29th EPS Conference on Plasma Phys. and Contr. Fusion Montreux, 17-21 June 2002 ECA Vol. **26B**, P-4.109 (2002)

All-Orders, Full-Wave RF Computations Using Localized Basis Functions

L. A. Berry, E. F. D'Azevedo, E. F. Jaeger, D. B. Batchelor, and M. D. Carter
Oak Ridge National Laboratory*, Oak Ridge, TN USA

Introduction. Many radio frequency (RF) problems for high-temperature plasmas require high-resolution, full-wave electromagnetic (EM) fields that resolve the finite Larmor radius interaction to all orders in $k_\perp \rho_i$ where $k_\perp$ is the perpendicular wavelength and ρ_i the ion Larmor radius. Examples include high-harmonic fast waves and Ion Bernstein Waves (IBWs), mode conversion, and alpha particle interactions. Such calculations have become possible in 2-D as a result of advances in large-scale parallel computing [1,2]. However, our ability to carry out high-resolution calculations in 2-D, and even moderate-resolution calculations in 3-D, is limited by computing power. The inversion of large, dense matrices is the limiting numerical step and, because matrix dimensions scale as either the square (2-D) or cube (3-D) of the resolution, linear increases in computer memory and computational capabilities will provide only incremental improvements in resolution. Matrix size and density are driven by the use of Fourier basis functions for the solution. The use of this basis set follows from our ability to analytically calculate the plasma conductivity for plane waves. In this paper we will outline the development in 1-D of an adaptive mesh technique that utilizes localized basis functions. This algorithm has the potential for computing higher resolutions in 2-D and 3-D with no increase in computational effort.

Localized Basis Functions. The ability of variable-resolution basis functions to efficiently represent EM fields is illustrated in Figure 1. The particular problem is a minority (25% H in D) fast wave configuration for an ASDEX-U like geometry. The real part of E_x is shown as a function of normalized major radius for both the "exact" solution with 256 Fourier modes and an interpolated solution with 74 cubic B-splines. A compressional Alfvén wave is incident from the right and mode-converts to a much shorter wave length IBW at the ion-ion hybrid resonance. Fifty uniformly spaced cubic splines were used with an additional 24 added in the region with propagating IBW. The need for variable resolution follows from the scale lengths of the two waves that are excited in this problem.

We have examined a variety of possible basis functions with emphasis on Daubechies wavelets and B-splines. For wavelets, the wavelet transform was employed, and the most significant coefficients retained up to the specified number of wavelets in the representation. For splines, knot locations were derived from the inverse of the (n-1)th derivative of an n-spline. (For a cubic spline n=3.) We found that cubic or quadratic splines provided the best accuracy for the ASDEX-U mode conversion problem. Figure 2 presents the results of cubic spline fits in terms of energy conservation for representations with 32 to 256 cubic splines. We conclude that about 64 splines are needed to obtain global energy conservation within ~1% for this particular problem. Errors in the relative power coupled to the different plasma species are less sensitive to the number of basis functions. We conclude that the number of electric field unknowns for a given resolution can be potentially reduced by a factor of two to three per dimension. To realize this gain, efficient techniques are required for evaluating the plasma current for the chosen basis functions and for adaptively providing increased resolution without an *á priori* knowledge of the solution.

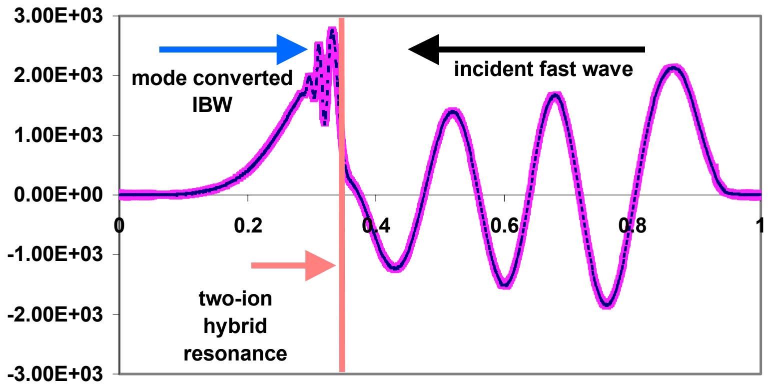

Figure 1 Re(E_x) as a function of normalized major radius. The "exact" solution, foreground dashed line, is indistinguishable from the less exact 74-spline fit, background solid line.

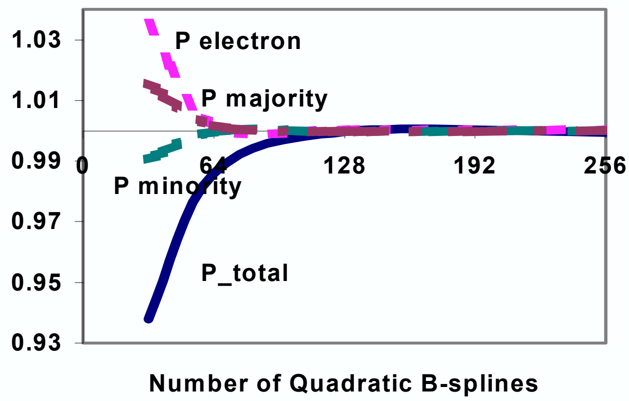

Figure 2 P_total is the fraction of antenna power that is absorbed by the plasma. The species fractions indicate changes in the mix of absorbed power relative to the "exact" solution.

