

# Landau Damping in Formation and Propagation of Dust Ion Acoustic Perturbations in Complex Plasmas

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1. Low-frequency dust ion acoustic (DIA) perturbations can propagate in cosmic plasma environments, dusty plasma of Earth's mesosphere. They have a relevance to low-frequency noise in the F-ring of Saturn [1]. In laboratory complex plasmas the investigation of linear [2] and nonlinear [3 – 5] DIA waves has been performed. The significant problem in description of both linear and nonlinear (shocks, solitons) DIA waves is the correct calculation of the Landau damping (or growth) rates. In particular, as it has been mentioned in [3], Landau damping effect can prevent the formation of DIA shocks. The Landau rates for the DIA perturbations have been found in [6]. However, this consideration has not taken into account the effects of dust particle charging. The derivation (with taking into account these effects) of the dielectric function [7, 8], which enables to calculate the Landau rates, leads to the results different from those [6]. Moreover, the final results for the dielectric function in [7, 8] are not equivalent. Here we calculate the Landau rates for DIA perturbations on the basis of completely kinetic description of complex plasmas. The results are applied to understand the role of Landau damping in the formation and propagation of DIA shocks.

2. To calculate the Landau rates for the DIA waves we follow the standard procedure (see, e.g., [9]). We start from the kinetic equations for electrons and ions

$$\left( \frac{\partial}{\partial t} + \mathbf{v} \cdot \frac{\partial}{\partial \mathbf{r}} + e_{\alpha} \mathbf{E} \cdot \frac{\partial}{\partial \mathbf{p}} \right) f_{\mathbf{p}}^{\alpha} = S_{\alpha} - \int \sigma_{\alpha}(q, \mathbf{v}) |\mathbf{v}| f_{\mathbf{p}}^{\alpha} f^d(q) dq \quad (1)$$

as well as for the dust grains (which are considered to be immobile)

$$\frac{\partial f^d}{\partial t} + \frac{\partial}{\partial q} \left( I_{\text{ext}} + \sum_{\alpha} I_{\alpha} \right) f^d = 0, \quad (2)$$

where  $f_{\mathbf{p}}^{\alpha}(t, \mathbf{r})$  is the distribution function of the particles of the kind  $\alpha (= e, i)$ ,  $f^d(t, \mathbf{r}, q)$  is the distribution function of the dust grains,  $e_{\alpha}$  is the charge of the particle of the kind  $\alpha$ ,  $q$  is the dust particle charge,  $\mathbf{E}$  is the electric field,  $S_{\alpha}$  describes any external source

of plasma particles,  $\sigma_\alpha(q, \mathbf{v})$  is the cross-section characterizing the interaction of dust grains and electrons/ions (see, e.g., [8]),  $I_\alpha(t, \mathbf{r}, q) = \int e_\alpha \sigma_\alpha(q, \mathbf{v}) |\mathbf{v}| f_\mathbf{p}^\alpha d^3 \mathbf{p} / (2\pi)^3$  is the current of the particles of the kind  $\alpha$  on dust grain,  $I_{\text{ext}}$  includes all other currents (e.g., the photoelectric current, the secondary emission current, etc.). The distribution functions are normalized as follows

$$n_\alpha = \int f_\mathbf{p}^\alpha(t, \mathbf{r}) \frac{d^3 \mathbf{p}}{(2\pi)^3}, \quad n_d = \int f^d(q) dq, \quad (3)$$

where  $n_\alpha$  and  $n_d$  are the densities of the particles of the kind  $\alpha$  and the dust particles, respectively.

Furthermore, we separate regular and fluctuating parts of the distribution functions, use the Fourier representation, and find linear (in the electric field  $\mathbf{E}$ ) responses of the distribution functions. Using these responses and the Poisson's equation we obtain the expression for the dielectric function. The final result is

$$\begin{aligned} \varepsilon_{\mathbf{k}, \omega} = 1 + \sum_\alpha \int \frac{4\pi e_\alpha^2}{|\mathbf{k}|^2 (\omega - \mathbf{k} \cdot \mathbf{v} + i\nu_{d,\alpha})} \left( \mathbf{k} \cdot \frac{\partial \Phi_\mathbf{p}^\alpha}{\partial \mathbf{p}} \right) \frac{d^3 \mathbf{p}}{(2\pi)^3} \\ + \frac{4\pi}{|\mathbf{k}|} \frac{n_d}{(\omega + i\nu_q)} \beta_{\mathbf{k}, \omega} \left( -1 + i \sum_\alpha \int \frac{e_\alpha |\mathbf{v}| \Phi_\mathbf{p}^\alpha}{(\omega - \mathbf{k} \cdot \mathbf{v} + i\nu_{d,\alpha})} \frac{\partial \sigma_\alpha(q, \mathbf{v})}{\partial q} \Big|_{q=q_0} \frac{d^3 \mathbf{p}}{(2\pi)^3} \right), \end{aligned} \quad (4)$$

