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Critical Behaviour of Fast Particles Induced by NBI in Sawtooth Stability Modelling

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The question of extended sawtooth quiescent periods and the relationship to strong kinetic stabilisation is a crucial issue for future large tokamaks. A particularly relevant issue addressed here is the response of neutral beam injected (NBI) ions on internal kink stability. Such a study is of interest for a number of reasons, not least because strongly sheared toroidal plasma rotation of up to 30 kHz has been measured in NBI discharges in JET [1], and correlations between changes in sheared plasma rotation and sawtooth stability have been predicted [2] and observed [3]. Also of interest is the effect of the injection angle, and resulting pitch angle distribution, on fast ion stabilisation. In particular it is seen that fast ion stabilisation of sawteeth in JET NBI plasmas could be dominated by fluid effects rather than kinetic effects. This follows because fluid effects are found to be strongly stabilising where there are large fractions of passing ions, which unlike trapped ions, spend more time in the inner region of good curvature. Such speculations can be made, for the first time, because all contributions significantly affected by anisotropy are evaluated accurately and included in the evaluation of the potential energy δW of the ideal internal kink instability.

The analysis is split into the response of the core plasma 'c' and the hot minority NBI ions 'h' on internal kink stability. The temperature of the core plasma is assumed to be low enough such that thermal ions do not give rise to kinetic effects, and the distribution function of the core plasma is assumed to be isotropic in pitch angle. However, NBI ions are in general anisotropically distributed and are fast enough to give rise to kinetic effects, which in turn are strongly dependent on the plasma rotation driven by NBI. The plasma anisotropy is limited by the empirically justified assumption that the isotropic core pressure is around two thirds of the total pressure. The following analysis therefore seeks to obtain the fast ion response of NBI in a weakly anisotropic plasma which is flowing at sub-sonic speeds.

The equilibrium hot ion pressure tensor can be described [4] by the double adiabatic model: $\underline{\underline{P_h}} = P_{h\perp}\underline{\underline{I}} + (P_{h||} - P_{h\perp})\hat{\mathbf{e}}_{||}\hat{\mathbf{e}}_{||}$, where $\underline{\underline{I}}$ is the unit dyadic and $\hat{\mathbf{e}}_{||}$ the unit vector parallel to the magnetic field. If the equilibrium is perturbed by a MHD mode with displacement vector $\boldsymbol{\xi}$, a potential energy term $\delta W_{hf} = \frac{1}{2}\int d^3x\, \boldsymbol{\xi}^* \cdot (\boldsymbol{\nabla} \cdot \underline{\underline{\delta P_h}})$ (where f denotes 'fluid') arises as a consequence of a convective perturbation of the fast ion distribution function: $\delta F_h = -\boldsymbol{\xi} \cdot \boldsymbol{\nabla} F_h$. It can be shown that [5]:

$$\delta W_{hf} = -\frac{1}{2}\int d^3x \left[\boldsymbol{\xi}\cdot\boldsymbol{\nabla}\left(P_{h\perp} + P_{h||}\right) - \left(P_{h\perp} + P_{h||} + C_h\right)\frac{\boldsymbol{\xi}\cdot\boldsymbol{\nabla}B}{B}\right]\frac{\boldsymbol{\xi}^*\cdot\boldsymbol{\nabla}B}{B}, \tag{1}$$

where $C_h = \pi m_h \int_{-\infty}^{\infty} d\mathcal{E} \int_0^{\infty} d\mu\, (B(\mu B)^2/|v_{||}|)\partial F_h/\partial\mathcal{E}$ is defined for a hot ion distribution $F_h(r, \mathcal{E}, \mu)$ with mass m_h. Equation 1 quantifies the effect of anisotropy on the internal kink mode. In particular the second term in the square brackets of Eq. 1 was identified by Mikhailoskii [6] as being a possible candidate for an anisotropically driven instability. The first term in Eq. 1 also quantifies the effect of anisotropy, but this time through the poloidal dependence of the pressure components $P_{h\perp}, P_{h||}$.

The leading order contributions to the toroidal plasma rotation arise from a finite pressure gradient of the thermal ions and from an equilibrium radial electric field E. It is appropriate to assume that poloidal flow is strongly damped [7], such that the toroidal rotation of the core plasma is given by $\Omega = \Omega_E + \omega_{*pi}$ with $\Omega_E = qE/B_0r$ and $\omega_{*pi} = -qP_i'/eZn_iB_0r$, where eZ, n_i and P_i are respectively the charge, density and pressure of thermal ions and $' \equiv d/dr$. The common source of rotation for the different plasma species is that arising from the equilibrium electric field $\Omega_E(r)$. Hence, for NBI scenarios, one can envisage a mechanism whereby the momentum of the injected ions initially rotates the plasma, which in order to satisfy the force balance equation, must establish a radial electric field.

