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Bifurcations in three classes of turbulent fluctuations with different scale-lengths

S.-I. Itoh[a], A. Kitazawa[a], M. Yagi[a] and K. Itoh[b]

[a] *Research Institute for Applied Mechanics, Kyushu University, Kasuga 816-8580, Japan*
[b] *National Institute for Fusion Science, Toki, 509-5292, Japan*

Abstract Cases where three kinds of fluctuations having the different typical scale-lengths coexist are analyzed and the statistical theory of strong turbulence in inhomogeneous plasmas is developed. The nonlinear interplay through them induces a quenching or suppressing effect, even if all the modes are unstable when they are analyzed independently. Variety in mode appearance takes place: one mode quenches the other two modes, and causes their subcritical bifurcations, or one mode is quenched by the other two modes, etc. Analysis reveals that the nonlinear stability boundary (marginal point) and the amplitude of each mode may substantially shift from the conventional results of independent analyses.

Introduction Progress has been made in the field of theory and modelling of plasma turbulence. Methodologies of turbulence theories have been advanced. (See, for a review, e.g. refs.[1-4].) Emphasis has been made on the importance of the nonlinear interplay between separated fluctuations having the different typical scale lengths. Interactions between the modes with different scale lengths have a large impact on the evolution of the turbulence [5-11]. In this paper, we analyze the turbulence composed of three different kinds of collective modes with different scale lengths. The nonlinear interplay is analyzed, and the transitions among them are investigated. A phase diagram is summarized.

Model The dynamical equations of fluctuation fields are given in a form as

$$\frac{\partial}{\partial t}\boldsymbol{f} + \mathcal{L}^{(0)}\boldsymbol{f} = \mathcal{N}\left(\boldsymbol{f},\boldsymbol{f}\right) + \tilde{\boldsymbol{S}}_{th} \qquad (1)$$

where a reduced set of equations are used and we employ $\boldsymbol{f}^{\mathrm{T}} = \left(\phi, J_{\|}, V_{\|}, P_e, p_i\right)$ and

$$\mathcal{N}\left(\boldsymbol{f},\boldsymbol{f}\right) = -\left(\nabla_\perp^{-2}\left[\phi, \nabla_\perp^2\phi\right], \left(1-\xi\nabla_\perp^{-2}\right)^{-1}\left[\phi, J_{\|}\right], \left[\phi, V_{\|}\right], \left[\phi, P_e\right], \left[\phi, p_i\right]\right) .$$

The nonlinear terms are expressed as $\mathcal{N}\left(\boldsymbol{f},\boldsymbol{f}\right) = -\boldsymbol{\Gamma}\boldsymbol{f} + \mathcal{D}\boldsymbol{f} + \tilde{\boldsymbol{S}}_{\mathrm{self}} + \tilde{\boldsymbol{S}}_{\mathrm{shorter}}$

where $-\boldsymbol{\Gamma}\boldsymbol{f}$ is the drag term, $\mathcal{D}\boldsymbol{f}$ is the drive term, $\tilde{\boldsymbol{S}}_{\mathrm{self}}$ and $\tilde{\boldsymbol{S}}_{\mathrm{shorter}}$ are the nonlinear noise terms.

The nonlinear dispersion relations are given as

$$\det\left(\lambda^m \boldsymbol{I} + \mathcal{L}^{(0)} + \boldsymbol{\Gamma}^l_{(m)} + \boldsymbol{\Gamma}^l_{(l)} + \boldsymbol{\Gamma}^l_{(h)}\right) = 0 \tag{2a}$$

$$\det\left(\lambda^l \boldsymbol{I} + \mathcal{L}^{(0)} + \boldsymbol{\Gamma}^l_{(l)} + \boldsymbol{\Gamma}^l_{(h)} - \mathcal{D}^l_{(m)}\right) = 0 \ , \tag{2b}$$

