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Role of the Negative Resistance Behavior in the Self-organization Process in Plasmas

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1. Introduction

In the present paper, by a SS we understand such a state *that breaks the spatial-temporal symmetry* of the previous state of the system and of the external constraints [1, 2, 3]. To create a state with a lower thermodynamic probability, which corresponds to a self-organized structure (SS), it is necessary either removing from the system the entropy produced, or providing such a mechanism by which the free energy of the system is not entirely transformed into disordered motion, but a significant part of it being stored in orderly motion. This latter mechanism means a decrease of the entropy production rate, being consistent with an instability process. The entropy expelling mechanism is triggered once the entropy goes over a certain threshold value. The ordered motion into which the free energy is stored emerges from the non-linear phenomena and, depending on the reactive properties of the structure, is periodic or quasi-periodic (self-induced oscillations) [4]. As a consequence, the SS exhibits a temporal rhythm which breaks the temporal symmetry of the previous state. Sometimes the period is large enough that the temporal symmetry breaking is no longer observed, but this does not mean that it is not present. The self-induced oscillations sometimes could have a chaotic pattern. This chaotic behavior is another form of manifestation of the non-linear systems in which a strong instability develops [5, 6, 7]. The presence of a dissipative process, whose rate depends on the amplitude of one of the system's parameters, could have the effect of a feedback controlling the chaotic behavior [8]. Therefore, when one raises the problem of the appearance of a SS in a system, one has to investigate two aspects: I) the instability that ensures the bifurcation and II) the mechanisms of the entropy production. In this paper we approach this problem theoretically in the case of a low-temperature diffusing plasma column. The necessary condition for an instability to be triggered and the implications of Glansdorff-Prigogine universal evolution criterion (UEC) are investigated. We show that, to have an instability which provokes a spatial-temporal symmetry breaking that leads to a self-organized structure, it is necessary that the reactions devolve in such a way that the plasma column conductivity is negative, i.e. a ***negative resistance behavior.***

2. Theoretical approach

As can be observed generally in experiments, the appearance of a self-organized structure is accompanied by the formation of a plasma double layer as the boundary of the structure. This always looks brighter than the surrounding plasma, which means that excitation and de-excitation processes are present and their rates are higher in this region than in the surrounding ones. Therefore, one has to take into account the contribution of these processes to the entropy production. On the other hand, the electric field, appearing as a result of the space charge separation within double layer, can accelerate electrons up to energies corresponding to the ionization level. After an ionizing collision, such an electron could recombine with an ion, usually resulting in an excited neutral atom. To describe a self-organization process in laboratory experiments, the mathematical model of a low temperature non-isothermal plasma ($T_e >> T_i$) in which elementary processes take place will be considered. A multi-fluid picture will be used together with the Poisson equation. The time variation of the density of the α-species particles in the reaction r is expressed by a reaction function $R_{\alpha r}$, which is given using the BGK approximation. Usually the following reactions are considered to play an important role in the appearance mechanism and in the dynamics of a SS: *excitations*, *de-excitations, ionizations and recombination*. Plasma is under the action of an external electric field taken as control parameter (here introduced by μ). Thus, the system of equation describing plasma reads:

$$\frac{\partial n_\alpha}{\partial t} = -\nabla \cdot (n_\alpha \vec{w}_\alpha) + \sum_\alpha R_{\alpha r} \qquad (1,a)$$

$$\frac{\partial}{\partial t}(n_\alpha \vec{w}_\alpha) = -\nabla \cdot [n_\alpha \vec{w}_\alpha)(\vec{w}_\alpha] - \frac{K_B}{m_\alpha}\nabla(n_\alpha T_\alpha) + n_\alpha \vec{f}_\alpha + \sum_\alpha R_{\alpha r}\vec{w}_{\alpha r} \qquad (1,b)$$

$$\frac{\partial}{\partial t}(\rho_\alpha u_\alpha) = -\nabla \cdot [\rho_\alpha u_\alpha \vec{w}_\alpha + \vec{J}_\alpha] - n_\alpha K_B T_\alpha (\nabla \cdot \vec{w}_\alpha) + \rho_\alpha \vec{f}_\alpha \cdot \vec{w}_\alpha + \sum_\alpha R_{\alpha r} U_{\alpha r} \qquad (1,c)$$

$$\nabla \cdot \vec{E} = \frac{1}{\varepsilon_o}\left(\sum_\alpha n_\alpha q_\alpha + \mu\right) \qquad (1,d)$$

