29th EPS Conference on Plasma Phys. and Contr. Fusion Montreux, 17-21 June 2002 ECA Vol. **26B**, P-5.059 (2002)

Kinetic and Fluid Simulations of the Collisionless Slab Ion Temperature Gradient Driven Turbulence

H. Sugama and T.-H. Watanabe
National Institute for Fusion Science / Graduate University for Advanced Studies
Toki 509-5292, Japan
W. Horton
Institute for Fusion Studies, The University of Texas at Austin
Austin, Texas 78712, USA

1. INTRODUCTION

In recent years, gyrokinetic and gyrofluid (or gyro-Landau-fluid) simulations [1] of plasma turbulence driven by microinstabilities such as the ion temperature gradient (ITG) mode [2] have been actively done in order to predict the anomalous transport coefficients in magnetically confined plasmas from the first principle. In the gyrofluid model, some closure relations are assumed to construct a truncated system of fluid equations from the gyrokinetic equation and their validity in nonlinear or turbulent regimes is not clear because conventional gyrofluid closure models such as the Hammett-Perkins (H-P) model [3] are originally derived so as to accurately reproduce gyrokinetic dispersion relations for linear modes. In fact, there exist some cases, in which the gyrokinetic and gyrofluid simulations show disagreements in their nonlinear results such as the saturated fluctuation levels and the turbulent transport coefficients.

In our previous work [4], we have presented the nondissipative closure model (NCM), which takes into account the time reversal symmetry of the collisionless kinetic equation. The NCM relates the parallel heat flux to the temperature and the parallel flow in terms of the real-valued coefficients in the unstable wave number space. The NCM was derived such that the closure relation is valid both for the unstable normal-mode solution and its complex-conjugate solution as well as for any linear combination of these solutions. A fluid system of equations with the NCM used reproduce the exact nonlinear kinetic solution of the three-mode ITG problem [5] while the H-P model fails in representing that solution. Then, the next question is whether the NCM can successfully describe strongly turbulent states of collisionless kinetic systems with a higher number of degrees of freedom. In the present work, in order to answer this question, we do both fluid and kinetic simulations of the two-dimensional slab ITG turbulence and investigate how accurately the fluid simulation using the NCM or the H-P model can reproduce results of the collisionless kinetic simulation under the same conditions.

2. BASIC EQUATIONS

The collisionless electrostatic gyrokinetic equation for ions in the uniform magnetic field $\mathbf{B}$ is written in the wave number ($\mathbf{k}$) space as

$$\begin{aligned} \partial_t f_{\mathbf{k}} + ik_\parallel v_\parallel f_{\mathbf{k}} - \frac{c}{B}\sum_{\mathbf{k}'+\mathbf{k}''=\mathbf{k}} [\mathbf{b}\cdot(\mathbf{k}'\times\mathbf{k}'')]\Psi_{\mathbf{k}'} f_{\mathbf{k}''} \\ = i\left[\omega_{*i}\left\{1+\eta_i\left(\frac{m_i v_\parallel^2}{2T_i}-\frac{1}{2}\right)\right\} - k_\parallel v_\parallel\right] F_M \frac{e\Psi_{\mathbf{k}}}{T_i}, \end{aligned} \tag{1}$$

where $F_M = n_0(m_i/2\pi T_i)^{1/2}\exp(-m_i v_\parallel^2/2T_i)$ is a background Maxwellian part of the ion distribution function, $f_{\mathbf{k}} = f_{\mathbf{k}}(v_\parallel, t)$ is a perturbation part integrated over the $\mathbf{v}_\perp$-space with the Maxwellian $\mathbf{v}_\perp$-dependence $\propto \exp(-m_i v_\perp^2/2T_i)$ assumed, and $\Psi_{\mathbf{k}} \equiv \phi_{\mathbf{k}}\exp(-b_{\mathbf{k}}/2)$ $[b_{\mathbf{k}} \equiv k_\perp^2 T_i/(m_i\Omega_i^2)]$. Here, inhomogeneities in the background density n_0 and temperature T_0 are taken into account only through $\omega_{*i} \equiv (cT_i/eB)\mathbf{k}\cdot\mathbf{b}\times\nabla\ln n_0$ and $\eta_i \equiv d\ln T_i/d\ln n_0$ while n_0 and T_i in other places as well as ω_{*i} and η_i are regarded as constants.

