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Numerical Treatment of the Ion Acoustic Instabilities in an Ion Beam Dusty Plasma Considering Dust Charge Fluctuations

J. Puerta, J. Silva, C. Cereceda and E. Castro
Departamento de Fisica, Universidad Simón Bolivar
Apdo. 89000, Caracas, Venezuela
e-mail: jpuerta@usb.ve, jsilva@lasimon.net, cereceda@usb.ve, ecastro@usb.ve

ABSTRACT

Dust grains immersed in a plasma can reach high charge states and therefore exhibit charge fluctuations due to waves launched in the plasma. In this paper, we look for a dispersion relation for a one dimensional dust ion-beam plasma system which can be suitable to calculate the growing rates and phase velocities of the ion and dust acoustic waves propagating in the system. Using an hybrid model, we treat the electrons as a fluid while ions and beam are described kinetically. We analyze the strong effects of the charge fluctuations on the onset of the instability. The dispersion relation is solved by means of our algebraic method described in previous works [1].

1. INTRODUCTION

Nowadays there has been a positive growing of interest in the physics of dusty plasma that means, solid particles with diameters between 1 to 100 μm. Under usual circumstances the dust particulates could achieve high states of negative charge , 10^3 - 10^4 electron charges. The dusty plasma system can support a vast variety of physical phenomena, in our case we are interested to calculate numerically the ion-beam instability using a simple hybrid model, that means that we treat the electrons as a fluid and the ions kinetically in order to find a suitable dispersion relation.

2. MODEL

Weak ion acoustic perturbation of a dusty plasma in the presence of an ion beam with the electrons as neutralizing background can be treated in the following description of our model [2].

For the electrons:

$$m_e n_e \left(\frac{\partial \vec{v}_e}{\partial t} + (\vec{v}_e . \nabla)\vec{v}_e \right) = -e n_e \vec{E} \qquad (1)$$

$$\frac{\partial n_e}{\partial t} + \nabla . (n_e \vec{v}_e) = \frac{dn_e}{dt}|_{charging} \qquad (2)$$

where the charging process is defined by [5]

$$\frac{dn_e}{dt}|_{charging} = -\frac{1}{q_e}(\frac{d\rho_{de}}{dt}) \;\; ; \;\; \rho_{de} = Q_{de}n_e \tag{3}$$

with ρ_{de} is the charge density due to the electron charging current.

For Dust:

$$\frac{\partial n_d}{\partial t} + \nabla.(n_d\vec{v}_d) = 0 \tag{4}$$

$$m_d n_d(\frac{\partial \vec{v}_d}{\partial t} + (\vec{v}_d.\nabla)\vec{v}_d \;) = -Z_d e n_d \vec{E} \tag{5}$$

Ambipolar field (without collisions):

$$e(\vec{E}_i + \vec{E}_i) = \frac{\bigtriangledown(n_i T_i)}{n_i} + \frac{\bigtriangledown(n_b T_b)}{n_b} \tag{6}$$

Quasineutrality:

$$n_e = n_i - Z_d n_d + n_b \tag{7}$$

Dust charging equation[3,4,5]:

$$e\frac{\partial Z_d}{\partial t} = I_e + I_d + I_b \tag{8}$$

From [1] - [8] we obtain the perturbation relations:

$$eE^1 = \frac{ik(n_e^1 + Z_d^1 n_d^0 + Z_d^0 n_d^1 - n_b^1)T_i^0}{n_e^0 + Z_d^0 n_d^0 - n_b^0} + \frac{ikn_b^1 T_b^0}{n_b^0} \tag{9}$$

$$n_e^1 = \frac{en_e^0(ik + n_e^0\pi a^2)E^1}{\omega^2 m_e(Z_d^0 - 1)} \tag{10}$$

$$n_d^1 = \frac{kn_d^0 Z_d^0 E^1}{i\omega^2 m_d} \tag{11}$$

Now from equation [8] we get

$$ei\omega Z_d^1 = I_e^1 + I_i^1 + I_b^1 \tag{12}$$

with

$$I_e^0 + I_i^0 + I_b^0 = 0 \tag{13}$$

From eq. (13) we can obtain Z_d^0. The basic equations for the ions and for the beam are given by the linearized Vlasov equation

$$\frac{\partial f_\alpha^1}{\partial t} + \vec{v}\cdot\frac{\partial f_\alpha^1}{\partial \vec{r}} + \frac{\vec{F}}{m}\cdot\frac{\partial f_\alpha^0}{\partial \vec{v}} = 0 \tag{14}$$

with α = ions, ion-beam.

