

Dissipative drift ballooning instabilities in tokamaks

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Abstract

The linear stability of high-toroidal-number drift-ballooning modes in tokamaks is investigated with a model that includes resistive and viscous dissipation, and assumes the mode frequency to be comparable to both the sound and diamagnetic frequencies. The coupled effect of ion drift waves and electron drift-acoustic waves is shown to be important, resulting in destabilization over an intermediate range of toroidal mode numbers. The plasma parameters where the assumed orderings hold would be applicable to the edge conditions in present day tokamaks, so these instabilities might be related to the observed quasi-coherent edge-localized fluctuations.

Experimental observations in the Alcator C-Mod tokamak indicate a well defined transition that separates ELM free H-mode behavior, in which the edge plasma is fluctuation free, from Enhanced D_α (EDA) behavior, in which a localized instability (the quasi-coherent mode) is present in the edge pedestal. It is possible that these observations could be explained by a linear stability threshold. Even though ideal-MHD ballooning modes might be near their stability threshold, electron and ion diamagnetic corrections are likely to be important, as are interactions with acoustic waves. Dissipative effects (especially resistive effects) are also likely to be important. In this paper we investigate the linear stability of drift ballooning modes using a simple model for the plasma equilibrium (the $s - \alpha$ model), but including a variety of non-ideal physical effects. We derive a simple eigenmode equation for drift ballooning modes by considering an optimal ordering in which $\omega \sim \omega_s \sim \omega_{*j} \sim \mu k_\perp^2 \sim \eta_\parallel k_\perp^2$, where ω is the mode frequency, $\omega_s \equiv C_s/Rq$ is the frequency for sound propagation over a connection length with $C_s^2 = (T_i + T_e)/m_i$ and R the major toroidal radius, $\omega_{*j} = (\mathbf{k} \times \mathbf{b}) \cdot \nabla p_j / (NeB)$ is the diamagnetic frequency for species j , $\mu = 0.3\nu_{ii}\rho_i^2$ is the classical perpendicular viscosity with $\rho_i = (T_i/m_i\omega_{ci}^2)^{1/2}$ the ion Larmor radius, and η_\parallel is the longitudinal Spitzer

resistivity. The $s - \alpha$ equilibrium model with $s = rq'/q$ and $\alpha = -2Rp'q^2/B^2 \sim O(1)$, is employed and a two length scale averaging formalism is developed, using $\omega/\omega_A \ll 1$ as the expansion parameter, with $\omega_A = V_A/Rq$, $V_A^2 = B^2/(Nm_i)$. The resulting eigenmode equation contains the following physical effects: (1) ideal-MHD instability drive (through the boundary condition), (2) resistive diffusion leading to resistive ballooning modes, (3) sound wave propagation, (4) two-fluid diamagnetic effects which modify sound waves into electron drift-acoustic modes and introduce finite ion Larmor radius effects, and (5) perpendicular ion viscosity. The two different classes of resistive ballooning modes, viz those driven locally (in radius) by the geodesic curvature (the Carreras-Diamond¹ modes) and those driven by the ideal-MHD energy (characterized by the stability index Δ'_B) are both described by the eigenmode equation.

A simple set of reduced two-fluid equations has been given by Zeiler et al.². We further simplify these equations by assuming that the equilibrium ion and electron temperatures are constant, and ions and electrons are isothermal. We also specialize them to the $s - \alpha$ equilibrium model. Finally we adopt our "drift ordering", that assumes all the frequencies, ω , ω_s , ω_{*j} , $k_\perp^2\mu$, $\eta_\parallel k_\perp^2$ to be comparable and small compared to the Alfvén frequency ω_A . It follows that the resistivity, the viscosity and the inertia only become important at large values of the extended ballooning variable θ . A two length scale analysis is performed by introducing the stretched variables $Z = \epsilon_\eta s\theta$ where $\epsilon_\eta = [\eta_\parallel n^2 q^2 / (r^2 \omega_A)]^{1/3}$, and $X^2 = i\epsilon_\eta \omega_A Z^2 / (\omega - \omega_{*e})$ in the secular terms of the ballooning equations. The result is an averaged ballooning equation of the form:

$$s^2 \frac{d}{dX} \left(\frac{X^2}{1 + X^2} \frac{dU_0}{dX} \right) - [X^2 Q + X^4 T] U_0 = 0, \quad (1)$$

with

$$Q(X) = i\hat{\omega}(\hat{\omega} - \hat{\omega}_{*i})(\hat{\omega} - \hat{\omega}_{*e}) [1 + 2q^2 P(X)] - \frac{\alpha^2}{2} [1 - P(X)], \quad (2)$$

$$P(X) = \frac{\omega_s^2}{\omega_s^2 - [\omega + 4\hat{\mu}(\omega - \omega_{*e})X^2](\omega - \omega_{*e})}, \quad (3)$$

and

$$T(X) = i\hat{\mu}(\hat{\omega} - \hat{\omega}_{*e})^2(\hat{\omega} - \hat{\omega}_{*i})[1 + 8q^2 P(X)]. \quad (4)$$