For analysing the hot ion response of 'sawtooth modes' with mode frequency ω, it is usually assumed that $\omega_{*h} \gg \omega$ and $\langle\omega_{mdh}\rangle \gg \omega$, where ω_{*h} and $\langle\omega_{mdh}\rangle$ are respectively the hot ion diamagnetic and bounce averaged magnetic drift frequencies. This is appropriate because, in contrast to 'fishbone modes', which generally satisfy [8] $\omega \sim \langle\omega_{mdh}\rangle$, sawteeth are believed to have mode frequencies of the order of ω_{*pi}. However, when toroidal rotation is present, the mode frequency is Doppler shifted [2]: $\omega \rightarrow \omega - \Omega_E(r_1) = \tilde{\omega}$. The dispersion relation for the internal kink mode can then be written in such a way that strongly subsonic sheared toroidal plasma rotation only appears in the kinetic contribution [2]. The present paper will be concerned with differential rotation $\Delta\Omega_E(r) = \Omega_E(r) - \Omega_E(r_1)$ of variable magnitude up to $\Delta\Omega_E \sim \langle\omega_{mdh}\rangle$, and the following orderings apply: $\langle\omega_{mdh}\rangle \sim \omega_{*h} \gg \tilde{\omega} \sim \omega_{*pi}$. The hot kinetic contribution to the internal kink mode is then:

$$\delta W_{hk} = -2^{5/2}\pi^3 m_h \frac{\xi_0^2}{R_0}\int_0^{r_1} dr\, r B_0 \int_{1/B_{\max}}^{1/B_{\min}} d\alpha \frac{I_q^2}{K_b}\int_0^\infty d\mathcal{E}\,\mathcal{E}^{5/2}\frac{\partial F_h}{\partial \mathcal{E}}\left[\frac{\omega_{*h} + \Delta\Omega_E}{\langle\omega_{mdh}\rangle + \Delta\Omega_E}\right], \qquad (2)$$

where $\alpha = \mu/\mathcal{E}$ is a pitch angle variable, and K_b and I_q are defined for example in Ref. [2]. Assuming that the mode does not lie inside the gap in the Alfvén continuum $0 < \tilde{\omega} < \omega_{*pi}$, the ideal stability criterion is given by $\delta W_T + \delta W_{hf} + \delta W_{hk} > 0$, where δW_{hk} denotes the real part of Eq. (2) and δW_T is the toroidal contribution to stability as described for example by Bussac *et al* [9].

The hot ion distribution function that is appropriate for this study will be invariant to rapid gyro-motion and finite orbit widths. Thus it must be expressible in the form $F_h(\mathcal{E}, \mu, r)$. The absolute velocity dependence of the NBI population is approximately described by a simple slowing down distribution $\sim 1/v^3$. However, the velocities of the minority ions will be focused to some extent in the direction of injection. A model distribution function [10] which includes these effects is as follows:

$$F_h(\mathcal{E}, \mu, r) = \frac{c(r)}{\mathcal{E}^{3/2}}\exp[-(\lambda - \lambda_0)^2/\Delta\lambda^2] \qquad (3)$$

for $0 \le \mathcal{E} \le \mathcal{E}_m$ and $F_h = 0$ for $\mathcal{E} > \mathcal{E}_m$. The pitch angle $\lambda = B_0\mu/\mathcal{E}$ is valid for $0 \le \lambda \le 1/(1-\epsilon)$, with $B = B_0(1-\epsilon\cos\theta)$. The coefficient $c(r)$ assumes the role of normalising the distribution function. It is proportional to $P_h(r) \equiv \left(\langle P_{h\perp}\rangle + \langle P_{h||}\rangle\right)/2$ where angular brackets denote averaging over poloidal orbits (both passing and trapped). The parameter λ_0, which describes the mode (or central) pitch angle is to leading order defined in terms of the angle of injection χ (from perpendicular) as $\lambda_0 \approx \cos^2\chi$. Hence for perpendicular injection ($\chi = 0$) it is clear that $\lambda_0 = 1$, which indicates that the distribution function is peaked in trapped space. For azimuthal injection ($\chi = \pm\pi/2$) the central pitch angle is $\lambda_0 = 0$. The spread of the distribution in pitch angle is governed by the parameter $\Delta\lambda > 0$. For example, on letting $\Delta\lambda \rightarrow 0$, Eq. (3) becomes a delta function (with discontinuity located at λ_0) such as that used by Chen *et al* [8] to model the effects of NBI on fishbones. The other extreme is $\Delta\lambda \gg 1$ which yields an isotropic distribution, independent of λ_0, such as that used in Ref. [11] to analyse the effects of slowing-down alpha particles on the internal kink mode.

