29th EPS Conference on Plasma Phys. and Contr. Fusion Montreux, 17-21 June 2002 ECA Vol. **26B**, P-5.021 (2002)

# Linear theory of the mirror instability in non-Maxwellian space plasmas

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In most space plasmas where collisions occur very seldomly, the particle velocity distributions of charged particles (electrons and ions) frequently deviate substantially from the canonical Maxwellian distribution. The simplest non-equilibrium distribution under such conditions is generally assumed to be the bi-Maxwellian distribution. It exhibits different temperatures parallel and perpendicular to the ambient magnetic field $\mathbf{B}$. Such anisotropic distributions give rise to the excitation of collective modes which when reacting on the particle distributions usually induce further changes on the distribution function. In a high-$\beta$ plasma, when the ion-temperature anisotropy $A_i = T_{i\perp}/T_{i\|} - 1$ exceeds a certain critical limiting value one encounters, at very low frequencies $\omega \simeq 0$, the magnetic mirror mode instability. This electromagnetic instability has attracted considerable interest in recent past because of its probable importance in the contribution to low-frequency magnetic plasma turbulence. Measured particle distributions in near-Earth space do in the overwhelming majority of cases considerably deviate from the bi-Maxwellian shape. They frequently exhibit long suprathermal tails on the distribution function or, in other cases, possess loss cones. Since the mirror instability is favored by higher perpendicular than parallel pressures it seems reasonable to generalize the theory of the mirror instability to arbitrary distribution functions and even to the inclusion of measured distribution functions.

We start from the perpendicular plasma force balance equation

$$\delta p_\perp + \frac{B\delta B_\|}{\mu_0} = -\frac{k_\|^2}{k_\perp^2}\left(1 + \frac{\beta_\perp - \beta_\|}{2}\right)\frac{B\delta B_\|}{\mu_0}, \tag{1}$$

as has been given, e.g., by *Pokhotelov et al.* [2000]. Here $\delta p_\perp$ is the variation of the perpendicular plasma pressure, $B = \mid \mathbf{B} \mid$ is the magnitude of the ambient magnetic field $\mathbf{B}$, $\delta B_\|$ is the compressional magnetic field perturbation, $k_\perp$ and $k_\|$ are the components of the wave vector $\mathbf{k} = (k_\perp, k_\|)$ perpendicular and parallel to the ambient field, respectively, and $\mu_0$ is the free space permeability. Moreover, $\beta_{\perp,\|} = nT/(B^2/2\mu_0)$ is the plasma beta, i.e. the ratio kinetic to magnetic energy density, with indices $\perp$, $\|$ indicating whether perpendicular or parallel pressures are meant and $n$ is the plasma density. The perturbed quantities in Eq. (1) are assumed to vary exponentially both in time and space as $\exp(-i\omega t + i\mathbf{k}\cdot\mathbf{r})$, where $\omega \ll \omega_{ci}$ is the wave frequency, which is assumed to be much less than the ion cyclotron frequency $\omega_{ci}$. The variation in the perpendicular plasma pressure associated with the mirror mode can be obtained using the general expression for the perturbed particle distribution. The latter can be calculated from the gyroperiod-averaged Vlasov equation [*Pokhotelov et al.*, 2001].

$$\delta F_j = -\frac{\mu\delta B_\|}{B}\frac{\partial F_j}{\partial\mu} + q_j\Psi\frac{\partial F_j}{\partial W} - \frac{\omega(q_j\Psi + \mu\delta B_\|)}{(\omega - k_\| v_\|)}\frac{\partial F_j}{\partial W}, \tag{2}$$

where $F_j(W,\mu)$ denotes the particle distribution function of the $j$th species, $W$ and $\mu$ are particle energy and magnetic moment, $v_\|$ is the parallel particle velocity, $m_j$ is the $j$th species particle mass, and $q_j$ is the particle charge, which in a simple two-component plasma is equal to $-e$ for electrons and $+e$ for the protons. $\Psi$ is the potential of the parallel electric field defined through $E_\| = -ik_\|\Psi$. The physical content of the different terms in Eq. (2) is the following: The first term on the right corresponds to the “mirror effect”, i.e., it represents the change in $F_j$ that is associated with the exclusion of particles from the regions of increased magnetic field by the quasi-static compressive magnetic field perturbation $\delta B_\|$. The second term arises from the particle acceleration in the parallel electric wave field $E_\|$. Finally, the last term describes resonant wave-particle interactions contributing to the mirror mode. The variation in the perpendicular plasma pressure $\delta p_\perp$ associated with the mirror mode can be obtained taking the second velocity moment of $\delta F_j$. Inserting $\delta p_\perp$ into Eq.(1) we obtain in the mirror approximation limit, $\omega \ll k_\|(n^{-1} < v_\|^2 F_i >)^{1/2}$, the wanted local linear dispersion relation of the magnetic mirror mode

