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Profile Consistency Theory Based on the Constant Magnetic Entropy Concept:Theory and Observation

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1. Summary of theory.

After the pioneering observation of Coppi [1] on the profile consistency, variation principles based on energy minimization have been proposed in the literature to identify the “natural “or ”optimal” tokamak equilibria [2]. However one may observe that the approach based on minimum energy suffers of the conceptual difficulty that the tokamak is an open thermodynamic dissipative system and the relaxation processes themselves are basically dissipative, so the system is not conservative. It seems therefore more appropriate to look for a variation principle in the framework of thermodynamics. The difficulty here is that we have to deal with collisionless or quasi-collisionless systems, so the equilibrium can be far from maxwellian. The problem is how to construct an entropy functional and a thermodynamic formalism that embody from the beginning the very existence of a collisionless magnetic equilibrium. The line of attack to this problem is offered by the information theory. In the framework of this theory the existence of a collective magnetic equilibrium is introduced “a priori” as a “testable” constraint whose probability in the space of all possible magnetic configurations suitably constrained is determined by the entropy principle and is to be confirmed by the observations. This leads to the following entropy functional

$$S \propto -\int_{\Omega} j^2 dV + \left(\mu^2 c / 4\pi\right) \int_{\Omega} \vec{j} \bullet \vec{A}\, dV \qquad (1)$$

where $\vec{j}$ is the current density, $\vec{A}$ the vector potential, μ is a Lagrange multiplier to be determined and Ω is the plasma volume under consideration. While S should be maximum in an isolated system (e.g. a pinch surrounded by a perfect conductive shell) it can only be stationary in an open system in which the external injection of power is balanced locally by internal dissipation. Applying Maxwell equations and adding the contributions p_A and p_L of auxiliary power and losses one obtains from (1)

$$dS / dt = \int_{\Delta\Omega} \left(\left(\vec{E} / \mu^2 \right) \bullet \nabla^2 \vec{j} + \vec{E} \bullet \vec{j} + p_A - p_L \right) dV = 0 \qquad (2)$$

where $\vec{E}$ is the electric field due to external induction and $\Delta\Omega$ is any toroidal ring in the plasma. Eq.(2) implies $\left(\vec{E} / \mu^2 \right) \bullet \nabla^2 \vec{j} + \vec{E} \bullet \vec{j} + p_A - p_L = 0$. On one hand the comparison of (2) with the stationary power balance implies a relation of the first term of (2) with the thermal diffusivity. On the other, the compatibility with the Grad-Shafranov (G-S) equation implies a strong restriction on the functions $p(\psi)$ and $F(\psi)$ of the G-S expression of the toroidal current density. In Ref [4] the predictions on the profiles were compared with TCV data in purely ohmic sawtoothing discharges (in this case $\Delta\Omega$ is taken in the confinement region) with ohmically relaxed current ($j \propto T^{3/2}$). The theory has been extended to situations with ECRH and with $q \geq 1$ in the plasma. Just as in the ohmic case, in presence of the $q = 1$ surface, the normalized (on $q = 1$) pressure profile (consistent with G-S) is given by a function $p_N(x)$ depending on only one unknown parameter K, related to the profile width $< p_N >$. $K < 1$ implies the minimum of the plasma energy $U = (3/2)\int nTdV$ and the profile concavity. This does not hold for $q \neq 1$ in the plasma, U being at a maximum. When $n(\psi)$ and $q(\psi)$ are monotonic it is possible to choose K and η such that $p_N(x)$ above can be closely approximated by taking $n(\psi)/\hat{n} = (1/q(\psi))^{\eta}$ (Turbulent Equipartition, TEP) and $T \propto j^{2/3}$ (see [5] where $\eta \approx 1$ in the ohmic case). Thus, the privileged states are entirely characterized only by $P_{aux}, B, R, q(0), q(0.95) or\ I_p, s, \bar{n}, \tau_E, V(=2\pi RE), \rho_{inv}$ where s is the outer radius of the confinement region and V the loop voltage. In particular, when τ_E is given by ITER89-P the magnetic state depends on the combination $P_0 \equiv P_{Aux}/\bar{n}^{0.77}BRs$. Thus the profile consistency with auxiliary heating does not mean necessarily a unique rigid profile but that possibly different profiles are invariant with respect to a certain family of parameter transformations (e.g. those leaving P_0 invariant).

