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On the Sensitivity of Polarimetric Measurements of the Current Density Profiles in Tokamaks

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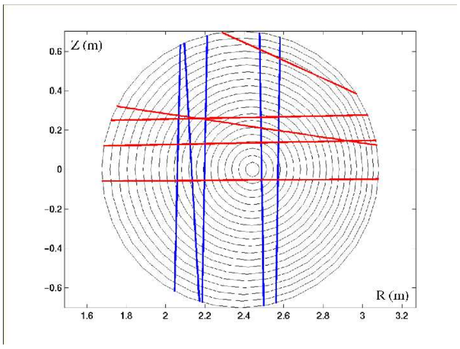
Introduction

Heat transport analysis on advanced heating scenarios shows that the shape of the current profile is an essential ingredient to obtain improved confinement regime Ref [1]. Precise measurement of the current profile is required to correctly characterise the heat transport dependance upon the current profile, furthermore control of the confinement regime is directly correlated with the control of the current profile.

Our study tries to answer to the following correlated questions :

1. What's the best design of the measurement apparatus to obtain a given precision on the current profile ?
2. With this apparatus what kind of current profile we can efficiently control ?



The study is restricted to the case of the determination of the current profile trough polarimetric measurement of the Faraday angles. On Tore Supra the present interfero-polarimeter is composed of 5 nearly vertical chords and we plan to add 5 nearly horizontal chords to improve the poloidal section coverage and the spatial resolution (see figure).

The purpose of this paper is to define the relation between the following parameters of the apparatus design :

1. precision of the Faraday angles measurement
2. required precision of density current profile
3. number of chords
4. functionnal form (harmonic richness) of the current profile.

Linearisation

We recall that the current density profile j(ρ) can be deduced from the faraday angles measurement α(c) on chord c using the following approximated expressions :

$$\alpha\ (c) = \int_c \vec{ds}\, C \lambda^2 n_e(s) \vec{B}(s)$$

$$\vec{ds} = (dR, dZ)$$

$$\vec{B} = (B_R, B_Z) = \frac{-\mu_0 I(\rho)}{2\pi R\tau \left(\frac{g_{\theta\theta}}{R\tau}\right)_0} \left(\frac{\partial R}{\partial \theta}, \frac{\partial Z}{\partial \theta}\right)$$

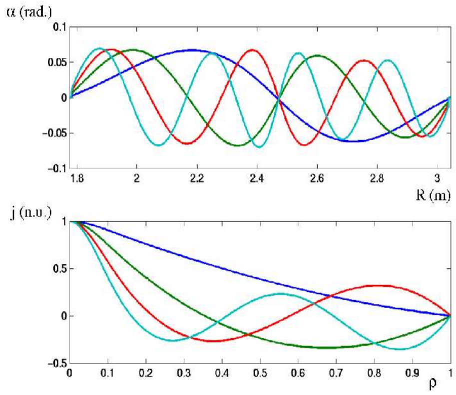
$$I(\rho) = \int_0^\rho d\rho\, \tau_0(\rho) j(\rho)$$

α(c) are the Faraday angles.
c is the chord index.
τ is the jacobian of the transformation (R,Z)->(ρ,θ).
g is the metric tensor.
λ is the wavelength.

A first approximation of the Shafranov shift and ellipticity profiles shapes fixes the plasma geometry. In this first approximation (which could be used as a first step of an itteration procedure), we can write a simple linear relationship between Faraday angles and the integrated current profile: $\alpha(c)=\int_c d\rho\, G(\rho) I(\rho)$, where G(ρ) is function of the geometry and of the electronic density only.

Inversion method

To deduce I(ρ) from the measured Faraday angles α(c) on polarimetric chords we first develop the current profile j(ρ) on a set of functions assumed to be sufficiently complete to describe accurately this current profile. The choice of the set seems to be arbitrary but the final result is sensitive to this choice as we shall see later. We have tested various sets of functions (Bspline, Bessel functions and polynomials).

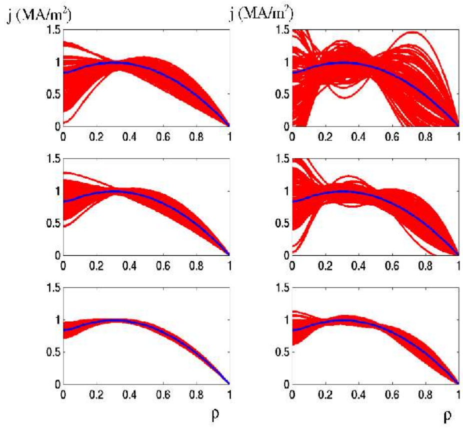


The amplitude components of the current profile are obtained by minimising a chosen distance between the measurements $\alpha_{mes}(c)$ and the searched fitted angles $\alpha_{fit}(c)$. A numerically accurate method to determine these amplitudes is to firstly orthonormalise (using Singular Value Decomposition algorithm) the initial function set. This method is best adapted to discuss the questions raised here.

