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Permittivity Tensor and RF Dissipation in Plasmas of Low Aspect Ratio Toroidal Devices

F.M.Nekrasov, A.G.Elfimov*, C.A. de Azevedo, A.S. de Assis+ and J.P.Neto

LNCC, Departmento de Matemática Aplicada, 25651-070, Petrpolis, RJ, Brasil.

Instituto de Física, Universidade de São Paulo, 05315-970, SP, Brazil

+*Instituto de Matemática-GMA, Universidade Federal Fluminense, Niterói, RJ, Brazil*

Introduction Recently, several experiments in spherical or low aspect ratio tokamaks (LART) (where share of trapped particles is very large) have produced efficient plasma confinement in high beta regimes and, nowadays, the radio frequency (RF) plasma heating and current drive studies are very important parts of experimental and theoretical programs on LART (see, for example, NSTX program[1] where fast wave heating and current drive is proposed). In LART, electrons moving along magnetic field suffer strong velocity modulation (bouncing effect) that can affect RF dissipation. The analysis of wave heating and current drive by RF fields can be carried out on the basis of Vlasov-Maxwell set of equations. In Ref.2, the Vlasov's equation was solved analytically and parallel dielectric tensor was found via Jacobi functions for a LART model with concentric circular magnetic surfaces. The numerical calculations of imaginary part of the parallel tensor component had demonstrated that bounce effect can strongly affect RF dissipation in the LART. Here, using the same procedure as in Ref.2, the complete set of the permittivity tensor components for solving Maxwell equations in LART is presented.

Plasma model and Vlasov equation. In magnetized plasmas, $\rho_L \ll L$ (where $\rho_L = v_T/\Omega_c$ is Larmor radius and v_T, Ω_c, and L are thermal velocities, cyclotron frequency and characteristic space-scales, respectively) the Vlasov equation can be simplified sufficiently with approximate integrating of particle trajectory equations. In this case, we get so called gyro-kinetic equation. Using approximations, $\rho_L/L \ll 1$ and $\omega/\Omega_c \ll 1$, we get so called drift equation. Here, in contrast to those methods, we use directly a traditional method (like in Chu-Goldberger-Low hydrodynamics for magnetized plasma) in the Vlasov equation. To perform this procedure, we introduce new 6-dimensional space $\vec{r}\,', \vec{v}$ where $\vec{r}\,' = \vec{r}, v_1 = (\vec{n} \cdot \vec{v})$, $v_2 = (\vec{\tau} \cdot \vec{v})$ and $v_3 = (\vec{h} \cdot \vec{v})$ are the velocity projections on unit vectors related to the geometry of the stationary magnetic field $\vec{B}_0$. The unit vectors $\vec{n}, \vec{h}$ are directed along the normal to a magnetic surface and along the magnetic field, respectively, and the binormal vector $\vec{\tau}$ is defined by the vector product $\vec{\tau} = \vec{h} \times \vec{n}$. Then, we introduce spherical coordinates (v, σ, γ) instead of the Cartesian coordinates $v_1 = v \sin\gamma \cos\sigma$, $v_2 = v \sin\gamma \sin\sigma$, $v_3 = v \cos\gamma$ that are defined in the regions: $0 \le v < \infty$, $0 \le \gamma \le \pi$, $0 \le \sigma \le 2\pi$.

Further, using the trasformations, $\partial/\partial t + \nu = -\mathrm{i}\omega + \nu = -\mathrm{i}\Omega$ with the Krook form of the collision term $St[f] = -\nu f$, the linearized Vlasov equation can be written for Fourier components f_l of the distribution function ($f = \sum_l f_l \exp il\sigma$) in the form:

$$(\hat{D} - \mathrm{i}\Omega) f_0 + (\hat{D}^+ + \mathrm{i}\omega^+) f_1 + (\hat{D}^- - \mathrm{i}\omega^-) f_{-1} + (v \sin\gamma \frac{\partial}{\partial\gamma} + 2v_3)(K^+ f_2 + K^- f_{-2}) = R_0$$

$$\left(1 + \mathrm{i}\mu \frac{\hat{D} + \mathrm{i}l\omega_0}{\Omega + \mathrm{l}\Omega_{\mathrm{ci}}}\right) f_l + \frac{\mathrm{i}\mu}{\Omega + l\Omega_{ci}} \left(\left[\hat{D}^- - \mathrm{i}(l-1)\omega^-\right] f_{l-1} + \left[\hat{D}^+ + \mathrm{i}(l+1)\omega^+\right] f_{l+1} + \right.$$