In these expressions $\hat{\mu}$ is a normalized perpendicular viscosity and the notation $\hat{\omega}_j$ denotes the dimensionless frequency $\hat{\omega}_j = \omega_j / (\epsilon_\eta \omega_A)$. Equations (1-4) describe the coupling

of visco-resistive ballooning modes to drift-acoustic waves. The novel feature of the foregoing two-scale analysis, lies in the ordering $\omega/\omega_s \sim 1$. In much previous analyses either the ordering $\omega/\omega_s \ll 1$ was assumed with the result that the geodesic curvature (Carreras-Diamond) drive was lost, or the limit $\omega/\omega_s \gg 1$ was taken so that sound wave propagation was neglected. Our equations provide a bridge between these two extremes, not only for studying visco-resistive ballooning modes, but also for investigating finite Larmor radius effects on ideal modes. The relevant dispersion relations are obtained by asymptotically matching the solution of Eqs.(1-4) to the solution in the ideal-MHD region:

$$U_0(Z \rightarrow 0) = 1 + \frac{\epsilon_\eta \Delta'_B}{Z}. \quad (5)$$

Considering drift effects on dissipation-free modes, we obtain the result that the coupling to drift-acoustic waves modifies the conventional result so that, assuming ideal instability, a range of intermediate toroidal mode numbers is always unstable. Specifically, the finite Larmor radius stabilization of ideal modes is lost when the ion drift wave is resonant with one of the electron drift acoustic wave branches: $\omega_{*i}(\omega_{*i} - \omega_{*e}) = (1 + 2q^2)\omega_s^2$. At this point the dispersion relation reduces to:

$$\frac{\gamma}{\omega_A} = -\frac{2sq}{\Delta'_B [3(1 + 2q^2)]^{1/2}}, \quad (6)$$

so that the growth rate of the ideal instability is actually enhanced by a factor $(2/3^{1/2})q$ above the value it would have in the absence of diamagnetic effects.

Zero-viscosity drift-resistive modes are driven unstable by the pressure gradient α and, for negligible Δ'_B drive or $\epsilon_\eta \Delta'_B \ll 1$, their dispersion relation is given by $Q = 0$ or

$$(\omega - \omega_{*i})[(1 + 2q^2)\omega_s^2 - \omega(\omega - \omega_{*e})] - \frac{i}{2}\alpha^2\omega_A^3 = 0. \quad (7)$$

In the $\omega_s \rightarrow 0$ limit the pressure gradient drives instability of the low frequency branch of the electron drift acoustic wave³, but the growth rate is strongly enhanced when the low frequency branch of the electron drift acoustic wave is degenerate with the ion drift wave. This is the same mechanism responsible for the loss of finite Larmor radius stabilization of ideal modes.

Perpendicular viscous effects enter Eqs.(1-4) in two ways: the term T contains the viscous drag on the perpendicular velocity and the factor P contains the drag on the

parallel flow. Our expression for $\hat{\mu}$ assumes its classical value and is therefore much less than unity. So we can expand $P(\hat{\mu}) = P_0 + \hat{\mu}P_1$ and approximate Eq.(1) by

$$s^2 \frac{d}{dX} \left(\frac{X^2}{1+X^2} \frac{dU_0}{dX} \right) - [X^2 Q_0 + X^4 T_0] U_0 = 0 \quad (8)$$

with $Q_0 = Q(P_0)$ and $T_0 = T(P_0) - 2\hat{\mu}\alpha^2 P_0^2 [(\omega - \omega_{*e})/\omega_s]^2$. Now we can obtain a variational expression for the dispersion relation using the trial function

$$U_0(X) = X^{1/2} K_{1/4}(\sigma X^2) \left\{ 1 - [\epsilon_\eta \Delta'_B - (\sigma/2)^{1/2} \Gamma(3/4)/\Gamma(5/4)] X \right\}, \quad (9)$$

where $K_{1/4}$ is the modified Bessel function and σ is a variational parameter. This choice of trial function incorporates the correct boundary condition matching the ideal-MHD solution as $X \rightarrow 0$, and reduces to the exact eigenfunction in the inviscid limit $T_0 = 0$. For negligible Δ'_B drive, the resulting variational dispersion relation is $Q_0 + 2.46 T_0^{2/3} = 0$. We have obtained numerical solutions of this variational dispersion relation. They show that viscosity does not suppress the intermediate- n instability associated with the coupling of ion drift and electron drift-acoustic waves.

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