$$\det\left(\lambda^h \boldsymbol{I} + \mathcal{L}^{(0)} + \boldsymbol{\Gamma}^h_{(h)} - \mathcal{D}^h_{(m)} - \mathcal{D}^h_{(l)}\right) = 0 \ , \tag{2c}$$

where λ^m, λ^l and λ^h are nonlinear eigenvalues. The indices m, l and h stand for the macro, semi-micro and micro mode fluctuations, respectively. Equation (2), the FD relation and the renormalization relation for eddy damping rate form a closed set of equations that determines the fluctuation levels (I^m, I^l, I^h), the decorrelation rates (λ^m, λ^l, λ^h) and the eddy-damping rates (γ_v^m, γ_v^l, γ_v^h) simultaneously. A derivation has been discussed in [5] . The controlling parameters are the driving source terms, $\left(D^m, D^l, D^h\right)$, and the critical levels of fluctuations for suppressing turbulence, $\left(I_{eff}^{l\leftarrow m}, I_{eff}^{h\leftarrow l}, I_{eff}^{h\leftarrow m}\right)$. They are defined as follows: $D^m = \left(1 + \left(\omega_E/\omega_{Ec}^m\right)^2\right)^{-1} \gamma_0^m k_0^{m-2}$, $D^l = \left(1 + \left(\omega_E/\omega_{Ec}^l\right)^2\right)^{-1} \gamma_0^l k_0^{l-2}$, and $D^h = \left(1 + \left(\omega_E/\omega_{Ec}^h\right)^2\right)^{-1} \gamma_0^h k_0^{h-2}$, where γ_0 stands for the each nonlinear growth rate without coupling and nonlinear noise, k_0^m, k_0^l and k_0^h are typical wavenumbers, and ω_E is the $E \times B$ shearing rate. $\omega_{E\,c}^m$, $\omega_{E\,c}^l$ and $\omega_{E\,c}^h$ are the critical values for the suppression of the modes.

The coupled equations are given in a normalized form as

$$\left(1 - x^2\right)^2 = \left(\sigma \hat{D}^l/\hat{D}^m\right)^2 \left(1 + \frac{x}{2}\right)\left(1 + \hat{D}^{m2} x^2\right)^{-2} \tag{3a}$$

$$y^2 = \left(\frac{\hat{D}^m}{\sigma \hat{D}^l}\right)^2 \left(1 - x^2\right)^2 - \frac{\hat{D}^h}{\hat{D}^l} \frac{\hat{D}^m}{\sigma \hat{D}^l} \left(1 - x^2\right) z \tag{3b}$$

and

$$\left[z^2 \left\{ 1 + \left(\hat{D}^m \omega_{Ec}^l/\omega_{Ec}^h\right)^2 x^2 + \left(\frac{\hat{D}^m}{\sigma}\right)^2 \left(1 - x^2\right)^2 - \frac{\hat{D}^h \hat{D}^m}{\sigma} \left(1 - x^2\right) z \right\} - 1 \right]^2$$

$$= \frac{x^2}{4} + \frac{1}{4} - \frac{\sigma \hat{D}^h}{4\hat{D}^m} \frac{z}{\left(1 - x^2\right)} \;, \tag{3c}$$

where $x = \frac{\sqrt{I^m}}{D^m}$, $y = \frac{\sqrt{I^l}}{D^l}$, $z = \frac{\sqrt{I^h}}{D^h}$, and D^h and D^l are normalized to the characteristic level to suppress the micro mode and $\sigma = \frac{\sqrt{I_{eff}^{h \leftarrow l}}}{\sqrt{I_{eff}^{l \leftarrow m}}} = \frac{\omega_{Ec}^h \left(k^m\right)^2}{\omega_{Ec}^l \left(k^l\right)^2}$.