Taking the velocity moments of Eq. (1), we obtain fluid equations,

$$\partial_t n_{\mathbf{k}} + ik_\parallel n_0 u_{\mathbf{k}} - i\omega_{*i} n_0\left(1-\frac{b_{\mathbf{k}}}{2}\eta_i\right)\frac{e\Psi_{\mathbf{k}}}{T_i} - \frac{c}{B}\sum_{\mathbf{k}'+\mathbf{k}''=\mathbf{k}}[\mathbf{b}\cdot(\mathbf{k}'\times\mathbf{k}'')]\Psi_{\mathbf{k}'}n_{\mathbf{k}''} = 0, \quad (2)$$

$$n_0 m_i\partial_t u_{\mathbf{k}} + ik_\parallel\left(T_i n_{\mathbf{k}} + n_0 T_{\mathbf{k}} + n_0 e\Psi_{\mathbf{k}}\right) - \frac{n_0 m_i c}{B}\sum_{\mathbf{k}'+\mathbf{k}''=\mathbf{k}}[\mathbf{b}\cdot(\mathbf{k}'\times\mathbf{k}'')]\Psi_{\mathbf{k}'}u_{\mathbf{k}''} = 0, \quad (3)$$

$$n_0\partial_t T_{\mathbf{k}} + ik_\parallel\left(2n_0 T_i u_{\mathbf{k}} + q_{\mathbf{k}}\right) - i\omega_{*i}\eta_i n_0 e\Psi_{\mathbf{k}} - \frac{n_0 c}{B}\sum_{\mathbf{k}'+\mathbf{k}''=\mathbf{k}}[\mathbf{b}\cdot(\mathbf{k}'\times\mathbf{k}'')]\Psi_{\mathbf{k}'}T_{\mathbf{k}''} = 0, \quad (4)$$

where $n_{\mathbf{k}}(t) \equiv \int_{-\infty}^{\infty} dv_\parallel f_{\mathbf{k}}(v_\parallel, t)$, $n_0 u_{\mathbf{k}}(t) \equiv \int_{-\infty}^{\infty} dv_\parallel f_{\mathbf{k}}(v_\parallel, t)v_\parallel$, $n_0 T_{\mathbf{k}}(t) \equiv \int_{-\infty}^{\infty} dv_\parallel \times f_{\mathbf{k}}(v_\parallel, t)(m_i v_\parallel^2 - T_i)$, and $q_{\mathbf{k}}(t) \equiv \int_{-\infty}^{\infty} dv_\parallel f_{\mathbf{k}}(v_\parallel, t)(m_i v_\parallel^3 - 3T_i v_\parallel)$.

Assuming the adiabatic electron response and using the quasineutrality condition give

$$\exp(-b_{\mathbf{k}}/2)n_{\mathbf{k}} - n_0\frac{e\phi_{\mathbf{k}}}{T_i}[1-\Gamma_0(b_{\mathbf{k}})] = \frac{e\phi_{\mathbf{k}}}{T_e} \qquad (\text{for } k_\parallel \neq 0), \quad (5)$$