Formulation of Configuration-Space Algorithm. Sauter and colleagues have developed an approach to calculating the plasma response in configuration space; however, their technique is computationally intense and cannot be readily extended to higher dimensions [3]. We have developed an alternative implementation for the configuration space algorithm that retains the advantages of variable resolution and less dense matrices, but with improved computational efficiency. This approach evaluates the plasma response for a particular basis function in Fourier space, and then transforms it into configuration space for developing the solution to the wave equation. In Refs. 1 and 2 an algorithm for the all-orders full wave model was developed with Fourier basis functions that utilized collocation to generate the set of linear equations from which the electromagnetic fields could be obtained. Schematically, these equations can be represented as $\vec{A} \bullet \vec{x} = \vec{b}$ where $\vec{x}$ is the solution vector for the EM field Fourier coefficients, $\vec{A}$ is the collocation coefficient matrix, and $\vec{b}$ is the corresponding vector of antenna currents. We can find the solution in configuration space, $\vec{X}$, by $\vec{X} = \vec{F} \bullet \vec{x}$ where $\vec{F}$ is the inverse Fourier transform. If we now let $\vec{y}$ be the amplitudes of a solution in some new basis set, then $\vec{X} = \vec{W} \bullet \vec{y}$ where $\vec{W}$ (not necessarily square) transforms the new basis to configuration space. These two expressions for $\vec{X}$ can be combined to generate the linear equations for $\vec{y}$, $\vec{W}^T \bullet \vec{F} \bullet \vec{A} \bullet \vec{F}^{-1} \bullet \vec{W} \bullet \vec{y} = \vec{W}^T \bullet \vec{F} \bullet \vec{b}$. The premultiplication by $\vec{W}^T \bullet \vec{F}$ both reduces the number of equations to match the basis set unknowns and approximately preserves the condition number characteristics of the original coefficient matrix. The set of matrix operations outlined above produces essentially the same linear equation set as if we had employed a Galerkin solution to the original equation set as presented in Ref. 3.

Adaptive Mesh Algorithm. Since, in general, we have little detailed knowledge of those areas of the plasma (especially in 2-D and 3-D) that will require high resolution, we will evolve an improved mesh for a particular problem by starting with a uniform mesh, and then refine it by examining a sequence of solutions for local convergence. Wave-equation residuals at intermediate points on a given mesh are one possibility. Global convergence can be monitored by energy conservation between antenna and absorbed power. We choose to obtain a solution with quadratic and cubic splines on an initially uniform mesh, and then to examine the difference between these solutions at interpolated mesh points. If we assume the solution with cubic basis functions is more accurate, additional resolution is then needed where these two solutions, the quadratic and cubic spline, have relatively large

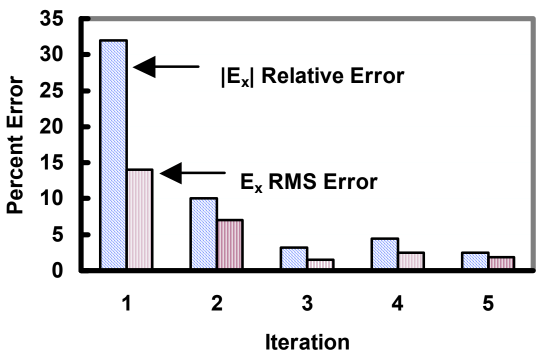

Figure 3. Error evolution with mesh refinement.

differences. Conversely, less resolution is needed where the differences at interpolated points is relatively small. This analysis allows us to obtain a refined mesh with the same number of basis functions by relocating points. Additional basis functions could be added following the same rationale. Results for the ASDEX-U minority configuration are summarized in Figure 3. We start with a uniform 64-point mesh. The relative maximum error and RMS errors between the "exact" 256-mode solution and the cubic spline solution are 32% and 14%, respectively. These both drop about a factor of ten after two mesh refinements. This result is encouraging since this 1-2% RMS error is within a factor of two of what might be expected given the ~1% discrepancy that we found (Fig. 2) for an optimized fit of 64 cubic splines to the exact solution.

Summary. We conclude that the proposed multi-resolution, adaptive algorithm has the potential for reducing the number of basis functions required in plasma EM calculations by a factor of two to three per dimension. This is particularly true for applications with both short and long wavelength features such as mode conversion. In the future, we plan to extend the evaluation from 1-D to 2-D (possibly 3-D) and to evaluate other metrics for adaptive mesh refinement. Even though multiple solutions are required in order to obtain a refined mesh, each solution will be much faster, given the N^3 scaling of direct matrix inversion, and there should be a net improvement. Even when there is no savings in execution time, higher-resolution analysis will be possible because computer memory often limits problem size for in-core inversion techniques for dense matrices. Less dense matrices may also allow the use of sparse-matrix techniques that will enable the solution of higher resolution problems.

*Oak Ridge National Laboratory is managed by UT-Battelle, LLC, for the U.S. Dept. of Energy under contract DE-AC05-00OR22725.

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