$$D(\omega,\mathbf{k}) \equiv \Delta + \frac{\left(\sum_j q_j \langle \mu B\partial F_j/\partial W\rangle\right)^2}{2p_{i\perp}\sum_j q_j^2 \langle \partial F_j/\partial W\rangle} - \frac{i\pi^2\omega}{k_\| p_{i\perp} m_i}\int_0^\infty dW\,(W - W_0)^2\frac{\partial F_{res}}{\partial W} = 0 \tag{3}$$

where

$$\Delta = A - \frac{1}{\beta_{i\perp}} - \frac{k_\|^2}{k_\perp^2\beta_{i\perp}}\left(1 + \frac{\beta_\perp - \beta_\|}{2}\right) \tag{4}$$

and the parameter $A$ denotes the generalized temperature anisotropy factor defined as

$$A = -\frac{\sum_j < \mu^2 B^2 \partial F_j/\partial W >}{2p_{i\perp}} - \frac{p_\perp}{p_{i\perp}}, \tag{5}$$

We have also defined a reference energy $W_0$ through

$$W_0 = \frac{e\sum_j q_j \langle \mu B\partial F_j/\partial W\rangle}{\sum_j q_j^2 \langle \partial F_j/\partial W\rangle}. \tag{6}$$

The summation appearing in these expressions is carried out over all particle species, and $\langle ... \rangle$ denotes the averaging in the velocity space. Here we used the following definitions for the generalized perpendicular pressure of the $j$th species $p_{j\perp} \equiv \langle \mu B F_j \rangle$ and perpendicular plasma beta of the $j$th species $\beta_{j\perp} = p_{j\perp}/(B^2/2\mu_0)$. The derivative term $\partial F_{res}/\partial W = \partial F_{res}/\partial W|_{\mu=W/B}$, describes the contribution of resonant particles with velocity $v_\parallel = 0$. These particles do not move along the magnetic field line, and thus for them the condition $\mu = W/B$ holds. Expression (3) is the general dispersion equation for the mirror mode. It is valid for arbitrary velocity distributions of both ions and electrons. It can be used for the study of the mirror mode in collisionless space plasmas containing any particle distribution that may give rise to the mirror instability like particle distributions exhibiting suprathermal energy tails and loss-cone distributions. It is also applicable to the analysis of the mirror instability of multi-component plasmas.

The single root of the dispersion relation for $\omega$ is purely imaginary whether the plasma is stable or not. The instability growth rate is thus defined as

$$\gamma = \frac{k_\parallel p_{i\perp} m_i}{\pi^2 D} \left[ K - \frac{k_\parallel^2}{k_\perp^2 \beta_{i\perp}} \left( 1 + \frac{\beta_\perp - \beta_\parallel}{2} \right) \right], \tag{7}$$

where the two quantities $K$ and $D$ are given by

$$K = A - \frac{1}{\beta_{i\perp}} + \frac{\left( \sum_j q_j \langle \mu B \partial F_j / \partial W \rangle \right)^2}{2 p_{i\perp} \sum_j e_j^2 \langle \partial F_j / \partial W \rangle}, \tag{8}$$

$$D = - \int_0^\infty dW [(W - W_0)^2] \frac{\partial F_{res}}{\partial W}. \tag{9}$$

The new final general threshold condition for marginal mirror mode growth rate then reads $\beta_{i\perp} K \geq 0$. An important contribution to the threshold comes from the electron terms contained in $A$ and from the last term in Eq. (8). From Eq. (7) one can see that the mirror mode has indeed long parallel length scale with the short scales being suppressed. The growth rate of the fastest growing mode is

$$\gamma_{\max} = \frac{2}{3^{3/2}\pi^2} \frac{1}{D} \frac{k_\perp p_{i\perp} m_i \beta_{i\perp}^{1/2} K^{3/2}}{[1 + \frac{1}{2}(\beta_\perp - \beta_\parallel)]^{1/2}}. \tag{10}$$

and occurs at a ratio of parallel to perpendicular wave numbers given by

$$\left( \frac{k_\parallel}{k_\perp} \right)^2_{\max} = \frac{K}{3} \frac{\beta_{\perp i}}{1 + (\beta_\perp - \beta_\parallel)/2}. \tag{11}$$

for fixed perpendicular wave number $k_\perp$.