Analysis of Bifurcation and Phase Diagram If nonlinear interactions between different fluctuations are neglected, independent solutions $x = 1$, $y = 1$, and $z = 1$ are given. Three classes of fluctuations can be independently unstable. Equation (3a) is a closed equation for x and the solution is controlled by two parameters, $\hat{D}^m$ and $\sigma\hat{D}^l/\hat{D}^m$, i.e., the magnitude of the drive of the macro mode and that of the semi-micro mode.

A case of intermediate drive for macro mode is shown here. Figure 1 illustrates the amplitudes x, y and z as a function of the driving parameter $\hat{D}^l$ for a fixed value of $\hat{D}^h$. (Examples of the parameters are chosen as $\hat{D}^m \simeq 1$, $\omega_{Ec}^h/\omega_{Ec}^l = 2$, $\left(k^m/k^l\right)^2 = 1/10$ and $\sigma = 1/5$.) The global mode suppresses other modes if $\hat{D}^l$ is small. When $\hat{D}^l$ increases, branches of fluctuations, in which both the semi-micro , y , and micro mode, z , fluctuations are subcritically excited near the critical point together with macro mode, appear. The nonlinear interplay induces the subcritical excitation. As $\hat{D}^l$ increases, the second bifurcation takes place, and the third and fourth of the fluctuation states are allowed. When $\hat{D}^l$ further increases, the macro mode is quenched by the other two modes.

Figure 2 illustrates the phase diagram on the $\left(\hat{D}^l, \hat{D}^h\right)$ plane. It contains a butterfly catastrophe that governs the state of fluctuations. Figure 1 shows the solutions on the disection of figure 2 at $\hat{D}^h = 4.8$. This new catastrophe is induced by the nonlinear interactions among three different classes of fluctuations.

In summary, we have analyzed the turbulence composed of three kinds of collective modes with different scale lengths, and various kinds of turbulent states were found. The transitions among turbulent states were investigated. A phase diagram was summarized. A butterfly type catastrophe as well as a cusp type catastrophe were revealed. It is stressed that the condition for the appearance of one kind of fluctuations with one characteristic scale length is strongly influenced by the presence of other kinds of fluctuations.

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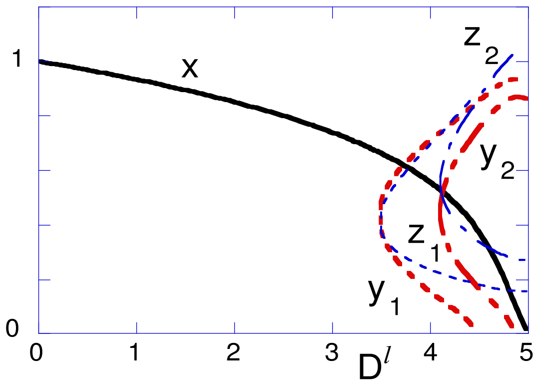

Fig.1 Amplitudes of the macro mode, semi-micro and micro mode fluctuations as a function of the drive of the semi-micro mode $\hat{D}^l$. Other parameters, $\hat{D}^m$, $\hat{D}^h$ and σ, are fixed as $\hat{D}^m = 1$, $\hat{D}^h = 4.8$ and $\sigma = 1/5$.

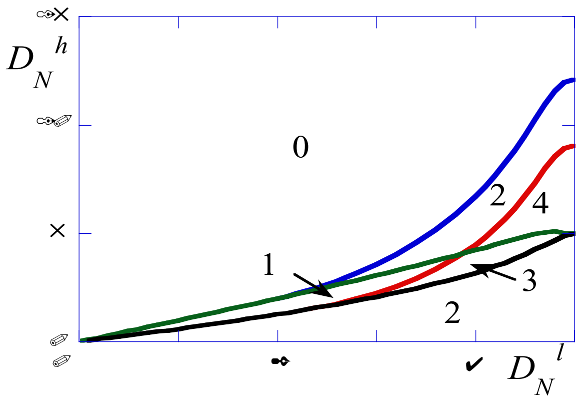

Fig.2 Phase diagram

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