated by long scale fluctuations responsible for small angle scattering all over the plasma. Consequently we should conclude that the fluctuation reflectometry spectrum in non-linear regime is as well not localised to the cut off vicinity and provides information on the turbulence frequency spectrum and level of density perturbations in a wide region between antennae and cut off. On contrary, the role of long scales is suppressed in the decay of the coherence, as it is seen from expression (8) for parameter κ_r. That means the decay of coherence in non-linear regime provides us with the local information from the cut off vicinity. However, as it is seen from (6) and (8), κ_r is much greater then the turbulent wave number. It's rather determined by the relative turbulence level.

4. Conclusion

According to the 1D model, the correlation reflectometry data in linear regime is strongly affected by the small angle scattering reducing diagnostic's localization and complicating the coherence length estimation. In non-linear regime the reflected signal spectrum broadening is also non-localised to the cut-off and determined by the wide region of plasma in which the turbulence is situated. In spite of this, the coherence decay in radial correlation reflectometry deep in non-linear regime is only sensitive to the turbulence level in the cut-off. The physical reason for this effect is provided by the phase mismatch produced by the part of plasma between cut offs evanescent for a smaller probing wave frequency. The coherence decays quickly if this phase mismatch's average absolute value exceeds π, thus making two signals independent.

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