As for an example of a non-Maxwellian equilibrium including finite electron temperature effects we consider the velocity distribution in the form used for instance by *Leubner and*

*Schupfer* [2001]

$$F_j = C_j \left(\frac{\mu B}{T_{j\perp}}\right)^{l_j} \left(1 + \frac{W}{\kappa_j T_{j\|}} - \frac{A_j \mu B}{\kappa_j T_{j\perp}}\right)^{-\kappa_j - 1}, \qquad (12)$$

which is a generalization of the usual $\kappa$-distribution to anisotropic empty loss-cone distributions. The velocity distribution (12) accounts for the presence of two basic effects. The first is the loss-cone effect related to the shape of the distribution function in the region of small magnetic moments of the particles and the second effect is the effect of $\kappa$ which causes a suprathermal tail on the distribution function. The analytical and the numerical results confirm amomg others our expectations that emptying the loss cone and stretching the distribution function to higher energies which both increase the perpendicular pressure and pressure anisotropy attribute free energy and therefore lower the instability threshold (Figure 1). Accounting for the electron temperature anisotropy $A_e$, it increases the mirror effect causing a decrease in the instability threshold and a steep increase on the growth rate.

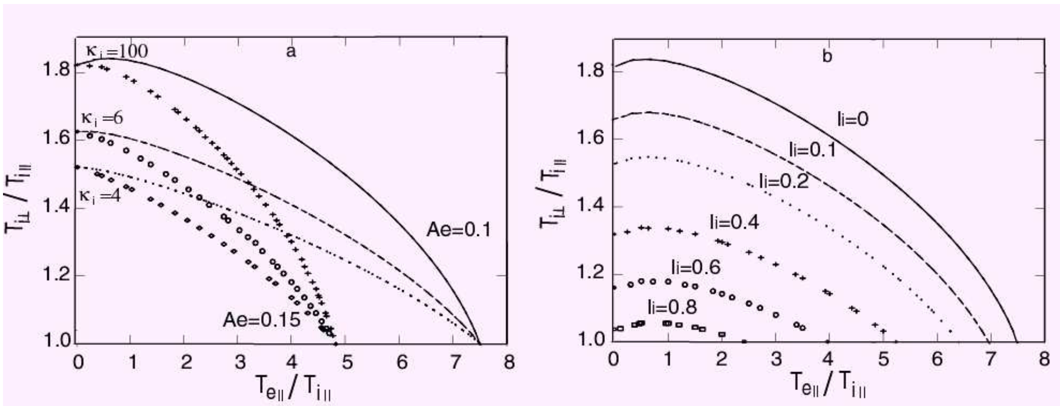


Figure 1: The general mirror instability threshold in terms of the ion anisotropy $T_{\perp i}/T_{\| i}$ as a function of the parallel electron-to-ion temperature ratio $T_{e\|}/T_{i\|}$ for kappa- and DGH-distributions and for fixed parameters $\kappa_i = \kappa_e = 100$, $l_e = l_i = 0$, $T_{e\perp}/T_{i\|} = 1.10$ and $\beta^M_{i\perp} = 1.2$ unless specified otherwise. (a) Dependence on $\kappa_i$. The case $T_{e\perp}/T_{i\|} = 1.10$: solid line $\kappa_i = 100$; dashed line $\kappa_i = 6$; dotted line $\kappa_i = 4$. The $T_{e\perp}/T_{i\|} = 1.15$: crosses $\kappa_i = 100$; circles $\kappa_i = 6$; diamonds $\kappa_i = 4$. (b) Dependence on $l_i$. From top: solid line $l_i = 0$, dashed $l_i = 0.1$, dotted $l_i = 0.2$, crosses $l_i = 0.4$, circles $l_i = 0.6$, squares $l_i = 0.8$.

**References**

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