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# THEORY OF ELECTRIC BARRIER FORMATION IN ECR POINT IN INHOMOGENEOUS MAGNETIC FIELD

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**Abstract**. In this paper the formation of the electric thermal barriers in kind of set of solitary hollow and hump of electric potential near ECR points on ends of cylindrical trap is investigated theoretically. It is shown that the solitary hollow is excited by resonant electrons and the electric potential hump is excited by ion flow. The hollow reflects electrons and hump reflects ions.

**Solitary hollow excitation**

In [1] the formation of a thermal barrier for plasma particles was observed experimentally in a neighborhood of a point of local electron cyclotron resonance (ECR) in a magnetic field, monotonically falling down along a system axis deep into it. In this paper the mechanism of such barrier formation for plasma electrons and ions is investigated theoretically. This mechanism can provide the formation of trap for plasma. In ECR point the transversal electron velocity is increased strongly. At motion of electrons from the system they are reflected from the magnetic barrier back inside the system. Further behavior of reflected electrons and electrons, initially moved inside the system, is the similar one. Namely, at electron motion along the inhomogeneous magnetic field their transversal velocity decreases and longitudinal velocity increases. The increase of longitudinal velocity results that electrons in that area, where they have penetrated, form uncompensated negative charge. It is a hole of electrical potential, from which the electrons are reflected. The increase of electron longitudinal velocity in a neighbourhood of ECR point results also that nonequilibrium state occurs, i.e. average electron velocity with respect to the ions. The reflection of current-carrying electrons from the hole of the electrical potential results in growth of its amplitude. Further we shall show, that the hole is excited on an ion mode with velocity, close to zero.

A plasma with electron current relative to ions is considered. This plasma is nonequilibrium. Perturbations are excited. At certain conditions the perturbations could be solitary types. Therefore properties of electrostatic potential dip, $\varphi$, of solitary kind are investigated. The dip reflects electrons with energy smaller than the dip depth. This leads to dip depth (amplitude, $-\varphi_o$) growth.

We use hydrodynamic equations for densities $n_\pm$ and velocities $V_\pm$ of positive and negative ions and Vlasov equation for electron distribution function $f_e$.

Due to reflection of resonant electrons with nonsymmetric relative to dip velocity V distribution function from potential dip the potential jump $\Delta\varphi$ is formed near the dip.

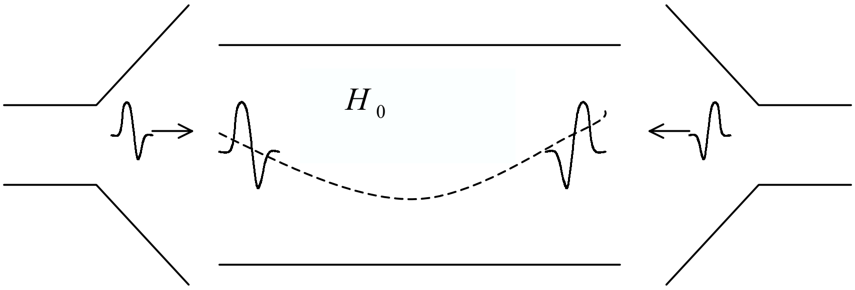


Fig. 1. Scheme of the thermal electrical barriers formation for plasma electrons and ions in ECR points on ends of the magnetized cylindrical trap

We use slow evolution of dip for its description. In zero approximation, taking into account that the resonant electrons are reflected from the dip, one can derive from Vlasov equation the expression for electron distribution function

$$f_e=f_{oe}[-(V^2-2e(\varphi\pm\Delta\varphi)/m_e)^{1/2}\pm V_o],\ V \gtrless A(\varphi)\mathrm{sign}(z),\ \ A(\varphi)=[2e(\varphi_o+\varphi)/m_e]^{1/2}. \quad (1)$$

We use the normalized values: $\phi\equiv e\varphi/T_e$, $N_-\equiv n_{o-}/n_{o+}$, $N_e\equiv n_{oe}/n_{o+}$, $Q_\pm=q_\pm/e$, $V_{s\pm}=(T_e/M_\pm)^{1/2}$. We normalize x on Debye radius of electrons $r_{de}$, $V_o$ on $V_{the}$, time t on plasma frequency of positive ions $\omega_{p+}^{-1}$, velocity of solitary perturbation $V_c$ on ion-acoustic velocity $(T_e/M_+)^{1/2}$. $T_e$, is the temperature of electrons, $n_{o-}$, $n_{o+}$ are unperturbed densities of negative and positive ions, $q_\pm$ is the charge of positive and negative ions.