2. Comparison with experimental data

With the purpose of checking the reliability of the theoretical predictions, the experimental profiles from the FTU tokamak (R=0.93m, a=0.3m B=4-8T) obtained for different heating conditions have been analyzed following the above theory. Three cases are here presented: a purely ohmic one, and two EC heated shots with off axis power deposition. Two 140 GHz gyrotrons were used, with total

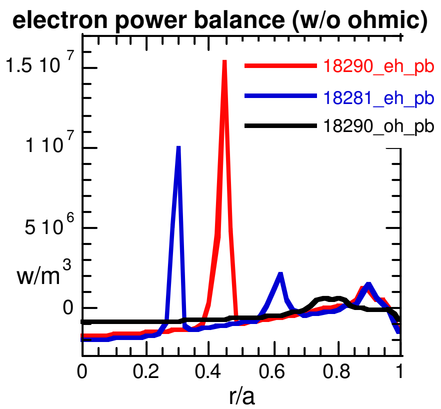

Fig.1. Net power density on the electrons, calculated with EVITA power balance code.

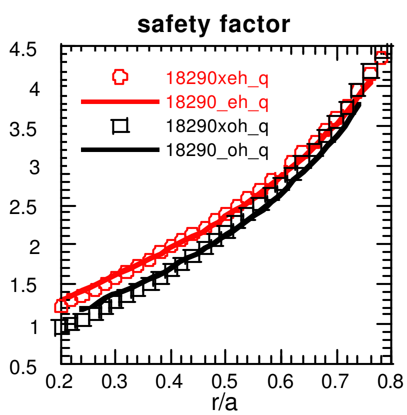

Fig. 2. Safety factor from EVITA (symbols, label x) and from theory (lines). eh =add. heating, oh= ohmic

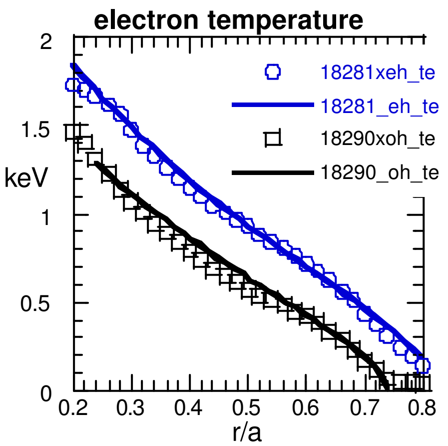

Fig.3. ECE meas. (symbols) and from theory (lines)