It appears evident that, because of the limited number of chords of the instrumental apparatus, the number of components of the set has to be limited too. That is to say that we only have access to a subset of the space of current profiles (i.e. to a limited low-harmonic range of this space). Our minimisation procedure is therefore going to be limited to a low-order subset of the chosen function set (consisting of the base functions ranging from the first to the K_L-th). This is equivalent to the requirement that the residual vector of Faraday angles has to be orthogonal to this low-order harmonic subset: a requirement that is probably more intuitive than a minimisation procedure. So, the amplitude of the k-th component $a(k)$ of the fitted profile is simply the projection of the measurement onto the k-th component of the reduced orthonormal function set. Using the Euclidean distance in the minimising procedure (i.e. the Euclidean scalar product to define the projections) this leads to :

$$a(k)=\sum_c \alpha_{set}(k;c)\,\alpha_{mes}(c)$$

The fitted current profile $j_L(\rho)=\sum_{k=1}^{K_L} j_k(\rho)$, where $j_k(\rho)=a(k)\, j_{set}(k;\rho)$, can be considered as the sum of the first K_L moments of the current profile. Of course the uncertainty on these moments increases with their index k. As progressively higher order moments contain progressively less meaningful information, the number of harmonics taken into account in the fit procedure has to be adapted to the diagnostic apparatus in order to avoid undesired oscillation effects. This is quantitatively illustrated in the following figure. It shows fits to the computer-generated Faraday angles based on a mildly hollow current density profile (blue line) correponding to a total plasma current I_p=0.94 MA, when we take into account two (K_L=2, left column) or three (K_L=3, right column) harmonics. Every red line



corresponds to a different extraction of the random noise used in the generation of the "measured" Faraday angles. The first row corresponds to the presently working diagnostic with a measurement error of 0.2°; the second one to its future implementation with the 5 planned nearly horizontal chords added and the same error, and the third one to the same implementation and a measurement error reduced to 0.06°.

Errors propagation

In order to evaluate quantitatively the accuracy in the profile measurement with a single figure of merit we proceed as follows. We assume the error $\varepsilon\sigma_c$ on the measured Faraday angles α_{mes} on the N_c chords to be statistically distributed on a gaussian around the true physical Faraday angles $\alpha_{phys}(c)$ with a standard deviation σ_c :

$$\alpha_{mes}(c) = \alpha_{phys}(c) + \epsilon\,\sigma_c$$

where ε is a normally distributed random variable of standard deviation equal to 1.

The fitted quantity $\alpha_{fit}(c)$ is calculated onto the K_L first low harmonic order of $\alpha_{phys}(c)$.

$$\alpha_{phys}(c) = \alpha^L_{phys}(c) + \alpha^H_{phys}(c)$$

Giving for the fitted current profile :

$$j(\rho) = \sum_c V^L(\rho;c)\,\alpha_{mes}(c) = \sum_c V^L(\rho;c)\,[\alpha^L_{phys}(c) + \epsilon\,\sigma_c]$$

with

$$V^L(\rho, c) = \sum_{k=1}^{K_L} j_{set}(k;\rho)\,\alpha_{set}(k;c)$$

Assuming that there is no correlations between errors on different chords and that the errors on Faraday angles are the same for all chords , we deduce the following expression for the standard deviation on current density :

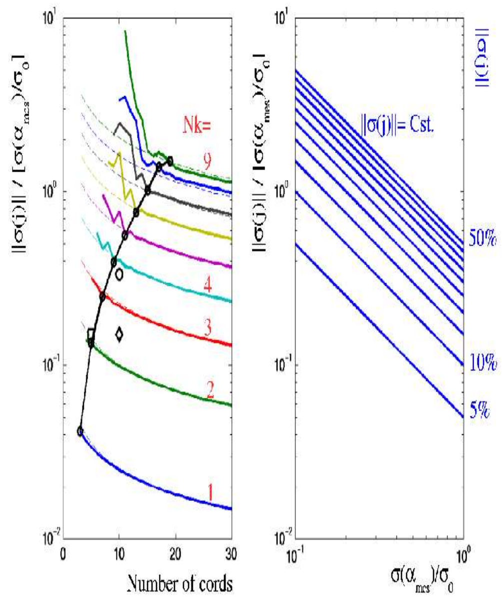
$$\sigma_j(\rho) = \sqrt{\sum_c (V^L(\rho;c))^2}\,\sigma_\alpha$$

This standard deviation can be scaled using, for example, the following global norm :

$$\|\sigma_j\| = \frac{1}{I_p} \int_0^1 d\rho\, \tau_0(\rho)\,|\sigma_j(\rho)|$$

Apparatus design

The following figure resumes the relations between the parameters of the apparatus (number of chords and errors on the Faraday angles), the number of components used in the data processing and the figure of merit $\|\sigma_j\|$.



As an example the actual design of the interfero-polarimeter of Tore Supra is able with a precision of $\sigma_0 = 0.2°$ on Faraday angles to determine the 2 first order harmonics of the density current with a precision scaled to 15% of the plasma current (black square). The future design with a precision of 0.06° on Faraday angles is able to determine with a precision scaled to 10% a 3rd order harmonic (black circle). The result is nearly the same using the Bessel set as initial set. Polynomial set leads to much higher errors amplification.

When the number of cords N_c is higher than 2 K_L + 1 (the black points on the figure correspond to the limit $N_c = 2\ K_L + 1$), the uncertainty $||\sigma_j||$ on current density appears to scale with N_c and K_L as $K_L^2 / N_c^{1/2}$ (dashed lines).

Current density control

For a given diagnostic apparatus, the harmonic order that can be accurately defined is limited. As a direct consequence the current density that can be efficiently controled by this apparatus has to obey to the same limitation. That is to say that the spatial resolution bandwidth of the control system is limited by the diagnostic apparatus. If we try to control high order harmonic parts of the density current, it's amplitude grows up and leads to unwanted oscillations.

Conclusions

The fitting method developed :

1. Is a useful tool for the design of the diagnostic apparatus for a desired precision on density current profile.
2. Can be adapted to an existing design to avoid unphysical oscillations on fitted profiles.
3. Supplies estimates of uncertainties on different moments of current density profiles.
4. Can be used to specify the class of density current profiles that can be efficiently controlled.

The fitting method applied in this paper to polarimetric measurements of the current profiles can be used on others diagnostics. This is the same method used on Tore Supra for the determination of temperature and density profiles as well.

[1] G.T. Hoang, et al., Nuclear Fusion, Vol. 38, No. 1 (1998)