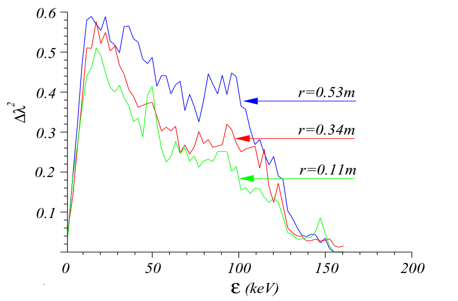

Figure 1: Showing Transp simulations for JET discharge 53595 which yield $\Delta\lambda(\mathcal{E},r)$ and also λ_0 (not shown here).

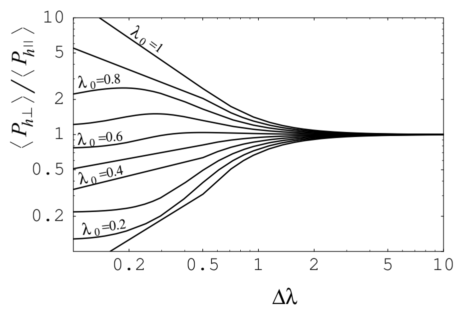

Figure 2: The anisotropy A_h verses $\Delta\lambda$ for different λ_0.

It is possible to parameterise the distribution function of Eq. (3) by employing the Transp code [12] to post-processes the distribution function from experimental data. One can then fit Eq. (3) to the computed distribution function and obtain the parameters λ_0 and $\Delta\lambda$. Figure 1 demonstrates this procedure for JET discharge 53595. In particular, while the central pitch angle is $\lambda_0 \approx 0.5$ for this discharge, the pitch angle width $\Delta\lambda$ exhibits a functional dependence on $\mathcal{E}$ and r. This is to be expected because preference in pitch angle will become less (but still significant) as the minority ions slow down due to collisions. However, for this paper, it is sufficient to employ an effective pitch angle width which is an average value over energy and radius. Hence, for discharge 53595 the effective pitch angle width is approximately $\Delta\lambda = 0.5$, and is in fact close to the actual width within the range $0 < r < r_1 \sim 0.4m$

and $40keV < \mathcal{E} < 80keV$. Evaluating the poloidal averaged second moments of Eq. (3) facilitates the plot of Fig. 2 which shows $A_h = \langle P_{h\perp}\rangle / \langle P_{h||}\rangle$ versus $\Delta\lambda$ for different λ_0. It can be seen that the isotropic limit $A_h = 1$ is rapidly reached for $\Delta\lambda > 2$ regardless of λ_0, while $A_h \sim 10$ or $\sim 1/10$ can be obtained for sufficiently small pitch angle width ($\Delta\lambda \sim 0.15$) and $\lambda_0 > 0.9$ or $\lambda_0 < 0.2$ respectively. For JET discharge 53595, for which $\Delta\lambda \approx 0.5$ and $\lambda_0 = 0.5$, one finds that $A_h \approx 0.7$.