where Γ_0 is defined by $\Gamma_0(b_{\mathbf{k}}) \equiv I_0(b_{\mathbf{k}})\exp(-b_{\mathbf{k}})$ with the 0-th-order modified Bessel function I_0. For the fluctuations with $k_\parallel = 0$, the electron density perturbation is often assumed to vanish and then the quasineutrality condition gives $\exp(-b_{\mathbf{k}}/2)n_{\mathbf{k}} - n_0(e\phi_{\mathbf{k}}/T_i)[1-\Gamma_0(b_{\mathbf{k}})] = 0$. When using this condition for $k_\parallel = 0$ with Eqs. (1) and (5) for the two-dimensional slab ITG turbulence simulation, we have found that a large zonal flow component, $\phi_{\mathbf{k}}$ with $k_\parallel = 0$, is nonlinearly generated, suppresses linearly-unstable modes with $k_\parallel \neq 0$, and results in no turbulent transport [6]. Thus, efficiency of zonal flow generation and resultant transport coefficients are strongly influenced by what condition is used for the $k_\parallel = 0$ modes. In more practical cases of toroidal configurations, the zonal flow would be significantly reduced by the collisionless transit time magnetic pumping and by the collisional damping [7] although neither of these effects is included in Eq. (1). Here, in order to avoid the complexity brought about by the zonal flow and get finite turbulent transport, we put

$$f_{\mathbf{k}} = \phi_{\mathbf{k}} = 0 \qquad (\text{for } k_\parallel = 0), \quad (6)$$

and

$$n_{\mathbf{k}} = u_{\mathbf{k}} = T_{\mathbf{k}} = \phi_{\mathbf{k}} = 0 \qquad (\text{for } k_\parallel = 0), \quad (7)$$

in our kinetic and fluid simulations, respectively.

Now, a closed nonlinear kinetic system of equations are given by Eqs. (1) and (5) for $k_\parallel \neq 0$ and by Eq. (6) for $k_\parallel = 0$, which are used for kinetic simulation of the slab ITG turbulence in the present work. In order to obtain a corresponding closed fluid system, we need to express the parallel heat flow $q_{\mathbf{k}}$ in Eq. (4) in terms of the lower-order moment fluid variables $n_{\mathbf{k}}$, $u_{\mathbf{k}}$, and $T_{\mathbf{k}}$. In the H-P model [3], $q_{\mathbf{k}}$ is written in the diffusive form as

$$q_{\mathbf{k}} = -n_0 \chi_{\|} i k_{\|} T_{\mathbf{k}}, \tag{8}$$

where the parallel heat diffusivity is given by $\chi_{\|} = 2(2/\pi)^{1/2} v_t / |k_{\|}|$ with the ion thermal velocity $v_t \equiv (T_i/m_i)^{1/2}$. Then, Eqs. (2)–(5) for $k_{\|} \neq 0$, Eq. (7) for $k_{\|} = 0$, and Eq. (8) give a closed fluid system of equations in the H-P model. In the NCM, the parallel heat flow $q_{\mathbf{k}}$ in the unstable wave number region is given as

$$q_{\mathbf{k}} = C_{T\mathbf{k}} n_0 v_t T_{\mathbf{k}} + C_{u\mathbf{k}} n_0 T_i u_{\mathbf{k}} \qquad \text{(for linearly unstable modes)}, \tag{9}$$

while the dissipative closure relation as written in Eq. (8) should still be used in the stable wave number region. The real-valued coefficients $C_{T\mathbf{k}}$ and $C_{u\mathbf{k}}$ are determined by requiring that the closure relation in Eq. (9) should exactly reproduce the kinetic dispersion relation [4].

3. SIMULATION RESULTS

Here, we consider a rectangular domain of $L_x \times L_y$ in the x-y plane with a uniform external magnetic field $\mathbf{B} = B(\hat{\mathbf{z}} + \theta\hat{\mathbf{y}})$ ($|\theta| \ll 1$), where $\hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$ denote the unit vectors in the y- and z-directions, respectively. The system is assumed to be homogeneous in the z-direction ($\partial/\partial z = 0$). We employ the periodic boundary conditions in both x and y directions. Then, we can write $\mathbf{k} = k_x\hat{\mathbf{z}} + k_y\hat{\mathbf{y}} = 2\pi[(m/L_x)\hat{\mathbf{x}} + (n/L_y)\hat{\mathbf{y}}]$, $f_{\mathbf{k}} = f_{m,n}$, and $\phi_{\mathbf{k}} = \phi_{m,n}$ ($m, n = 0, \pm 1, \pm 2, \cdots$) and the parallel wave number is given by $k_{\|} = k_y\theta$. The background density and temperature gradients are assumed to exist in the x-direction, and their gradient scale lengths are given by $L_n = -(d \ln n_0/dx)^{-1} (> 0)$ and $L_T = -(d \ln T_i/dx)^{-1} (> 0)$, respectively.