$$\left. + vK^- \left[\sin\gamma \frac{\partial}{\partial\gamma} - (l-2)\cos\gamma\right] f_{l-2} + vK^+ \left[\sin\gamma \frac{\partial}{\partial\gamma} + (l+2)\cos\gamma\right] f_{l+2} \right) = R_l \qquad (1)$$

where $\omega_0 = \frac{v\cos\gamma}{2}[(\vec{n}\cdot\nabla\times\vec{n}) + (\vec{\tau}\cdot\nabla\times\vec{\tau}) - 2(\vec{h}\cdot\nabla\times\vec{h})]$; $\omega^{\pm} = \frac{1}{2}(\omega_1 \pm \mathrm{i}\omega_2)$;

$$\omega_1 = \frac{v\cos^2\gamma}{\sin\gamma}(\vec{n}\cdot\nabla\times\vec{h}) - v\sin\gamma(\vec{h}\cdot\nabla\times\vec{n}); \quad \omega_2 = \frac{v\cos^2\gamma}{\sin\gamma}(\vec{\tau}\cdot\nabla\times\vec{h}) - v\sin\gamma(\vec{h}\cdot\nabla\times\vec{\tau});$$

$$K^{\pm} = \frac{1}{2}(K_1 \pm) \mathrm{i} K_2);$$

$$K_1 = \frac{1}{2}[(\vec{\tau}\cdot\nabla\times\vec{n}) + (\vec{n}\cdot\nabla\times\vec{\tau})] \;; K_2 = \frac{1}{2}[(\vec{n}\cdot\nabla\times\vec{n}) - (\vec{\tau}\cdot\nabla\times\vec{\tau})];$$

$$\hat{R}_0 = -\frac{e}{M} E_3 \cos\gamma \frac{\partial F_0}{\partial v}, \quad \hat{R}_l = -\frac{e(E_1 - ilE_2)}{2M(\Omega + l\Omega_{ci})} \sin\gamma \frac{\partial F_0}{\partial v}, \quad l = \pm 1;$$

$$\hat{R}_l = 0, \quad l \ge 2; \quad F_0 = \frac{N}{(2\pi\, v_T)^{3/2}} \exp\left(-\frac{v^2}{2v_T^2}\right)$$

The projections of electric field $\vec{E}$ on the unit base vectors $(\vec{n}, \vec{\tau}, \vec{h})$ are marked by the indexes (1, 2, 3), respectively, and $\Omega_{ci} = e|B_0|/Mc$ is the ion cyclotron frequency. To describe the electron component of the plasma the values of e and M should be substituted by $-e$ and m. The next notations for differential operators in Eq.(1) are introduced

$$\hat{D} = v\cos\gamma(\vec{h}\cdot\nabla) - \frac{v}{2}(\nabla\cdot\vec{h})\sin\gamma\frac{\partial}{\partial\gamma}; \quad \hat{D}_1 = v\sin\gamma(\vec{n}\cdot\nabla) - v(\vec{\tau}\cdot\nabla\times\vec{h})\cos\gamma\frac{\partial}{\partial\gamma};$$

$$\hat{D}_2 = v\sin\gamma(\vec{\tau}\cdot\nabla) + v(\vec{n}\cdot\nabla\times\vec{h})\cos\gamma\frac{\partial}{\partial\gamma}; \quad \hat{D}^{\pm} = \hat{D}_1 \pm \mathrm{i}\hat{D}_2 \qquad (2)$$

Finally, the formal parameter μ in the front of small values ρ_L/L is replaced by 1.

Solution of Vlasov equation and evaluation of dielectric tensor. To find the solution of eq.(1) in the second order of ρ_L/L we take into account the harmonics $|l| \leq 3$,

$$f_{\pm 2} = -\mathrm{i}\mu \left[1 - \frac{\mathrm{i}\mu}{\Omega \pm 2\Omega_c}\left(\hat{D} \pm 2\mathrm{i}\omega_0\right)\right]\left[\sin\gamma \frac{\partial}{\partial\gamma}\left(\frac{vK^{\mp} f_0}{\Omega \pm 2\Omega_c}\right) + \frac{(\hat{D}^{\mp} \pm 2\mathrm{i}\omega^{\mp}) f_{\pm 1}}{\Omega \pm 2\Omega_c}\right] \qquad (3)$$

