29th EPS Conference on Plasma Phys. and Contr. Fusion Montreux, 17-21 June 2002 ECA Vol. **26B**, P-4.210 (2002)

Oscillations in a chain of rod-shaped particles in a plasma

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Abstract. Three dimensional rotatory modes of oscillations in a one-dimensional chain of rod-like charged particles or dust grains in a plasma are investigated. The dispersion characteristics of the modes are analyzed. The stability of different equilibrium orientations of the rods, phase transitions between the different equilibria, and a critical dependence on the relative strength of the confining potential are analyzed. The relations of these processes with liquid crystals are discussed.

The dynamic properties of the particle motion, formation of colloidal crystals and phase transitions in plasma-dust systems are important fundamental questions related to the general theory of self-organization in open dissipative systems [1]. The cases already studied, experimentally and theoretically, mostly correspond to spherical dust grains, but there is growing interest in the properties of colloidal structures composed of elongated (cylindrical) particles [2, 3] levitating in the sheath region of a gas discharge plasma.

Unlike point-like or spherical particles, elongated rotators exhibit a number of additional oscillations related to the new (rotational) degrees of freedom [4, 5]. The excitation and interactions of all these modes will strongly affect the lattice dynamics, leading in particular to new types of phase transitions, as well as affecting those phase transitions already existing in lattices composed of spherical grains. Here, we present a full three-dimensional analysis of the rotatory modes in the chain of rod-like particles, and analyse the critical dependence of the equlibrium and stability of such a chain.

Each rod-like particle is modeled as a rotator having two charges (and masses) concentrated on the ends of the rod, the upper charge being Q_a and the lower charge Q_b; the charges are constant and the masses are equal. The rod of length L, connecting these two charges, has zero radius and mass. We consider a one-dimensional infinite linear chain of rods, with their centers of mass evenly separated by the distance d in equilibrium, along the x-axis. The relevant forces are due to the external potentials and the interparticle interactions. The external potential Φ_{ext} is a combination of the potentials due to both gravitation and the external electrodes. The interparticle force is Coulombic in nature, and there is an exponential decay of the interparticle potential with the distance λ_D (the plasma Debye length). The Lagrangian is given by

$$L = \frac{m}{2}\sum_n \left(\frac{d\mathbf{R}^n}{dt}\right)^2 + \frac{I}{2}\sum_n \left[\left(\frac{d\phi^n}{dt}\right)^2 (\sin\theta^n)^2 + \left(\frac{d\theta^n}{dt}\right)^2\right]$$
$$-\sum_n \left(Q_a\Phi_a + Q_a\Phi_b + Q_b\Phi_a + Q_b\Phi_b + Q_a\Phi_{\text{ext}} + Q_b\Phi_{\text{ext}}\right), \qquad (1)$$

where the sum is over all the particles, and I is the common rotational inertia of each particle. Here θ is the angle the rod makes with the z (vertical) axis, and ϕ is the angle the projection of the rod onto the $x-y$ (horizontal) plane makes with the x-axis (the direction of the chain of particles).

The external potential can be approximated by a parabolic potential for small oscillations, whose minimum lies at the center of mass of a rod ($y=0, z=0$). The assumption of an infinite chain in the x-direction removes the need for a confining potential in that direction. The external potential is therefore assumed to act in both the z (vertical) and y directions, $Q_a\Phi_{\text{ext}}(\mathbf{a}^n)+Q_b\Phi_{\text{ext}}(\mathbf{b}^n)=k_y y^2+k_z z^2$, where (y,z) are the coordinates of the upper charge Q_a.

The equations of motion are extremely nonlinear; moreover, the θ and ϕ behaviour is coupled. In order to proceed we must *linearise* these equations about an equilibrium position. We shall call the equilibrium colatitudinal and azimuthal angles, about which we consider small perturbations, θ_0 and ϕ_0 respectively, which are common to all particles. The analysis then proceeds as follows: let ε be a small perturbation of θ, so that $\theta^n=\theta_0+\varepsilon^n$; and similarly for ϕ by letting η be a small perturbation from ϕ_0.