The stability of the internal kink mode is now determined for various NBI scenarios. The fluid contribution of Eq. (1) is obtained upon substituting the leading order eigenmode (the top hat), and together with the kinetic contribution of Eq. (2) is solved exactly upon assuming the model distribution function of Eq. (3). The following profiles and parameters typical of JET equilibria are employed: $a = 1.25$m, $R_0 = 3$m, $B_0 = 3$T. The safety factor profile is $q = q_0(1 + \lambda_q(r/a)^{2\nu_q})^{1/\nu_q}$ with $q_0 \equiv q(0) = 0.75$, $\lambda_q = 22.62$ and $\nu_q = 1.74$ which gives $r_1/a = 0.36$ and $q_a = 4.6$. The poloidal averaged pressure profile for the hot ions is given by $P_h(r) = P_{h0}[1-(r/a)^2]^2$ with P_{h0} the central pressure. This profile represents well the TRANSP simulations. One can write $P_{h0} = e\, n_{h0}T_{h0}$, where e is the absolute charge of the electron, n_{h0} is the density of hot ions and T_{h0} the corresponding temperature (units of electron volts) for a Maxwellian distribution. Quantities chosen here are $n_{h0} = 0.135 \times 10^{19}/m^3$ and $T_{h0} = 80$ keV. Such a temperature corresponds to that produced by the Octant 4 NBI system in JET. Note that although the anisotropy is varied in the following sections, $P_h(r) = \langle P_{h\perp} + P_{h||}\rangle /2$ remains unchanged throughout. The injection energy $\mathcal{E}_m$ arrives from the property $m_h\mathcal{E}_m/eT_{h0}(eV) \sim 1$. Hence $m_h\mathcal{E}_m/e = 80$ keV is employed. The charge coefficient Z_h is chosen to be unity, i.e. either hydrogen or deuterium is assumed. The pressure profile of the core plasma enters $\delta W_h = \delta W_{hf} + \delta W_{hk}$ through a parameter of the magnetic precession frequency [13]. The core plasma pressure profile assumed is $P_c = P_{c0}[1 - (r/a)^2]^3$, where $P_{c0} = e(n_{i0}T_{i0} + n_{e0}T_{e0})$, with $n_{i0} = n_{e0} = 4 \times 10^{19}$ and $T_{i0} = T_{e0} = 4.5$ keV. Hence the ratio of the core pressure and hot ion pressure $P_c/P_h = 10/3$ in the centre, which is a typical value for NBI experiments. For this choice of total pressure $P_c + P_h$ and q profile, Bussac's [9] toroidal term is very small ($\delta\hat{W}_T = -0.00066$) such that ideal stability is largely determined by $\delta\hat{W}_h = \delta W_h/(6\pi^2 R_0\xi_0^2\epsilon_1^4 B_0^2/\mu_0)$. The toroidal rotation profile which accurately fits the data of Ref. [1] is $\Omega_E(r) = \Omega_{E0}\left[1 - (r/a)^2\right]^{3/2}$. Hence for the parameters chosen, the differential rotation $\Delta\Omega_E(r) = \Omega_E(r) - \Omega_E(r_1)$ at the centre is $\Delta\Omega_E(0) \approx 0.2\,\Omega_{E0}$.

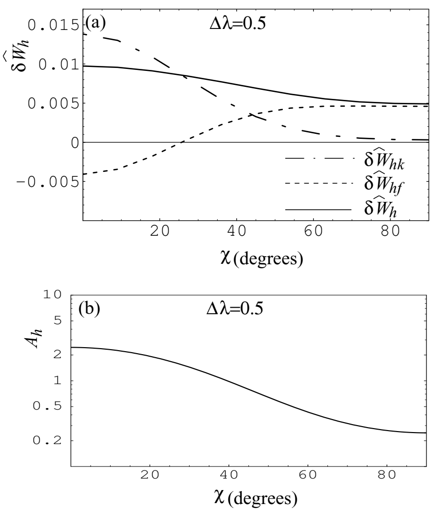

Figure 3: Showing (a) the hot ion response as a function of χ for $\Delta\lambda$=0.5 and zero plasma rotation. (b) Depicts a linear−log plot of the anisotropy A_h.

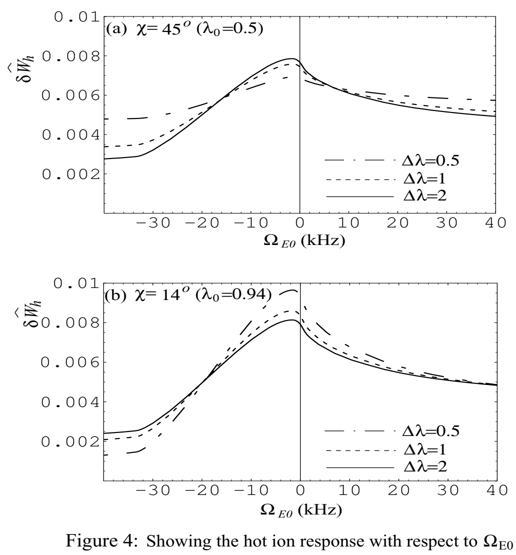

Figure 4: Showing the hot ion response with respect to Ω_{E0} for three differing pitch angle widths. (a) is for χ=45 degrees and (b) is for χ=14 degrees.