3. DERIVATION OF THE DISPERSION RELATION

Now, using the equations [8] - [12], we obtain an expression for $f^1_\alpha(\vec{v})$. Introducing these results into eq.(14) we obtain the dispersion relation. This can be transformed using the multipolar technique for the Z dispersion function in order to write down this dispersion relation in a polynomial form $\frac{\sum_i A_i \Omega_i}{\sum_j B_j \Omega_j} = 0$, where $\sum_j B_j \Omega_j \neq 0$ and $\Omega = \frac{\omega}{\omega_{pi}}$, and $K = \frac{k}{k_{De}}$. The A_i coefficients depends on the following dimensionless parameters:

$$\zeta_i = \left(\frac{\Omega}{k}\right)\sqrt{\frac{T_e}{2T_i}} \quad ; \quad \zeta_p = \left(\frac{\Omega}{K} - U_b\right)\sqrt{\frac{T_e}{2\,T_i}} \quad ; \quad k_d = \sqrt{\frac{n_e^0\,e^2}{\epsilon_0\,T_e}} \quad ;$$

$$\Gamma = \frac{e^2\,Z_d^0}{4\,\pi\,\epsilon_0\,a\,T_e}\;;\; f_1(u_0) = \left(\frac{\sqrt{\pi}}{4\,u_0}\right)\left(1+2u_0^2\right) erf(u_0)\;;\; f_2(u_0) = \left(\frac{\sqrt{\pi}}{2\,u_0}\right) erf(u_0)$$

$$M_d = \frac{m_d}{m_i} \quad ; \quad M_i = \frac{m_i}{m_e} \quad ; \quad \theta_b = \frac{T_i^0}{T_b^0} \quad ; \quad \theta_i = \frac{T_e^0}{T_i^0} \;.$$

where Γ is the coupling parameter. Solving the numerator we get the roots, and a careful analysis of these roots gives us the real and imaginary part of Ω.

4. CONCLUSION OF OUR RESULTS

Figures 1 to 4 show the growth rate ($\Omega_i = Im(\Omega)$) versus the ion streaming velocity U_b. In all figures the dimensionless wave number used is $K = 0.01$, the beam temperature to ion temperature ratio is $\theta_b = 10$. The equilibrium value of dust charge is calculated to be $Z_d^0 = 4 \text{ x } 10^4$. In figures 1 and 2 the electron temperature to ion temperature ratio is set equal to 10, and in figures 3 and 4 the grain radius is 0.1μm.

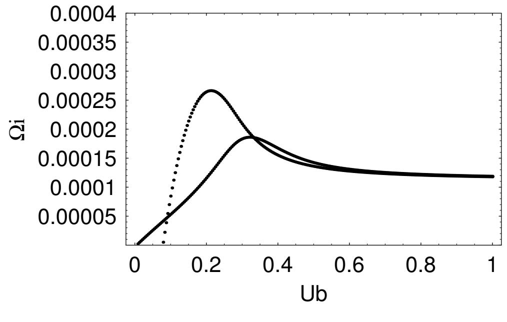

Fig. 1 Ω_i *vs.* U_b, $a = 0.001\mu m$.

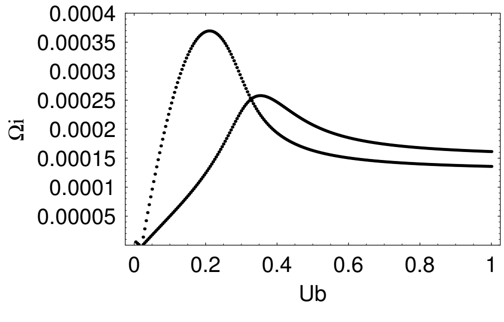

Fig. 2 Ω_i *vs.* U_b, $a = 0.1\mu m$.

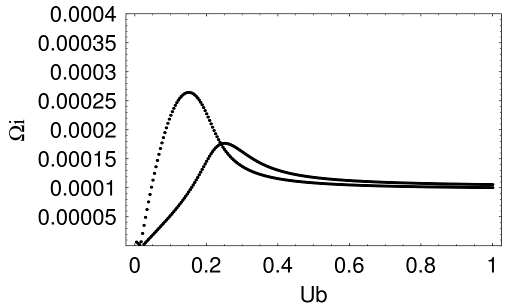

Fig. 3 Ω_i *vs.* U_b, $T_e/T_i = 20$.

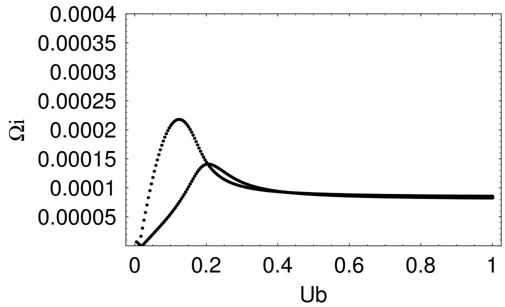

Fig. 4 Ω_i *vs.* U_b, $T_e/T_i = 30$.

We observe two unstable modes for beam velocities in the range between 0 to 1, in good agreement with the results found by the numerical simulation by Gyoo-Soo Chae et al. [5]. It is believed that this ion acoustic instability is due to dust charging fluctuations. For smaller grain radius the maximum instability goes lower, this may be explained by the decreasing of the current fluctuations with the reduction of the cross section for smaller grain radius (see Fig. 1-2). In the case $T_e/T_i = 20$ we observe that the growth rate is higher than for the case $T_e/T_i = 30$, which may be explained by the smaller cross section for higher electron temperature. This hybrid method appears to be well suited technique to analyze such complex systems.

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