This approximation is enough to take the bounce resonance effect in nondiagonal components of the dielectric tensor. Note that we study only electron Cherenkov dissipation which is valid in the condition $v_{Te} \ll |L(\Omega - \Omega_{ce})|$ then we can neglect Ω in comparison with the electron cyclotron frequency in denominators of Eqs.(1)(2). Representing the distribution function harmonics as $f_l = f_l^{(0)} + \mu f_l^{(1)}$ and using the definitions $4\pi\mathrm{i}j_\alpha/\omega = \hat{\varepsilon}_{\alpha\beta} E_\beta$, we have the next equations that can be used in calculations of anti-Hermitian parts of the dielectric tensor:

$$\hat{\varepsilon}_{11} E_1 = \mathrm{i}\frac{2\pi^2 e^2}{m\,\omega\,\omega_c}\int_0^\infty v^3 dv \int_0^\pi d\gamma \sin^2\gamma \hat{D}_2 \hat{T}\,(\hat{D}_2 + \omega_1)\frac{\sin\gamma}{\omega_c}\frac{\partial F_0}{\partial v} E_1;$$

$$\hat{\varepsilon}_{21} E_1 = -\mathrm{i}\frac{2\pi^2 e^2}{m\,\omega\,\omega_c}\int_0^\infty v^3 dv \int_0^\pi d\gamma \sin^2\gamma \hat{D}_1 \hat{T}\,(\hat{D}_2 + \omega_1)\frac{\sin\gamma}{\omega_c}\frac{\partial F_0}{\partial v} E_1;$$

$$\hat{\varepsilon}_{12} E_2 = -\mathrm{i}\frac{2\pi^2 e^2}{m\,\omega\,\omega_c}\int_0^\infty v^3 dv \int_0^\pi d\gamma \sin^2\gamma \hat{D}_2 \hat{T}\,(\hat{D}_1 - \omega_2)\frac{\sin\gamma}{\omega_c}\frac{\partial F_0}{\partial v} E_2;$$

$$\hat{\varepsilon}_{22} E_2 = \mathrm{i}\frac{2\pi^2 e^2}{m\,\omega\,\omega_c}\int_0^\infty v^3 dv \int_0^\pi d\gamma \sin^2\gamma \hat{D}_1 \hat{T}\,(\hat{D}_1 - \omega_2)\frac{\sin\gamma}{\omega_c}\frac{\partial F_0}{\partial v} E_2;$$

$$\hat{\varepsilon}_{23} E_3 = -\mathrm{i}\frac{4\pi^2 e^2}{m\,\omega\,\omega_c}\int_0^\infty v^3 dv \int_0^\pi d\gamma \sin^2\gamma\, \hat{D}_1 \hat{T}\,\cos\gamma \frac{\partial F_0}{\partial v} E_3;$$

$$\hat{\varepsilon}_{33} E_3 = -\mathrm{i}\frac{8\pi^2 e^2}{m\,\omega}\int_0^\infty v^3 dv \int_0^\pi d\gamma \sin\gamma\cos\gamma\, \hat{T}\,\cos\gamma \frac{\partial F_0}{\partial v} E_3;$$

$$\hat{\varepsilon}_{32} E_2 = \mathrm{i}\frac{4\pi^2 e^2}{m\,\omega}\int_0^\infty v^3 dv \int_0^\pi d\gamma \sin\gamma\cos\gamma\, \hat{T}\,(\hat{D}_1 - \omega_2)\frac{\sin\gamma}{\omega_c}\frac{\partial F_0}{\partial v} E_2$$

In above equations the operator $\hat{T}$ is the inverse operator of the operator $(-\mathrm{i}\Omega + \hat{D}_0)$. The parallel component of the dielectric tensor is studied in [2-4]. Here, we present the results of analyses for the anti-Hermitian parts of the tensor components,

$$\hat{\varepsilon}^{m,m}_{33(\text{unt.})} = \frac{\sqrt{2}\epsilon\omega_{pe}^2}{\sqrt{\pi}\Omega k_0 v_T}\sum_r \int_{\kappa_0}^1 \frac{\mathrm{d}\kappa}{\kappa^3}\int_{-\infty}^{\infty}\frac{u^4 \exp\left(-u^2/2\right)\,\left[Q^{m,r}_{\text{unt.1}}(u,\kappa)\right]^2 \,\mathrm{d}u}{u(r+m+n\bar{q}_t) - \Omega/\omega_{\text{unt.b}} - \mathrm{i}\nu_e/\omega_{\text{unt.b}}} \qquad (4)$$

$$\hat{\varepsilon}^{m,m}_{32(\text{unt.})} E_2^m = \mathrm{i}\frac{\sqrt{2}\omega_{pe}^2\epsilon}{\sqrt{\pi}\omega_H\Omega}\sum_r \int_{\kappa_0}^1 \mathrm{d}\kappa \frac{K(\kappa^2-\kappa_0^2)V^5_{\text{unt}.r,m} P^{m,r}_{\text{unt.1}} Q^{m,-r}_{\text{unt.1}}}{k_0\kappa_0\kappa^4|r+m+n\bar{q}_t|}\exp\left(-\frac{V^2_{\text{unt}.r,m}}{2}\right)\frac{\partial E_2^m}{\partial\rho}$$