Figure 3 (a) plots $\delta\hat{W}_h$, $\delta\hat{W}_{hk}$ and $\delta\hat{W}_{hf}$ as a function of χ for $\Delta\lambda = 0.5$. As expected, $\delta\hat{W}_{hk}$ is largest for perpendicular injection, for which nearly all the ions are trapped, and diminishes as χ approaches zero. Meanwhile, the fluid contribution is destabilising for perpendicular injection since the predominantly trapped ions are located in the region of poor curvature. For tangential injection it can be seen that the influence of passing ions, which spend most time in the region of good curvature, give rise to strong fluid stabilisation of the mode. The net result is that hot ion stability varies only moderately with respect to injection angle or indeed with the corresponding degree of anisotropy A_h, as shown in Fig. 3 (b). It is important to note that the approximate range of the total plasma anisotropy

$A = (P_c + \langle P_{h\perp} \rangle)/(P_c + \langle P_{h\|} \rangle)$ is $0.75 < A < 1.25$ over the full range of χ. For such a small range of anisotropy the effect on the toroidal contribution δW_T of the resulting slightly shaped flux surfaces is ignorable [14]. It is also interesting to compare Fig. 3 with $\delta \hat{W}_h = 0.0056$ which is obtained for the highly simplified analytical model employed in Ref. [15] for slowing down alphas and Ref. [16] for NBI ions. Note that $\delta \hat{W}_h$ in [15] and [16] agree if a correction factor of $2^{-1/2}$ is used in the former.

Finally Fig. 4 shows $\delta \hat{W}_h$ with respect to a realistic range of central toroidal plasma rotation. Figure 4 (a) is for $\chi = 45$ degrees, which corresponds to the injection angle on the Octant 4 NBI system in JET. It is seen that counter-rotation of -30 kHz reduces $\delta \hat{W}_h$ by more than a factor of two if the distribution function is close to isotropic ($\Delta\lambda = 2$). Co-rotation of 30 kHz also reduces $\delta \hat{W}_h$ but not by as much as counter rotation. The sensitive dependence of kink mode stability on plasma rotation weakens for decreasing $\Delta\lambda$ since, as seen in Fig. 3 (a), $\delta \hat{W}_h$ is then increasingly dominated by $\delta \hat{W}_{hf}$; a quantity independent of toroidal rotation. In contrast, for an injection angle of $\chi = 14$ degrees, Fig. 4 (b) shows that toroidal rotation can greatly diminish hot ion stability when $\Delta\lambda$ is small. This is because, for such an injection angle, the number of trapped ions increases for reducing $\Delta\lambda$. Hence, while both co and counter rotation reduces the stabilising kinetic contribution, fluid effects remain strongly destabilising for this case. An injection angle also of $\chi = 14$ degrees was employed in the PDX experiment where co and counter beam injection gave rise to contrasting sawtooth and fishbone characteristics [17].

In conclusion, a semi-analytical approach within the framework of ideal kinetic theory has been employed to model the effects of NBI ions on sawteeth. The analysis complements recent investigations into NBI stabilisation of JET sawtoothing plasmas [10,16]. Investigation of experimentally relevant scenarios have highlighted the importance of plasma rotation and anisotropy, both of which arise as a consequence of the injection of neutral beams. For the distribution function parameterised to model JET NBI discharges, where beam injection is approximately 45 degrees, it is shown that the anisotropic fluid term is more stabilising to the ideal internal kink mode than the kinetic term. The stabilising role of passing ions, often ignored in similar studies, is crucial. This would be even more evident for scenarios where the injection angle approaches azimuthal, which is seen to be a possible effective means of stabilising sawteeth. Both co and counter plasma rotation of realistic amplitude and shear are shown to have an important influence on hot ion stabilisation for NBI populations which have large trapped fractions. Extrapolation to more energetic particles indicates that toroidal rotation is not expected to significantly modify alpha or ICRF minority ion stabilisation of sawteeth, although the kinetic stability of thermal ions is predicted to be strongly modified by plasma rotation in fusion grade plasmas [2]. The analysis employed in the present paper will also assist in determining the effects of anisotropy on ICRF sawtoothing discharges. This would be particularly important for discerning the applicability of employing auxiliary heated ions in JET to predict the effects of isotropic alpha particles in ITER.

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