Equations (1) and (5) for $k_{\|} \neq 0$ and (6) for $k_{\|} = 0$ are used as governing equations for the kinetic (or Vlasov) simulation, in which the fine-scale phase space structures such as the ballistic mode are resolved by employing 8,193 grid points for discretization of the velocity space, $-5 \leq v_{\|}/v_t \leq 5$ and using the nondissipative time-integration scheme [8]. For comparison to the kinetic simulation, two types of fluid simulations using different closure models are done. Both fluid simulations are based on Eqs. (2)–(5) for $k_{\|} \neq 0$ and Eq. (7) for $k_{\|} = 0$. However, one of them employs the NCM given by Eq. (9) for linearly unstable modes and the H-P dissipative closure given by Eq. (8) for linearly stable modes while the other uses the H-P closure for all modes. Here, for all simulations, we use the conditions $T_e/T_i = 1$, $\eta_i = L_n/L_T = 10$, $L_x = L_y = 20\pi\rho_i$ ($\rho_i \equiv v_t/\Omega_i$: the ion thermal gyroradius), and $\Theta \equiv \theta L_n/\rho_i = 1$.

Figure 1 shows the normalized perpendicular heat diffusivity $\chi/(\rho_i^2 v_t/L_n)$ as a function of normalized time $v_t t/L_n$ obtained by the kinetic and fluid (NCM, H-P) simulations, where $\chi \equiv \mathbf{q}_\perp \cdot \hat{\mathbf{x}}/(n_0 T_i/L_T)$ and $\mathbf{q}_\perp \equiv \frac{1}{2} n_0 \sum_{\mathbf{k}} \mathrm{Re}[T_{\mathbf{k}}^* i(c/B)\mathbf{b} \times \mathbf{k}\Psi_{\mathbf{k}}]$. In the saturated state of turbulence, χ obtained by the kinetic simulation is in a good agreement with χ from the fluid simulation using the NCM although χ obtained by the fluid simulation using the H-P closure for all modes are significantly larger than them.

Figure 2 shows the ion distribution function $f_{\mathbf{k}}$ divided by $\phi_{\mathbf{k}}$ for $k_x = 0$ and $k_y = 0.4\rho_i^{-1}$ (which corresponds to the linearly most unstable mode). While $\mathrm{Re}[f_{\mathbf{k}}/\phi_{\mathbf{k}}]$ at nonlinear or turbulent stages ($v_t t/L_n = 210, 400$) are similar to that of the linear eigenfunction seen at an early stage ($v_t t/L_n = 50$) except for superposition of fine structures due to the ballistic mode, Im$[f_{\mathbf{k}}/\phi_{\mathbf{k}}]$ in the turbulent state has a different profile from that in the linear stage and oscillates around zero. Thus, the phases of $f_{\mathbf{k}}$ and $(n_{\mathbf{k}}, u_{\mathbf{k}}, T_{\mathbf{k}}, q_{\mathbf{k}}, \cdots)$ relative to $\phi_{\mathbf{k}}$ may take either a positive or negative sign. This is similar to the case of the three-mode ITG problem [4] and the NCM is applicable to this situation more properly than the H-P model.