$$P^{m,r}_{\text{unt.1}} = \frac{1}{\pi}\int_0^\pi \frac{\mathrm{d}x\, cos(rx - \sum_l g^l_{mr} sinlx)}{\sqrt{1-\kappa_0^2\,\mathrm{sn}^2\,x}} \quad Q^{m,-r}_{\text{unt.1}} = \frac{2K}{\pi^2}\int_0^\pi \mathrm{d}x\,\mathrm{dn}x\, cos(rx - \sum_l g^l_{mr} sinlx)$$

where $V_{\text{unt}.r,m} = \Omega/\omega_{\text{unt.b}}(r + m + n\bar{q}_t)$ is the dimensionless resonance velocity and $\bar{q}_t = q_t/\sqrt{1-\epsilon^2}$ is the tokamak safety factor. The periodic g^l_{mr} and elliptic $\text{dn}x, \text{sn}\,x$ functions with argument $2\pi\,x/K$ are presented in Ref.4. In Fig.1, in addition to analyses[4] of $\hat{\varepsilon}^{m,m}_{33(\text{unt.})}$, we show some specific cases of $\hat{\varepsilon}^{m,m}_{32(\text{unt.})}$ component for short wavelength that may be important for Landau and transit time magnetic pumping dissipation in LART.

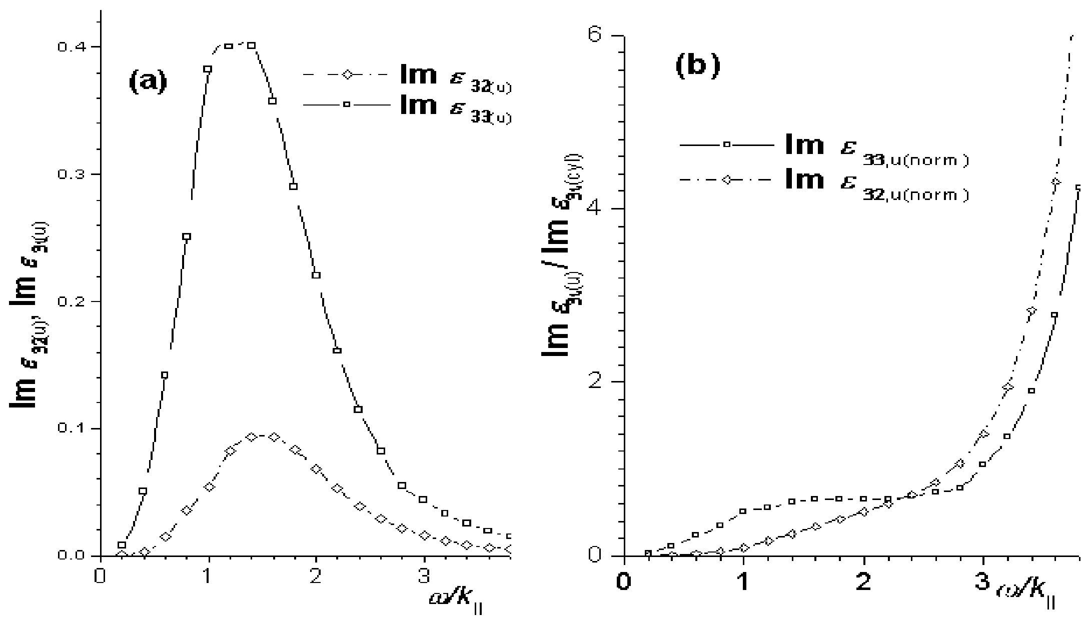

Fig.1. *Plot of the imaginary part of the tensor components,* $Im\varepsilon^{mm}_{32}$ *(solid line with squares) and* $Im\varepsilon^{mm}_{33}$ *(dash-dot line with diamonds) over* $\omega/k_{\|}v_{Te}$ *for untrapped electrons* ($m + nq_t = 13$ *and* $r/R = 0.5$) *that are normalized: (a) on* $k^2_{\|}\lambda^2_{De}$ *and (b) on cylindrical tensor components, respectively.*

In conclusion, we can say that the wave dissipation (Landau and TTMP) in untrapped electrons is strongly enhanced for waves with phase velocity larger than thermal velocity and strongly modified near rational magnetic surfaces (when $k_{\|} = 0$) because of strong modulation of parallel velocity of electrons. This phenomena have to be taken into account in RF plasma heating, current drive calculations and in analyses of drift instabilities at the rational magnetic surfaces in LART.

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