where  $\nu_{d,\alpha} = \int \sigma_\alpha(q, \mathbf{v}) |\mathbf{v}| \Phi^d(q) dq$ ,  $\Phi = \langle f \rangle$  is the regular part of the distribution function, the brackets denote the statistical averaging on the ensemble of the dust particles,  $\nu_q = \omega_{pi}^2 a (1 + z_0 + T_i/T_e) / \sqrt{2\pi} \nu_{Ti}$  is the grain charging rate,  $\omega_{pi}$  is the ion plasma frequency,  $a$  is the grain radius,  $z = Z_d e^2 / a T_e$ ,  $\nu_{Ti}$  is the ion thermal velocity,  $q = -Z_d e$ ,  $-e$  is the electron charge, the subscript 0 stands for the unperturbed plasma parameters,  $T_{e(i)}$  is the electron (ion) temperature,  $\beta_{\mathbf{k}, \omega} = S_{\mathbf{k}, \omega}(q_0) / (1 - \tilde{S}_{\mathbf{k}, \omega}^d(q_0) \chi_{\mathbf{k}, \omega})$ ,  $\chi_{\mathbf{k}, \omega} = i n_d / (\omega + i\nu_q)$ ,

$$\begin{aligned} S_{\mathbf{k}, \omega}(q) &= \sum_\alpha \int \frac{e_\alpha^2 \sigma_\alpha(q, \mathbf{v}) |\mathbf{v}|}{i(\omega - \mathbf{k} \cdot \mathbf{v} + i\nu_{d,\alpha})} \left( \frac{\mathbf{k}}{|\mathbf{k}|} \cdot \frac{\partial \Phi_\mathbf{p}^\alpha}{\partial \mathbf{p}} \right) \frac{d^3 \mathbf{p}}{(2\pi)^3}, \\ \tilde{S}_{\mathbf{k}, \omega}^d(q) &= \sum_\alpha \int \frac{e_\alpha^2 \sigma_\alpha(q, \mathbf{v}) |\mathbf{v}|}{i(\omega - \mathbf{k} \cdot \mathbf{v} + i\nu_{d,\alpha})} \frac{\partial \sigma_\alpha(q, \mathbf{v})}{\partial q} \Phi_\mathbf{p}^\alpha \frac{d^3 \mathbf{p}}{(2\pi)^3}. \end{aligned}$$

The expression (4) coincides in the limit of immobile dust with the formula for the dielectric function calculated in [8] (see Eq. (55) in [8]). The frequency of DIA waves  $\omega_{\mathbf{k}} = \omega_{\mathbf{k}}^s + i\gamma_{\mathbf{k}}^L$  and the Landau rate are calculated in the limit  $|\mathbf{k}| \nu_{Ti} \ll \omega \ll |\mathbf{k}| \nu_{Te}$  from the equation  $\varepsilon_{\mathbf{k}, \omega} = 0$ , where  $\nu_{Te}$  is the electron thermal speed. The results are well-known for  $\omega_{\mathbf{k}}^s$  (see, e.g., [1]) and new for  $\gamma_{\mathbf{k}}^L$ :

$$\begin{aligned} \omega_{\mathbf{k}}^s &\approx \frac{|\mathbf{k}| c_s \sqrt{n_i/n_e}}{\sqrt{1 + \mathbf{k}^2 \lambda_{De}^2}}, \\ \gamma_{\mathbf{k}}^L &\approx -\sqrt{\frac{\pi}{8}} \frac{m_e n_i}{m_i n_e} \frac{\omega_{\mathbf{k}}^s}{(1 + \mathbf{k}^2 \lambda_{De}^2)^{3/2}} - \nu_q \sqrt{\frac{\pi}{2}} \frac{Z_d n_d}{n_e z} \frac{(t+z)}{(1+t+z)(1 + \mathbf{k}^2 \lambda_{De}^2)}, \end{aligned} \quad (5)$$

where  $c_s = \sqrt{T_e/m_i}$  is the ion acoustic speed,  $t = T_i/T_e$ ,  $\lambda_{De}$  is the electron Debye length. The first term in  $\gamma_{\mathbf{k}}^L$  coincides with the Landau rate found in [6]. The second term in  $\gamma_{\mathbf{k}}^L$ , proportional to  $v_{q_0}$ , is associated with the dust particle charging processes. It is not taken into account in [6]. Nevertheless, in many real situations this second term (we denote it as  $\gamma^{L,q}/(1 + \mathbf{k}^2 \lambda_{De}^2)$ ) is dominant in Landau damping.

3. It is the situation of dominance of the term containing  $\gamma^{L,q}$  in the Landau damping that takes place in the experiments [3, 4] on DIA shocks. To evaluate the ratio of the second term to the first one in the expression (5) for  $\gamma_{\mathbf{k}}^L$  one has to use Fourier representation of shock profile. This allows us to obtain the typical wave vector. For the data [10] used to model the experiments [3] the typical wave vector corresponding to  $\varepsilon Z_d \equiv Z_d n_d/n_i = 0.75$  is  $|\mathbf{k}| \approx 0.12 \text{ cm}^{-1}$ . For this value of  $|\mathbf{k}|$  the term in  $\gamma_{\mathbf{k}}^L$  associated with the dust particle charging is one order of magnitude larger than the first one.