4. CONCLUSIONS

We have made a detailed comparison between kinetic and fluid simulations of the collisionless slab ITG turbulence. The validity of these closure models for quantitative prediction of the turbulent thermal transport is examined by comparing nonlinear results of the fluid simulations with those of the collisionless kinetic simulation of high accuracy. It is found from the kinetic simulation that the phase relation between the potential and the distribution function for the most unstable mode is different from that predicted by the linear unstable eigenfunction and is better described by the NCM than by the H-P model. This fact explains a reason for our simulation results that, in the steady turbulence state, the turbulent thermal diffusivity χ obtained by the H-P model is significantly larger than χ given by the NCM and that the latter is closer to χ found in the kinetic simulation.

REFERENCES

[1] A. M. Dimits, *et al.*, Phys. Plasmas **7**, 969 (2000).
[2] W. Horton, Rev. Mod. Phys. **71**, 735 (1999).
[3] G. W. Hammett and F. W. Perkins, Phys. Rev. Lett. **64**, 3019 (1990).
[4] H. Sugama, T.-H. Watanabe, and W. Horton, Phys. Plasmas **8**, 2617 (2001).
[5] T.-H. Watanabe, H. Sugama, and T. Sato, Phys. Plasmas **7**, 984 (2000).
[6] T.-H. Watanabe and H. Sugama, submitted to Phys. Plasmas.
[7] Z. Lin, *et al.*, Phys. Plasmas **7**, 1857 (2000).
[8] T.-H. Watanabe, H. Sugama, and T. Sato, J. Phys. Soc. Jpn. **70**, 3565 (2001).

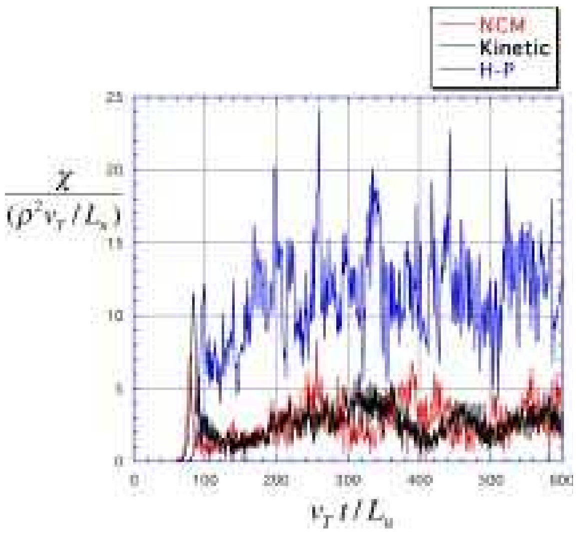

Fig.1. Normalized perpendicular heat diffusivity $\chi/(\rho_i^2 v_t/L_n)$ as a function of normalized time $v_t t/L_n$ obtained by the kinetic and fluid (NCM, H-P) simulations.

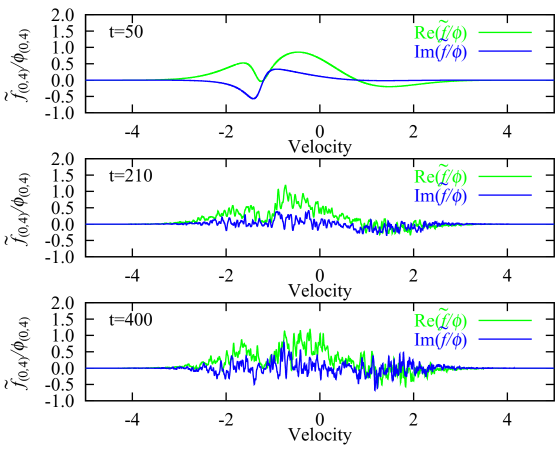

Fig.2. Ion distribution function $f_{\mathbf{k}}$ divided by $\phi_{\mathbf{k}}$ at $v_t t/L_n = 50, 210, 400$ for $k_x = 0$ and $k_y = 0.4\rho_i^{-1}$ (which corresponds to the linearly most unstable mode).