We next determine the common *equilibrium* orientation of the particles. The result is that equilibria exist in only the following orientations (unless $k_y=k_z$); $(\theta_0,\phi_0)=(0,\phi_0)$ (with ϕ_0 arbitrary) or $(\pi/2,0)$ or $(\pi/2,\pi/2)$, which we draw successively as shown in Fig. 1 (a), (b), and (c), respectively. If $k_y=k_z$, equilibrium exists for the orientation

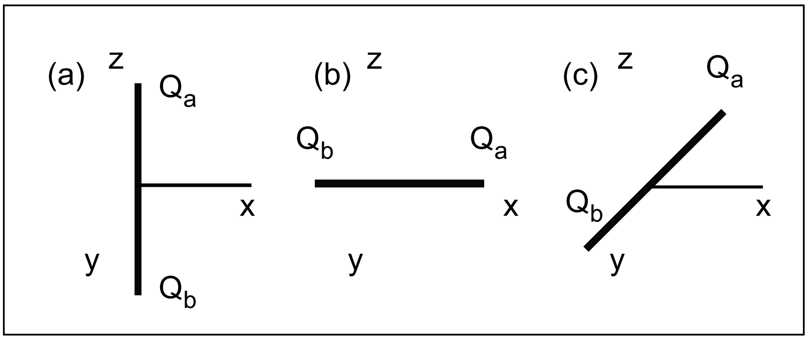

FIGURE 1. The three equilibrium orientations of the rod-like dust grains.

$(\theta_0,\pi/2)$, where θ_0 is arbitrary. Oscillations in θ about the second equilibrium have already been considered in Ref. [5], and oscillations in ϕ about that equilibrium can be obtained simply by exchanging k_y and k_z. Since ϕ is undefined for a vertically oriented rod, we concentrate here on the horizontal equilibrium case $(\pi/2,\pi/2)$.

The perturbation equation describes small oscillations in azimuthal angle ϕ^n about the equilibrium Fig. 1(c), $\theta=\pi/2$, $\phi=\pi/2$. To investigate the existence of an oscillatory solution we compute its Fourier transform and obtain:

$$I\omega^2=+L^2\left[Q_a\Phi_a''(d)-\frac{Q_a}{L_d}\Phi_b'(L_d)\right]\sin^2(kd/2)$$
$$+\frac{d^2L^2}{L_d^2}\left[Q_a\Phi_b''(L_d)-\frac{Q_a}{L_d}\Phi_b'(L_d)\right]\cos^2(kd/2)+\left[Q_a\leftrightarrow Q_b\right]-\frac{L^2}{2}k_y, \quad (2)$$

where $L_d^2 = L^2 + d^2$. For the Debye-Coulomb potential in equation (2), the product $Q_a\Phi_a'$ is negative, while $Q_a\Phi_a''$ is positive, as is true for any potential that falls off with distance. The result is that the coefficients of the oscillatory sine and cosine terms in Eq. (2) are always positive. Thus the dust particles would always exhibit stable oscillations, except for the presence of the term $-k_yL^2/2$ from the external potential, which acts to pull the grains away from this equilibrium to the x-axis (where $y = 0$). These competing terms may then give rise to regions of stable behaviour and regions of unstable behaviour. Recall that the moment of inertia is $I = mL^2/2$. Hence the factor L^2 cancels throughout and is only present, implicitly, through the quantity L_d. We may plot the dispersion relation, by selecting some typical values of the parameters involved: $m = 10^{-15}$kg, $Q = 10^3e$ to 10^4e, and $\lambda_D = 300\mu$m; the resulting dispersion relation is shown in Fig. 2. This plot clearly shows stable regions, corresponding to $\omega^2 > 0$, and unstable regions

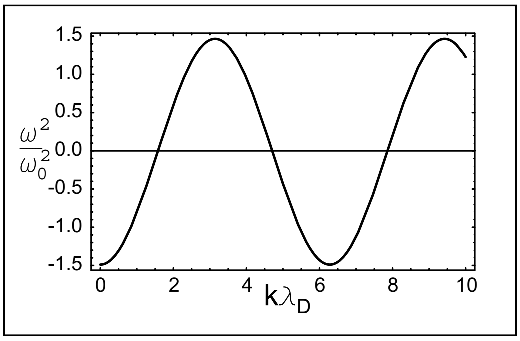

FIGURE 2. Normalised frequency squared versus normalised wavenumber for perturbations in the angle ϕ, travelling in the x direction. The equilibrium orientation is $(\theta_0 = \pi/2, \phi_0 = \pi/2)$. Here ω_0 is the dust plasma frequency, $d = \lambda_D$ and $k_y = 10^{-10}$kgs^{-2}.

corresponding to $\omega^2 < 0$ (i.e ω imaginary.)