Integrating (1), one can derive the electron density in first approximation on V

$$n_e\approx n_{oe}\exp(\phi)[1-(2\Delta\phi/\sqrt{\pi})\int_o^\beta dx\,\exp(-x^2)-2V_o(2/\pi)^{1/2}\int_o^\beta dx\,(x^2-\phi)^{1/2}\exp(-x^2)] \quad (2)$$

Far from the dip the plasma is quasineutral $n_e(z)|_{z\to\infty} = n_e(z)|_{z\to-\infty} =1$. From here one can derive, using (2), the expression for potential jump near the dip

$$\Delta\phi=V_o(2/\pi)^{1/2}(1-\exp(-\phi_o))/[1-(2/\sqrt{\pi})\int_o^{\sqrt{\phi o}} dx\exp(-x^2)] \quad (3)$$

From hydrodynamic equations one can obtain densities of positive and negative ions

$$n_{i\pm}=n_{\pm NL}+n_{\pm\tau},\ n_{\pm NL}=n_{o\pm}/[1-(\pm q_\pm)2\varphi/M_\pm V_c^2]^{1/2}, \quad (4)$$

$$\partial n_{\pm\tau}/\partial z=\pm 2(\partial\varphi/\partial t)(n_{o\pm}q_\pm/M_\pm V_c^3)[1-(\pm q_\pm)\varphi/M_\pm V_c^2]/[1-(\pm q_\pm)2\varphi/M_\pm V_c^2]^{3/2}$$

Substituting (2), (4) in Poisson equation one can derive nonlinear evolution equation

$$\partial^3_z\phi+\{Q_+^2V^2_{s+}(1-2\phi Q_+V^2_{s+}/V_c^2)^{-3/2}(1-\phi Q_+V^2_{s+}/V_c^2)+$$

$$+Q_-^2N_-V^2_{s-}(1+2\phi Q_-V^2_{s-}/V_c^2)^{-3/2}(1+\phi Q_-V^2_{s-}/V_c^2)\}2\partial_t\phi/V_c^3$$

$$+(\partial_z\phi/V_c^2)\{Q_+^2V^2_{s+}(1-2\phi Q_+V^2_{s+}/V_c^2)^{-3/2}+Q_-^2N_-V^2_{s-}(1+2\phi Q_-V^2_{s-}/V_c^2)^{-3/2}\}- \quad (5)$$

$$-\{\exp(\phi)-\text{sign}(z)V_o(2/\pi)^{1/2}\{(\phi_o/(\phi_o+\phi))^{1/2}\exp(-\phi_o)-\int_{\sqrt{-\phi}}^{\sqrt{\phi o}} dy(1-2y^2)\exp(-y^2)/(y^2+\phi)^{1/2}+$$

$$+(1-\exp(-\phi_o))[1-(2/\sqrt{\pi})\int_o^{\sqrt{\phi o}} dx\exp(-x^2)]^{-1}[\exp(-\phi_o)/(\phi_o+\phi)^{1/2}+2(\phi_o+\phi)\exp(-\phi_o)+$$

$$+4\int_{\sqrt{-\phi}}^{\sqrt{\phi o}} dy\, y(y^2+\phi)^{1/2}\exp(-y^2)]/\sqrt{\pi}\}\}\partial_z\phi=0$$

From (5) one can show that the dip propagates with slow velocity $V_c \approx (T_e/M_+)^{1/2}(n_+/n_e)^{1/2}(q_+/e)$. From (5) one can get also the growth rate of the dip amplitude

$$\gamma_{nl}\approx\omega_{p+}(V_o/V_{the})^{3/2}(q_+/e)(n_+/n_e)^{1/2}\{1+[1/3-(n_e/n_+)(e/q_+)](e\varphi_o/T_e)(\pi/2)^{1/2}(V_{the}/2V_o)\} \quad (6)$$

One can see that the dip of large amplitude is formed at electron current velocity $V_o$ larger than threshold one. The threshold decreases at decreasing $n_e/n_+$ and equal zero at $n_e/n_+<q_+/3e$. The maximum threshold is realized at $n_{i-}=0$.

**Electric potential hump excitation**

As the electrons of the plasma flow are reflected from the electrical potential hole and the ions pass through the hole freely, uncompensated volume charge of ions is formed behind the hole, in which field ions are decelerated. This volume charge forms perturbation in kind of the electrical potential hump. The ion flow strengthens this potential hump. Quasistationary properties of the hump are described in a neglect by nonequilibrium effect. The account of the nonequilibrium state results in excitation of the hump.

In linear approximation the excitation of the perturbation by the plasma flow, driven with respect to negative and motionless positive ions, is described by the following relation:

$$1+1/(kr_{de})^2-\omega^2_{p+}/(\omega-kV_{o+})^2-\omega^2_{p-}/\omega^2-\omega^2_{pq}/\omega^2=0. \quad (7)$$

Here $\omega$, k are the frequency and wave vector of perturbation; $\omega_{p\pm}$ are the plasma frequencies of negative and flow positive ions; $\omega_{pq}$ is the plasma frequency of the motionless positive ions; $r_{de}$ is the Debye radius of electrons; $V_{o+}$ is the velocity of the ion flow.