Where: $\vec{w}_\alpha = \vec{w}_\alpha(t,\vec{r})$ is the mean velocity of the α-species particle, m_α is the mass of a particle of species α, $T_\alpha = T_\alpha(t,\vec{r})$ is the local temperature of the α-species particle, $\vec{f}_\alpha = q_\alpha \vec{E}/m_\alpha$, $\vec{w}_{\alpha r}$ is the mean velocity of the α-species particle that enters or results from the reaction r, $u_\alpha = u_\alpha(t,\vec{r})$ is the specific internal energy of the α-species fluid, $U_{\alpha r} = m_\alpha u_{\alpha r}$ is the thermal energy of those particles that enter or result from the reaction r and could be evaluated conform [9], $\rho_\alpha = n_\alpha m_\alpha$, K_B is the Boltzmann's constant, $\vec{w}_\alpha)(\vec{w}_\alpha$ denotes the tensor product and $\vec{E} = -\nabla \Phi$ is the electric field due to the resulting potential Φ. The electron current density $\vec{j}_e$ has two components, a diffusion and a drift one: $\vec{j}_e = -q_e D_e \nabla n_e + \vec{C} \cdot \vec{E}$, $\vec{C}$ denoting the conductivity tensor. As the current-voltage characteristics suggest, one can assume that the ion current density has only a diffusion component: $\vec{j}_i = -q_i D_i \nabla n_i$. The flux of the excited atoms is given by: $\vec{j}_{xv} = -D_x \nabla n_{xv}$.

2.1. The instability appearance. We assume that, prior to the triggering of the instability, plasma is in a stationary state characterized by the values: $n_e = n_i = n_o$, $n_{xv} = n_{xvo}$. In the Poisson equation we assume that the variation of the electron density is mainly responsible for the variable local electric field. Under these assumptions, imposing the conditions necessary to obtain a temporal symmetry breaking through an absolute instability process [3] and introducing the scalar conductivity as:

$$C = \mathrm{Tr}\left[\vec{C} \cdot \vec{k})(\tilde{\vec{E}}\right] / \left(\vec{k} \cdot \tilde{\vec{E}}\right) \qquad (2)$$

We obtain from the system (1,a,b,c,d) a set of simultaneous inequalities for C which, in the case we neglect the diffusion processes (for the sake of simplicity), becomes:

$$C < -\varepsilon_o (b + c + g) \qquad (a)$$
$$C < \varepsilon_o \frac{ad - g(b + c)}{c + g} \qquad (b) \qquad (3)$$
$$C < 0 \qquad (c)$$

Where: $a = \alpha_i n_o$ (ionization frequency), $b = \beta_{r3} n_o$ (three-particle recombination frequency), $c = n_o(\beta_{r2} + \beta_{r3} n_o)$ (total recombination frequency), $d = \alpha_x n_a + \beta_{r2} - \alpha_i n_{x\ o}$ (effective excitation frequency), $g = \alpha_x + \alpha_i n_o$ (total excited atoms disappearance frequency). These inequalities must be simultaneously satisfied and they show that, in order to have a spatial-temporal symmetry breaking, it is necessary to have a negative differential resistance. The condition (3,a) also implies (3,c). In order to have a negative value for C in (3,b) it is necessary to put:

$$ad < g(b + c) \qquad (4)$$

This is in accordance with the phenomenology admitted in the case of the negative hysteresis. Taking into account the significance of *a, b, c, d, g,* the inequalities (3) show that, as the voltage increases, the electron production rate decreases because the total recombination frequency is greater than the ionization frequency and the total disappearance frequency of the excited atoms is greater than the excitation frequency. Thus, the current voltage characteristic will present a *N*-type negative differential resistance.

2.2. The entropy production. To deduce the conditions that enable the appearance of a self-organization phenomenon, we shall refer to the variation of the total entropy of the plasma. From the fundamental equation of the thermodynamics we deduce the acknowledged form of the α-component balance entropy equation:

$$\frac{\partial}{\partial t}(\rho_\alpha s_\alpha) = -\nabla \cdot \vec{J}_{S\alpha} + \sigma_\alpha \qquad (5)$$

Where:

$$\vec{J}_{S\alpha} = \frac{\vec{J}_{Q\alpha}}{T_\alpha} + \vec{J}_\alpha \left(\hat{S}_\alpha - \frac{\mu_\alpha}{T_\alpha} \right) \qquad (6,a)$$

$$\sigma_\alpha = \vec{J}_{Q\alpha} \cdot \nabla\left(T_\alpha^{-1}\right) - \vec{J}_\alpha \cdot \left(\nabla \frac{\mu_{\alpha s}}{T_\alpha} + \frac{\mu_\alpha}{T_\alpha} \cdot \frac{\nabla n_\alpha}{n_\alpha} + \frac{q_\alpha}{T_\alpha} \nabla \Phi \right) + \sum_r \frac{R_{\alpha r}}{T_\alpha} (U_{\alpha r} - 2\mu_{\alpha s}) \qquad (6,b)$$

