HINT Equilibrium Calculations for W7-AS

J. Geiger, T. Hayashi*

Max-Planck-Inst. f. Plasmaphysik, Euratom Ass., 17491 Greifswald, Germany

*National Institute for Fusion Science, Toki, Japan

Introduction

Interpretation of experimental data from stellarators needs the topological information of the 3-D flux surface structure in order to compare the data on a common flux coordinate system. The plasma equilibrium currents driven by the plasma $\beta$ change the magnetic configuration and vacuum flux surfaces can serve only as zeroth order approximation to this problem. Therefore, equilibrium codes calculating the finite-$\beta$ flux surfaces have to be applied to interpret the data correctly. However, MHD-equilibrium codes which assume the existence of nested flux surfaces, like the widely used NEMEC-code [1], fail if island structures are getting important. For W7-AS, this is the case in the high-$\epsilon$ configurations ($\epsilon \geq 5/11$) where natural islands lead to a corrugated separatrix. These configurations are of strong interest being used for experiments with the island divertor. At present, evaluations have to be done with the vacuum field, which is a good approximation for small $\beta$-values. However, the new Improved Confinement (HDH) regime reaches non-negligible $\beta$-values. These lead to a shift of the plasma as a whole, possibly displacing the strike-point patterns on the divertor targets and also deforming the $\epsilon$-profile. The different shear at the plasma boundary then alters the size of the boundary islands responsible for the separatrix which may even lead to ergodization of the plasma boundary. Thus, diagnostics as well as theoretical modelling of the plasma edge need appropriate equilibrium calculations including these effects. To understand the changes in the magnetic configuration due to the finite-$\beta$, we investigate the most simple configuration at the relevant $\epsilon$-value of $5/9$, i.e. without vertical field and without using the control coils to change the size of the boundary islands.

The HINT-code

We use the HINT-code [2] which, besides the PIES-code [3], is one of the advanced 3-D MHD-equilibrium codes not relying on the existence of nested flux surfaces. Therefore, it is able to handle islands in- and outside the plasma. The HINT-code solves the time-dependent resistive MHD-equations on a Eulerian grid. The pressure relaxation along field lines is treated in a separate way by averaging the pressure along the field line for a certain length (input parameter) and then interpolating it onto the grid. This decouples to a certain extent the pressure evolution from the magnetic field evolution depending on
the averaging length for the pressure. A major advance in the applicability of HINT is the possibility to include coil currents in the computational region. However, convergence problems due to the numerical scheme applied for the time integration occur if the coils are quite close to the plasma like in the case of the control coils at W7-AS. Therefore, we omit them in the investigations at the moment.

5/9-configuration
The magnetic configuration of W7-AS at the boundary \( \iota \)-value 5/9 which is used in divertor experiments is characterized by 9 large islands leading to a separatrix forming the plasma boundary (Fig. 1). Although the NEMEC-calculation cannot reproduce the separatrix correctly - NEMEC smoothes the boundary - the \( \iota \)-profile is generally consistent with the field line tracing results. A smaller boundary for the free-boundary NEMEC calculation gives a better agreement showing the effect of the indentation on the profile.

HINT-calculations
The input of the HINT-code consists of the magnetic vacuum field configuration on a spatial grid of 55 toroidal planes per half period (54 intervals) each with a grid size of 171 points in R and z. To adjust W7-AS to the normalizations of the HINT-code the magnetic configuration has been doubled in size. The rectangular regions in R and z have a size of 2.0 m x 1.2 m in the triangular plane (original size 1.0 m x 0.6 m). The local coordinate system used in HINT rotates helically so that the R-z-size in the elliptical cross section is 0.6 m x 1.0 m. We start the investigation with a pressure
profile form of \((1 - s^2)^2\) (Fig. 2) to have a broader profile with zero gradient at the edge. Due to the numerical solution method the pressure profile changes during the iteration process. The final pressure-profiles are shown in Fig. 2 compared with the initial profile form. The peaking factor of the evolved profiles is about 2.5 which can be seen in the table on the left. Increasing \( \beta \) leads in these cases to a more pronounced indentation of the separatrix which results in a decrease in the small radius \( a_{\text{eff}} \). The positions of the x-points seem not to move very much in the present calculations. There is, however a movement which is mostly due to the effect of the Shafranov shift, i.e. due to the effect that the outer flux tubes expand poloidally, the upper and lower x-points seem to move inward. The boundary value of \( \iota \) tries to approach 5/9. The profile develops a minimum which, in the course of increasing \( \beta \), heads towards the 10/19 resonance. For a slightly higher \( \beta \)-value one expects the resonance to appear, defining a new separatrix. Indeed, in a case with a more peaked profile, we see the 19 islands developing at the boundary within an overall 5/9 corrugation of the flux surfaces, forming a new separatrix and thus decreasing the confinement region. Due to the low shear at the boundary it is difficult to keep the pressure distribution consistent with the existing flux surfaces.

**Recalculating with fixed boundary VMEC**

Taking the boundary from the HINT calculations, it is possible to perform a fixed boundary VMEC-run calculating the equilibrium with the corresponding pressure profile to
check the consistency of the HINT equilibrium. We did this recalculation for the cases with $\beta_o \approx 0.5\%$ and 1\%. A difficulty in these VMEC calculations is that the number of Fourier coefficients (F.C.) is quite high to represent the corrugated boundary. We had to take 16 toroidal and 16 poloidal F.C.’s at least for both cases. In the 1\% case a better representation had to take 20 poloidal F.C.’s into account. The calculations need considerably more time due to the large number of iterations needed for convergence because of the small time step required to advance the F.C.’s carefully enough. The comparison of the flux surfaces from the VMEC-run and the HINT-calculation is shown in Fig. 4.

![Figure 4: Comparison of HINT Poincare-plots (black) with flux surfaces from fixed-boundary VMEC-calculation (green) for cases with $\beta_o=0.49\%$ (left) and 0.99\% (right).](image)

**Conclusion**

The first calculations show the applicability of the HINT-code to the high-$\beta$-configurations of W7-AS without the correction coils. The changes in the $\epsilon$-profiles as seen in the HINT results can be confirmed with fixed boundary VMEC-runs based on the HINT-boundaries. As a main change in the configuration we point to the increasing indentation of the separatrix with higher $\beta$-values which is a sign of an increasing boundary island width. Depending on the location of a profile diagnostic this can have a considerable influence on the transformation from real space to flux coordinates. In general, this is a good step forward since the principal changes of the magnetic configuration with $\beta$ is accessible with this code. However, further studies and comparisons with experiment and benchmarking with other codes like PIES are necessary to confirm the results.

**Acknowledgement**

The main author expresses his gratitude for the help he got from the Theory Group of Prof. T. Hayashi at NIFS, especially the Working Group providing the computing power.