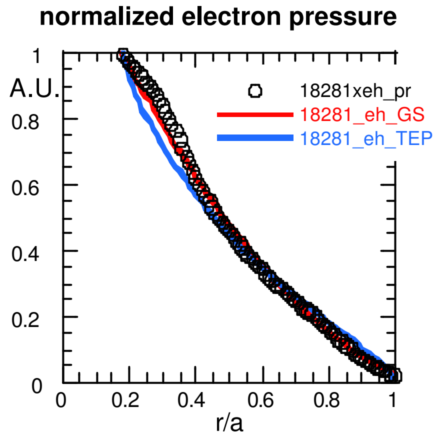

Fig.4. Symbols: experiments, red:G-S, blue: TEP

injected power up to 0.85 MW. In the shot 18290 the whole EC power was absorbed at r/a≈0.45, whereas in the shot 18281 the absorbed power was shared in two separated layers (r/a≈0.3 and r/a≈0.63) using poloidal steering of the beams. The description of the experimental conditions including a detailed interpretative transport analysis of these shots has been presented elsewhere [6]. In all three cases a moderate sawtooth activity is present in the plasma core. The analysis attains to the so-called confinement region of the plasma between the inversion radius of sawteeth and the scrape off layer in which radiative losses are dominant. For this group of FTU shots (B≈5.7T, I_p=0.4MA, ne=0.8-0.9 $10^{20}m^{-3}$ line averaged) the confinement region roughly extends from 0.2 to 0.8 r/a. The net power density balance on the electron population is assigned as showed in fig.1, in which electron (EC) heating, radiative and e-i losses are taken into account respectively in the terms p_A and p_L of eq.(2). Only a few additional physical parameters are necessary in order to obtain, accordingly with the above mentioned theory, the relevant plasma profiles. From the operative point of view, calculation of the profiles is performed with a simple code that numerically solves eq. (2), given as input B, R, ρ_{inv}, s, I_p, $q(0)$, a parameter μ which is univocally assigned from V if τ_E and $\overline{n}$ are assumed. Fig.2 shows the safety factor profile for both the ohmic and the EC cases obtained from the theory compared with EVITA interpretative code calculation. The two cases (two different temporal slices of the same shot) are in some way extremes: a very high local power density in the EC phase and a substantial equilibrium of non diffusive losses and ohmic input in the ohmic one, implying a marginal contribution of diffusive transport. The

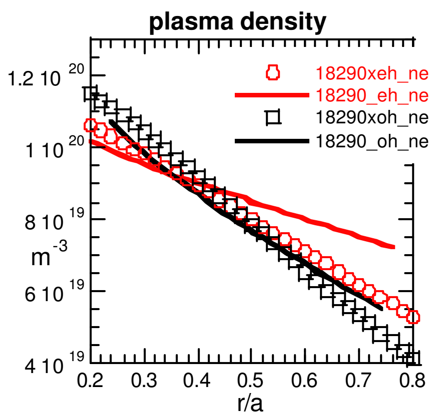

Fig.5. Density profile from DCN interferometry (symbols) and from TEP. oh:η =0.57; eh: η =0.3

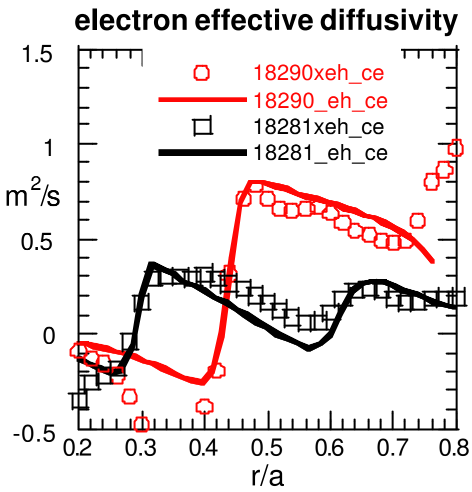

Fig.6 Symbols: EVITA power bal; lines: theory.

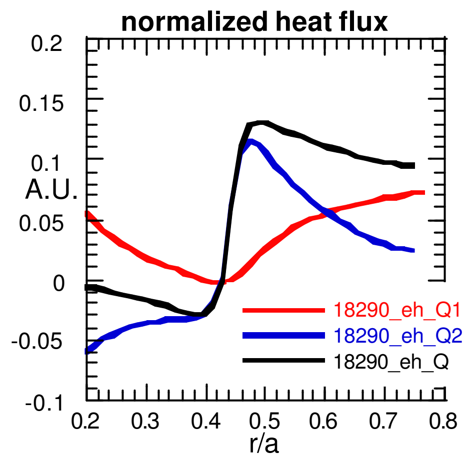

Fig.7. Addenda of the normalized heat flux Q from theory

electron temperature of fig. 3 is obtained from the calculated current density and Spitzer resistivity. As mentioned above, introducing the simple relation $n(\psi) \propto (q(\psi))^{-\eta}$ one can find an optimal value of η for which the normalized pressure obtained from the calculated temperature of fig.3 fits closely $p_N(x)$ and the experimental profile (fig.4). This gives also a good fitting of the density profile in the Ohmic case while for EC plasmas the description is not completely satisfactory (fig.5).

A power balance relationship between heat flux and temperature gradient can be derived from equation (2), consistent with the theory. The electron effective diffusivity is shown in fig.6, compared with EVITA calculation. The normalized heat flux Q can be written as the addition of the two terms Q1$\propto$ shear/q+β (where β is constant) and Q2 shown in fig. 7. Q1 is vanishing at the transition between low to high transport region, with the consequence that the transition is always associated with the fixed value -β of the ratio shear/q, where β is determined only by the poloidal magnetic configuration.

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