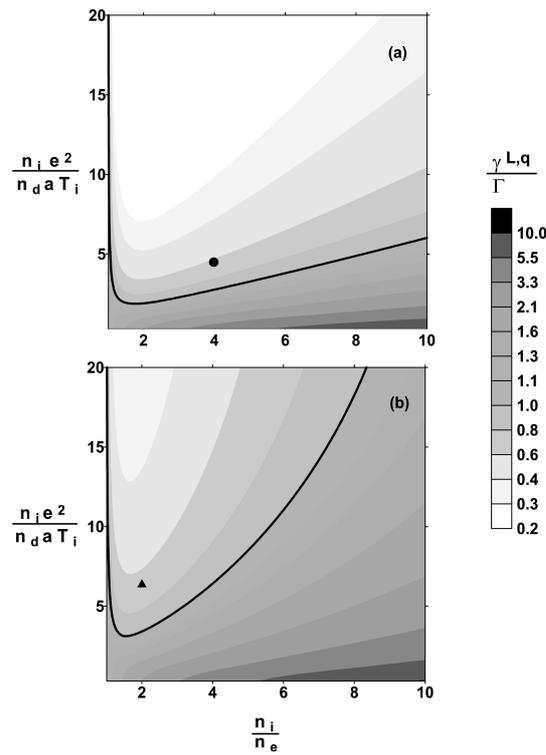
We note that it is possible to use the expression (5) for  $\gamma_{\mathbf{k}}^L$  for the cases of the experimental data [3, 4]. Indeed, the inequality  $\omega_{\mathbf{k}}^s \gg |\mathbf{k}|v_{Ti}$  assumes the fulfillment of the condition  $T_e/T_i \gg n_e/n_i$ . In complex plasmas  $n_e/n_i < 1$ . Furthermore, the inequality  $T_e > T_i$  is seemingly fulfilled in both the experiments: in [4]  $T_e \approx 1 - 1.5 \text{ eV}$ ,  $T_i < 0.1 \text{ eV}$ ; in [3] the way of plasma ionization assumes that  $T_i$  should be at least two times less than  $T_e$  (to obtain this it is necessary to apply the corresponding boundary condition [10, 11] on the hot plate of the installation [3]), besides the turbulent electron heating can take place.

Thus to evaluate the role of the Landau damping we have performed the comparison of  $\gamma^{L,q}$  with  $\Gamma = (v_{ch} + \tilde{v})/2$ , where definitions of  $v_{ch}$  and  $\tilde{v}$  are given in [10, 12].  $\Gamma$  is the damping rate of linear DIA waves which are solutions of the hydrodynamical set of equations used [10] to describe the experiments [3, 4]. The results are presented in Fig. 1(a) and Fig. 1(b) for the data of the experiments [3] and [4], respectively. We see that for the experiments [3, 4]  $\Gamma > \gamma^{L,q}$ . This means the validity of the hydrodynamical approach [10] for the description of these experiments. This fact is illustrated in Fig. 2 constructed analogously and for the same parameters of the experiment [3] as Fig. 1(b) in [10].

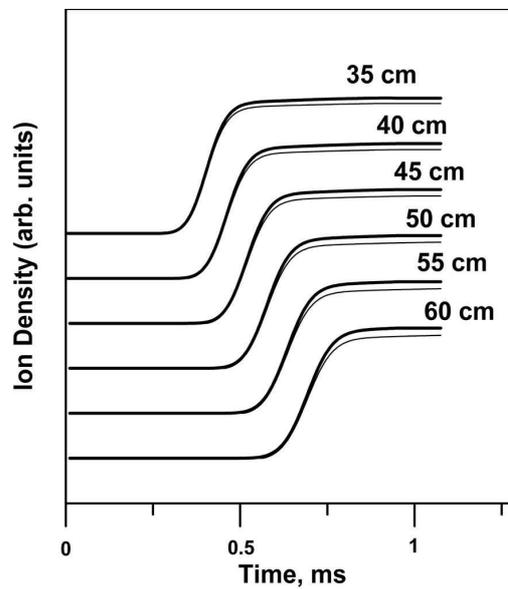
We note that for the experiments [3, 4] the Landau damping is not significant. However, the change in the plasma parameters (see Fig. 1) can increase its role significantly.

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**FIGURE 1.** Profiles of equal magnitudes of the ratio  $\gamma^{L,q} / \Gamma$ . The dark circle (a) corresponds to the plasma parameters of the experiment [3] and  $\epsilon Z_d = 0.75$ , while the dark triangle (b) fits the plasma parameters of the experiment [4] and  $\epsilon Z_d = 0.5$ .



**FIGURE 2.** Time evolutions of the ion density at different distances from the grid calculated on the basis of the hydrodynamical approach [10] (heavy curves) and with taking into account the Landau damping (light curves).