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Frequency Spectrum of Alfvén Waves Destabilized in Flashing Loops of the Solar Corona

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I. INTRODUCTION

Recently a novel non-linear mechanism of the heating of the plasma ions by the waves with $\omega = \omega_B/s$ (ω is the wave frequency, ω_B is the ion gyrofrequency, s is an integer) has been proposed [1]. It was shown in Ref. [1] that the particle motion becomes stochastic when the wave amplitudes exceed a certain threshold magnitude. This mechanism leads to the increase of the ion energy mainly across the magnetic field, which implies that it results in a state with the high-temperature ions having $T_\perp > T_\parallel$, where $T_\perp$ and $T_\parallel$ are the ion temperatures across and along the magnetic field, respectively. Such a state was observed in the lower Solar corona [2]. A possible source of the waves is an instability that can be excited in a plasma by a small population of the energetic ions. In this case the route to the heating is the following: energetic ions $\longrightarrow$ waves $\longrightarrow$ bulk ions. A laboratory plasma experiment on the NSTX spherical torus seems to confirm such a possibility [3].

In the present work, an Alfvèn instability excited by the energetic ions in a plasma of the Solar corona is considered. In fact, this work extends the analysis of Ref. [4] to clarify a question of the width of the spectrum of destabilized waves, $\Delta\omega$. Such a study is of importance because, according to Ref. [1], the threshold amplitudes of the waves are reasonable only for sufficiently large $\Delta\omega$.

II. GROWTH RATE AND THRESHOLD DENSITY OF THE INSTABILITY

We consider a plasma of the flashing loops of the Solar corona, which contains small amount of the energetic ions with the energy $\mathcal{E} \gtrsim 1MeV$ produced by some accelerating process during the flash. Flashing loops are actually the mirror systems that contain a plasma with the density $n \sim 10^{10} - 10^{11} cm^{-3}$ and the temperature $T \sim 0.1 - 1keV$. The magnetic field in the loops is $B \sim 10 - 100G$.

We assume that energetic ions are characterized by a strongly anisotropic velocity distribution and take their distribution function in the form:

$$F_\alpha(\mathbf{v}) \propto \frac{\delta(\chi - \chi_0)}{v^3}, \qquad (1)$$

where $\chi = v_\parallel/v$ is the particle pitch angle, $v_c \le v \le v_\alpha$, $v_c \sim \sqrt{2\mathcal{E}_c/M_\alpha}$, $\mathcal{E}_c \sim (M_i/M_e)^{1/3}T_e$, M is the particle mass. Being anisotropic, the energetic ions can destabilize the Alfvèn waves provided that their density exceeds a certain threshold magnitude, n_{cr}. The latter is determined by the equation $\gamma = \gamma_\alpha(n_{cr}) + \sum_{j=e,i}\gamma_j = 0$, where the first term describes the instability drive by the energetic ions, the other terms describe the wave damping by electrons and ions.

We use an approximation of homogeneous plasma with the equilibrium magnetic field $\mathbf{B} = (0,0,B)$ and consider Alfvèn waves satisfying the condition $v_{Ti} \ll \omega/k_\parallel \ll v_{Te}$ with v_T the thermal velocity and $k_\parallel$ the longitudinal wave number. Then $\gamma_i \ll \gamma_e$, and γ_e is given by the following expression:

$$\gamma_e = -\frac{1}{2}\sqrt{\frac{\pi}{2}}\frac{M_e}{M_i}\frac{v_{Te}}{v_A}\frac{\chi_0^2}{1-\chi_0^2}\widetilde{\omega}^3 z^2 \omega_B, \tag{2}$$

where $\widetilde{\omega} = \omega/\omega_B$, v_A is the Alfvèn velocity.

In order to calculate γ_α we use the perturbative approach. Then proceeding from a general expression for the dielectric permeability tensor and using Eq. (1) we obtain:

$$\gamma_\alpha = -\mathrm{sgn}(k_\parallel \chi_0)\frac{\pi}{2\Lambda}\frac{n_\alpha}{n_i}\frac{\omega_B}{\widetilde{\omega}}\frac{k_\parallel^2}{k_\perp^2}\sigma_A, \tag{3}$$

where $\Lambda = \ln(v_\alpha/v_c)$,

$$\sigma_A = \sum_{l_{min}}^{l_{max}}\left(J_l^2 - \frac{\xi_0}{1-\chi_0^2}\frac{\partial J_l^2}{\partial \xi_0}\right), \tag{4}$$