We now investigate the behaviour of the perturbation ε of the colatitudinal angle θ, recalling that ε and η completely decouple in the linear approximation. The Fourier transformed equation of motion leads to the following dispersion relation:

$$I\omega^2 = L^2\left[\frac{Q_a}{d}\Phi_a'(d) - \frac{Q_a}{L_d}\Phi_b'(L_d)\right]\sin^2(kd/2) + \left[Q_a \leftrightarrow Q_b\right] + \frac{L^2}{2}(k_z - k_y). \quad (3)$$

Once again note the oscillatory dependence on wavenumber. The first term on the right hand side is this time *negative*, since $Q\Phi'(\mathbf{r}) < 0$ and $d < L_d$. Thus the dust particle's mutual repulsion causes instability. Note the competing terms from the external potential. In the horizontal case one needs $k_z > k_y$ for the possibility of stability (of course wavenumber gaps are still possible). Note that similar dependence exhibit pairs of unbound spherical particles levitating in the confining potential in x- and z-directions [6]. We also recall that the interchange $k_z \leftrightarrow k_y$ gives the vertical equilibrium case. By selecting $k_z > k_y$ the dispersion relation will be qualitatively the same as in Fig. 2.

Instability implies that the rods may switch from the horizontal to the vertical equilibrium (or vice versa) depending on the sign of $k_z - k_y$. However, our analysis is only valid for small perturbation angles, and breaks down at large amplitudes. In order to examine what ensues if the rods lie near an unstable orientation we must consider what will happen physically: the rods will in fact move in opposite directions to move away from one another. Thus on the average, the even rods, say, will move clockwise and the odd rods counter-clockwise (or vice versa). Hence it may be that there exists some intermediate value of stability between the vertical and horizontal. Our analysis is different now from earlier, since we are allowing alternate rods to have opposite equilibrium orientations. By considering the vertical case, this corresponds to equilibria at θ_0 and $-\theta_0$ alternately. Let the even rods be described by perturbations ε^n in θ as before, and the odd neighbours by ζ^{n-1} and ζ^n. The first order perturbation equation is

$$I\ddot{\zeta}^n = \frac{L^2}{4}\left[-\frac{Q_a\Phi_a'(R_1)+Q_b\Phi_b'(R_1)}{R_1}+\frac{2Q_a\Phi_b'(R_2)}{R_2}\right]$$
$$\times\left(2\cos(2\theta_0)\zeta^n-\eta^n-\eta^{n-1}\right)-\frac{L^2}{4}(k_y-k_z)\cos(2\theta_0)\zeta^n. \qquad (4)$$

Thus equilibrium occurs if $\theta_0 = 0$ (the vertical case), or $\theta_0 = \pi/2$ (the horizontal case), or if the term in brackets vanishes. In the case where $k_z = k_y$, and when $Q_a = Q_b$, an equilibrium occurs at $\theta_0 = \pi/4$, when the charges are furthest away from one another, and neighbours are out of phase by $\pi/2$. In this case the right hand side of the perturbation equation (4) vanishes; this is because for an external potential symmetric about the x axis, an arbitrary rotation of θ_0 may be made, as long as neighbours are at right angles to each other. If the potential is not symmetric ($k_y \neq k_z$), equilibria may occur at intermediate angles different to $\theta_0 = \pi/4$.

Hence the described system of rods can undergo phase transitions from one state to another i.e vertical to horizontal and vice versa, or to an intermediate equilibrium, dependent on the easily adjustable external potentials. In fact it is a natural extension to see that this is also true of the third equilibrium case, corresponding to the equilibrium at $(\pi/2, 0)$. This process has a relation to processes in liquid crystals [7], where a state of matter exists between the solid and liquid phases, wherein rod shaped molecules exhibit a partial alignment, rather than a rigid array seen in crystals. The direction of this partial alignment (and phase) can be altered by an external influence.

ACKNOWLEDGMENTS

This work was supported by the Australian Research Council.

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