From (7) it is visible, that it is possible to select the flow velocity such, that:

$$V_{ph}=\omega/k\approx(V_{o+}/2^{4/3})[(n_-m_+q_-^2/\ n_+m_-q_+^2)+(n_{+q}q_{+q}^2/\ n_+q_+^2)]^{1/3}<<V_{s+}, \quad (8)$$

the perturbation is motionless, i.e. $V_{ph}<<V_{s+}$. $V_{s+}=(T/m_+)^{1/2}$ is the ion-acoustic velocity of the flow positive ions. Here $n_-$, $m_-$, $q_-$ ($n_+$, $m_+$, $q_+$) are the density, mass and charge of negative (positive) ions.

From (7) we derive the growth rate of the perturbation:

$$\gamma=(1.5)^{1/2}(V_{o+}/r_{de})[(n_-m_+q_-^2/n_+m_-q_+^2)+(n_{+q}q_{+q}^2/n_+q_+^2)]^{1/3}(V_{s+}^2q_+/V_{o+}^2e-1)^{1/2}. \quad (9)$$

At non-linear stage of the instability development the electrical potential $\varphi$ of the perturbation represents the solitary hump of finite amplitude $\varphi_o$. Let's consider properties of the solitary perturbation. As the densities of negative and fixed positive ions are small, we suppose, that the shape of the quasistationary perturbation is determined by dynamics of electrons and flow positive ions. The interaction of this perturbation with negative ions and fixed positive ions results in excitation of the perturbation.

Already at small amplitude of the perturbation adiabatic dynamics of electrons begins. Then the distribution function of the nontrapped electrons has the following kind:

$$f_e(v)=[n_{oe}/V_{te}(2\pi)^{1/2}]\exp(e\varphi/T_e-m_e v^2/2T_e). \quad (10)$$

For the trapped electrons the distribution function does not depend on the energy.

Integrating the electron distribution function on velocity, we obtain the following expression for electron density:

$$n_e=(n_o/(2\pi)^{1/2})(2/T)^{3/2}\int^{\infty}_{o}d\varepsilon(\varepsilon+e\varphi)^{1/2}\exp(-\varepsilon/T). \quad (11)$$

From hydrodynamic equations for positive ions it is possible to receive the following expression for their density:

$$n_+=n_{o+}/[1-2q_+\varphi/m_+(V_{o+}-V_h)^2]^{1/2}. \quad (12)$$

Here $V_h$ is the velocity of the solitary perturbation.

In result of the substitution (11), (12) into the Poisson equation we have the equation for spatial distribution of the electrical potential of the perturbation of any amplitude:

$$(\phi')^2=(8/3\sqrt{\pi})\int^{\infty}_{o}da e^{-a}(a+\phi)^{3/2}-4+(2v^2_{oh}/Q)[(1-2Q\phi/v_{oh}{}^2)^{1/2}-1]. \quad (13)$$

$Q=q_+/e$, $\phi=e\varphi/T$, «’»$=\partial/\partial x$, $x=z/r_{de}$, $v_{oh}=(V_{o+}-V_h)/V_{s+}$.

From the condition $\phi'|_{\phi=\phi o}=0$ and (13) the non-linear dispersing relation follows:

$$v_{oh}{}^2/Q=(A-2)^2/2(A-2-\phi_o),\ \ A=(8/3\sqrt{\pi})\int^{\infty}_{o}da e^{-a}(a+\phi)^{3/2}. \quad (14)$$

For flow velocity $(q_+/e)^{1/2}V_{s+}$ the perturbation is motionless.

Now we take into account in hydrodynamic equations for ions the following terms of expansion on small parameter $\gamma/kV_{tr-}$, $V_{tr-}=(q_-\varphi_o/m_-)^{1/2}$. At their substitution in the Poisson equation, we obtain the evolution equation:

$$2\omega^2_{p+}\partial^3\varphi/\partial t^3/(V_{o+}-V_h)^3=-(\omega^2_{p-}+\omega^2_{pq})\partial^3\varphi/\partial z^3. \quad (15)$$

From (15) the growth rate of the non-linear perturbation amplitude is followed:

$$\gamma_{NL}\approx\omega_{p+}(e\varphi_o/T)^{1/2}[(n_{o-}m_+q^2_-/n_{o+}m_+q^2_+)+(n_{oq}q^2_{+q}/n_{o+}q^2_+)]^{1/3}\ . \quad (16)$$

**References**

[1] T.Kaneko, Y.Miyahara, R.Hatakeyama, N.Sato. J. Phys. Soc. Jap. **69** (7), 2060 (2000).