$J_l = J_l(\xi_0)$ is the Bessel function of l-th order, l is the number of the cyclotron harmonic in the resonance condition $l\omega_B \approx k_\parallel v_\parallel$, $\xi_0 \equiv lz = lk_\perp k_\parallel^{-1}\sqrt{\chi_0^{-2}-1}$, l_{min} and l_{max} are determined by the condition:

$$\chi_0\widetilde{\omega}\frac{v_c}{v_A} < l < \chi_0\widetilde{\omega}\frac{v_\alpha}{v_A}. \tag{5}$$

In the limit case of $z \ll 1$ the $l = 1$ harmonic mainly contributes, which yields:

$$\gamma_\alpha = \frac{\pi}{8\Lambda}\frac{n_\alpha}{n_i}\frac{\omega_B}{\widetilde{\omega}}\frac{1+\chi_0^2}{\chi_0^2}. \tag{6}$$

It follows from Eqs. (2), (6) that the instability threshold equals to

$$\frac{n_{cr}}{n} = \frac{4\Lambda}{\sqrt{2\pi}}\frac{M_e}{M_i}\frac{\chi_0^4}{1-\chi_0^4}\frac{v_{Te}}{v_A}\widetilde{\omega}^4 z^2. \tag{7}$$

For instance, $n_{cr}/n \sim 10^{-5}$ for $v_{Te}/v_A = 20$, $\chi_0 = 0.3$, $\widetilde{\omega} = 0.5$, $z = 0.5$.

When $z \gg 1$, $\sigma_A(z)$ can approximated as follows:

$$\sigma_A(z) = -\sum_{l_{min}}^{l_{max}}\frac{2}{\pi(1-\chi_0^2)}\cos(2lz - \pi l). \tag{8}$$

The terms in Eq. (8) tend to compensate each other. However, there are certain points, z_*, where the contributions of all the terms accumulate. These points are

$$z_* = \pi \left(0.5 + \frac{p}{l} \right), \tag{9}$$

where p is an integer. The function $|\sigma_A(z)|$ has sharp maxima at $z_*(p/l)$ when $l_{max} \gg 1$, i.e., when $\widetilde{\omega}(v_\alpha/v_A)\chi_0 \gg 1$. The numerical calculations confirm this conclusion, see Fig. 1. The instability threshold for $z \gg 1$ is given by

$$\frac{n_{cr}}{n} = \frac{\Lambda}{2}\sqrt{\frac{\pi}{2}}\frac{M_e}{M_i}\frac{v_{Te}}{v_\alpha}\widetilde{\omega}^3\frac{\chi_0^3}{1-\chi_0^2}z^4. \tag{10}$$

Equation (10) shows that n_{cr} strongly grows with z. This implies that electron damping tends to stabilize the waves with large z.

Another stabilizing factor is the finite width of the pitch angle distribution of the energetic ions. To investigate its influence on the instability we replace $\delta(\chi - \chi_0)$ in the expression for F_α by $\pi^{-1/2}\Delta^{-1}\exp[(\chi - \chi_0^2)/\Delta^2]$ with Δ a parameter that describes the pitch-angle spread. Then we find that the instability persists when

$$\Delta \ll (\chi_0/l)(1-\chi_0^2)/(2z). \tag{11}$$

It follows from Eq. (11) that the effect of finite Δ grows with z.

As seen from Fig. 1, the growth rate significantly depends on l_{max}, which is determined by the magnitude of $\chi_0\widetilde{\omega}v_\alpha/v_A$. On the other hand, the waves of interest for the method of plasma heating suggested in Ref. [1] are characterized by $\widetilde{\omega} \lesssim 1/2$. Taking $\widetilde{\omega} = 1/2$ and $v_\alpha/v_A = 10$, $\chi_0 = 1/3$, we obtain $l < 10/6$, i.e., $l_{max} = 1$. Therefore, below we consider this case. One can see that a general condition of $l_{max} = 1$ is $1 \le \widetilde{\omega}\chi_0 v_\alpha/v_A < 2$. In particular, this condition yields $0.3 \le \widetilde{\omega} \le 0.6$ for the considered v_α/v_A and χ_0. For the frequencies in this range, $\sigma_A(z)$ does not depend on ω and the growth rate changes only by a factor of 2 $[\gamma_\alpha(z) \sim \omega^{-1}\sigma(z)]$. Writing Eq. (3) as

$$\gamma_\alpha = -\mathrm{sgn}(k_{||}\chi_0)\frac{\pi}{2\Lambda}\frac{n_\alpha}{n}\frac{\omega_B}{\widetilde{\omega}}\widetilde{\gamma}, \tag{12}$$

where $\widetilde{\gamma} \equiv (1-\chi_0^2)\sigma(z)/(\chi_0^2 z^2)$, we calculate $\widetilde{\gamma}$. The result is shown in Fig. 2. We observe that the growth rate is maximum for $z \lesssim 0.5$. This implies that when $k_{||}/k_\perp > 5.66$, the frequencies of the destabilized waves lie in the rather wide range. Their growth rate is determined by Eq. (12) with $\sigma(z)/z^2 \approx 0.25$.