And: $\vec{J}_{Q\alpha} = -\chi_{\alpha s} \nabla T_\alpha$ is the heat flux, $\vec{J}_\alpha = n_\alpha \vec{w}_\alpha$ is the particle flux, $\hat{S}_\alpha = m_\alpha s_\alpha$. In the previous formulae s_α, u_α, P_α, ρ_α, μ_α, $\tilde{N}_\alpha$ and T_α represent the specific entropy, the specific internal energy, the partial pressure, the mass density, the chemical potential, the specific number of α-species particles and the absolute temperature of the α-component. The entropy production rate σ_α is of particular importance in the following analysis. Generally one can write $\vec{J}_\alpha = -D_\alpha \nabla n_\alpha - n_\alpha M_\alpha \nabla \Phi$, M_α being the mobility of the α-species particles. The second term on the r. h. s. of (6,b) (introduced as $\sigma_{\sigma 2}$) becomes:

$\sigma_{\alpha 2} = (D_\alpha \nabla n_\alpha + n_\alpha M_\alpha \nabla \Phi) \cdot \nabla \frac{\mu_{\alpha s}}{T_\alpha} + \sigma_D + \sigma_\Phi + \sigma_{D\Phi}$. Where:

$$\sigma_D = \frac{\mu_\alpha D_\alpha}{n_\alpha T_\alpha} (\nabla n_\alpha)^2;\ \sigma_{D\Phi} = \left(\frac{q_\alpha D_\alpha}{T_\alpha} + \frac{C_\alpha \mu_\alpha}{n_\alpha q_\alpha T_\alpha} \right) (\nabla n_\alpha) \cdot (\nabla \Phi);\ \sigma_\Phi = \frac{C_\alpha}{T_\alpha} (\nabla \Phi)^2 \qquad (a) \qquad (7)$$
$$C_\alpha = n_\alpha q_\alpha M_\alpha \qquad (b)$$

Let us suppose that the system is close to the dynamical equilibrium corresponding to a self-organization state in which a SS is present. For such a SS the entropy is practically constant and we also suppose that the volume of SS remains constant. We observe that $\sigma_D > 0$ and if:

$$C_\alpha = -C_{0\alpha}\,, \quad C_{0\alpha} > 0 \tag{8}$$

That is a *negative differential resistance,* then σ_Φ could be negative. *We shall make the important assumption that a pulsating SS represents a succession of SSs continuously following each other. Each of the so defined structures is characterized by its spatial-distributions of the state parameters. These distributions appear as a result of the fulfillment of the self-organization conditions, i.e. they verify the universal evolution criterion (UEC)* of Glansdorff and Prigogine [10]. Considering that (8) is fulfilled and putting $\left(\sum_\alpha n_\alpha q_\alpha + \mu\right)/\varepsilon_o = Q$, under the above scenario for the pulsating SS the UEC becomes:

$$\left(\frac{-C_\alpha Q}{q_\alpha} + D_\alpha \nabla^2 n_\alpha\right)\left(\frac{q_\alpha}{K_B T_\alpha}\frac{\partial \Phi}{\partial t} + \frac{\partial(\ln n_\alpha)}{\partial t}\right) + \sum_r \frac{R_{\alpha r}}{K_B T_\alpha}\left(2q_\alpha \frac{\partial \Phi}{\partial t} - \frac{\partial U_{\alpha r}}{\partial t}\right) = 0 \tag{9}$$

3. Conclusions

Using a simple theoretical model we have shown that the instability, responsible for the bifurcation that leads to a spatial-temporal symmetry breaking, is due to those processes that provide negative differential resistance behavior. As the Glansdorff-Prigogine universal evolution criterion requires, the appearance of such a phenomenon is also necessary in order to make the transient regime leading to a self-organized state. The entropy expulsion out of the self-organized structure is assured principally through two processes: The first consists of the photon and particle fluxes leaving the structure. This explains why a SS looks brighter than the surrounding plasma. The second process is represented by the entropy losses through the particles disappearing in the reactions taking place within the structure. Therefore one could affirm that the appearance of a NR in plasma is an indication that a self-organized structure could be present while the relations (8) and (9) together constitute a criterion for the appearance of self-organized structures in low temperature plasmas.

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