III. SUMMARY

We have shown that the presence of a small population of the energetic ions with anisotropic velocity distribution in a plasma of flashing loops of the Solar corona may result in the destabilization of Alfvén waves. The instability arises because of the interaction of the energetic ions and the waves through the cyclotron resonance. Its threshold is lowest for $z < 1$. The characteristic time of the development of the instability is $\gamma^{-1} \sim 0.01 - 0.1s$. The wave spectrum of the destabilized waves can be rather wide. For instance, $\Delta\omega/\omega \sim 0.67$, for $\chi_0 = 1/3$, $v_\alpha/v_A = 10$ and $n_\alpha \gg n_{cr}$. This fact is favourable for the fulfillment of a condition of stochasticity of the bulk ion motion in the wave field obtained in Ref. [1]. However, our analysis tells nothing about the amplitudes of the destabilized waves, which are determined by nonlinear processes. Therefore, the carried out analysis is necessary but not sufficient to clarify whether the mechanism of Ref. [1] is responsible for the plasma heating in the flashing loops of the Solar corona.

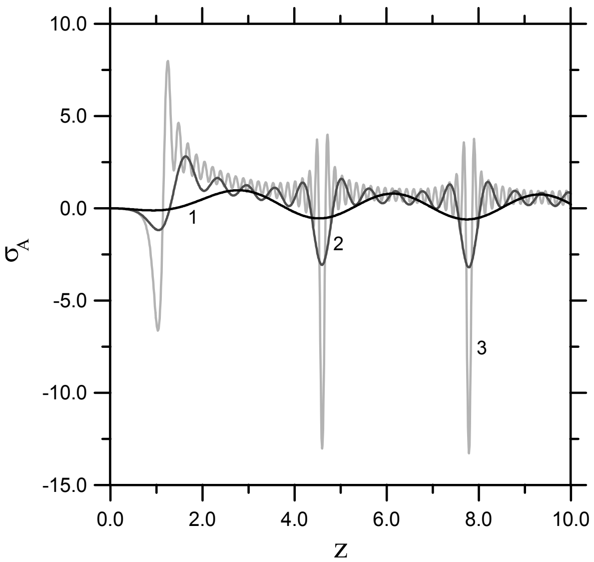

FIG. 1.

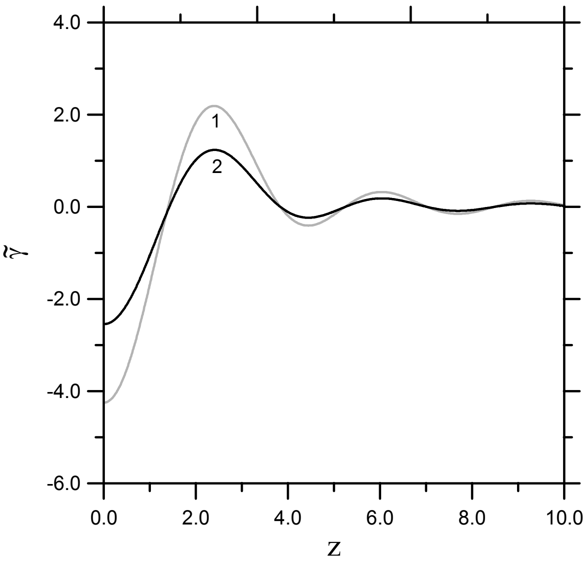

FIG. 2.

FIG. 1. σ_A versus z for $\chi_0 = 0.25$ and various magnitudes of l_{max}. $1 - l_{max} = 1,\ 2 - l_{max} = 5,\ 3 - l_{max} = 20$.

FIG. 2. Normalized growth rate of Alfvèn instability, $\tilde{\gamma}$, versus z for $v_\alpha/v_A = 10$ and various χ_0. $1 - \chi_0 = 0.25,\ 2 - \chi_0 = 0.33$.

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