Optimization of Stellarators with Respect to Neoclassical Transport in Real Space*

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Introduction

In the present work, a new code for optimizing stellarators with fixed coil design is developed. It is based on an existing code for calculating neoclassical transport with field line integration [1]. The advantage of optimizing a stellarator in real space variables is that there is no need neither for running an equilibrium code nor for a transformation to Boozer coordinates. It is possible to do an optimization in real space in case of a weak influence of the finite plasma pressure on the equilibrium.

The optimization of neoclassical transport properties is part of most of the design activities or configuration changes (e.g. inward shifted discharges in LHD and CHS). Some of the Heliac configurations (e.g. TJ-II) are not optimized with respect to neoclassical transport because this was not their main objective. To improve the performance of those configurations, the prospect of some minor configuration changes to improve the neoclassical confinement is checked. This in turn might improve the possibilities for the specific main tasks of those configurations. Since coil design cannot be changed for an existing device, the possibilities to control the magnetic field configuration are limited to the changes in coil currents or coil positions.

Method

The optimization is based on the field line integration technique which is employed in the NEO code [1] and has originally been developed for magnetic fields represented in real space coordinates. The magnetic fields computed directly from coil currents can be used for evaluation of neoclassical transport properties. Therefore, the optimization can be done without any restrictions to the complexity of the field. Of course, the field line integration technique in its real space representation is limited to vacuum fields, unless PIES equilibria are used. For configurations of interest (e.g. a Heliac like TJ-II; [2,3,4]) this causes no problem because the pertinent equilibria are only very weakly modified with finite plasma pressure.

Optimization Procedure

The chosen optimization procedure simulates the annealing process of solid state matter and is called simulated annealing [5,6]. This method uses stochastic processes [7] to "scan" the parameter space and selects "test configurations". From each test configuration a fitness parameter, which here is the energy contents, is calculated. Comparing the fitness parameters according to the simulated annealing process gives the desired solution.

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Fitness Parameter

As fitness parameter for the optimization, the total stored energy in the plasma volume in case of an energy source, which is localized at the magnetic axis, is used. The $1/\nu$-confinement regime is assumed. Furthermore, we assume cold ions and use the ambipolarity condition in order to eliminate the radial electric field. Therefore, the temperature profile is uniquely defined by the heat conductivity equation

$$\frac{1}{r}\frac{\partial}{\partial r} r\kappa_\perp \frac{\partial T}{\partial r} + Q(r) = 0\,, \tag{1}$$

and the boundary conditions $T(a) = 0$ and $\frac{dT}{dr}|_{r=0} = 0$. Here, $\kappa_\perp$ denotes the heat conductivity coefficient and the source term is $Q(r) = \frac{Q_0}{r}\delta(r)$. The heat conductivity can be expressed as $\kappa_\perp = A\hat{n}\epsilon_{eff}^{3/2}T^{7/2}$, where A is a constant which does not change during optimization, ϵ_{eff} is an effective ripple calculated by the field line integration technique [1], r is an effective radius (see next section), and $\hat{n}$ is a normalized plasma density. Integrating the temperature profile resulting from Eq. 1, one obtains the normalized stored energy as

$$\hat{W} = \int_0^a dr\, r\, \hat{n}(r) \left(\int_r^a \frac{dr'}{r'\,\hat{n}(r')\,\epsilon_{eff}^{3/2}(r')} \right)^{2/9}, \tag{2}$$

where the normalization factor stays constant during optimization.

Effective Radius

The effective radius is defined in Ref. [1] in differential form, $Sdr = dV$, where S is the area of the magnetic surface and V is the volume limited by the magnetic surface. The computation of this quantity requires the calculation of many magnetic surfaces. The accuracy is low, if just a few surfaces are used. Instead of this, one can introduce a different definition of an effective radius $r = 2V/S$, which can be calculated during a single field line integration. Dividing the volume V, limited by the magnetic surface, by the surface area S, one obtains

$$r = \frac{2V}{S} = \frac{2}{3}\frac{\int dS\mathbf{r}\cdot\frac{\nabla\psi}{|\nabla\psi|}}{\int dS} = \frac{2}{3}\frac{<\mathbf{r}\cdot\nabla\psi>}{<|\nabla\psi|>} = \frac{2}{3}\lim_{L_S\to\infty}\frac{\int_0^{L_S}\frac{dl}{B}\mathbf{r}\cdot\nabla\psi}{\int_0^{L_S}\frac{dl}{B}|\nabla\psi|}\,. \tag{3}$$

Here, $\nabla\psi$ is the vector normal to the flux surface, $\mathbf{r}$ is the radius vector, B is the module of the magnetic field, and L_S is a distance measured along the magnetic field line. For configurations like the TJ-II standard configuration the effective radius is close to the quantity defined in [1], since the shape of the cross-sections of magnetic surfaces changes only little with radius (see Fig. 1).

Preliminary Results

For the first run of the optimization, four currents (the toroidal coil current, I_{tor}, the helical coil current, I_{hel}, the horizontal coil current corresponding to the central coil, I_{hor}, and the horizontal coil current corresponding to the vertical field coils, I_{vert}) are varied. The limits of the variation of the currents was set to 10% of the value of the corresponding current to limit the number of possible configurations for a first run of the program. As a configuration with an enhanced energy confinement compared to the TJ-II standard configuration the configuration, which is marked with a blue point, at the upper left plot in Fig. 2 has been found. For calculation of the fitness parameter two models for the particle density are used, one where $n = constant$ (green) is assumed and for the other a parabolic particle density profile $n = 1 - \alpha * (r/a)^2$, with $\alpha = 0.8$, is assumed (red). It turns out, that $\hat{W}$ for the model with the parabolic profile for particle density approximately scales with 0.8 to $\hat{W}$, when constant particle density is assumed. In Fig. 3 the corresponding cross-sections of the TJ-II standard configuration and the marked configuration, mentioned above, are shown. The configuration with an enhanced energy confinement has a bigger plasma volume compared to the standard configuration.

Conclusion and Outlook

In the present work the stellarator optimization is set up directly in real space variables. A new formula for calculating an effective radius $r = r(\psi)$ is proposed.

Due to the big amount of computational time, a parallelization of the optimization will be done. First estimations of computational time have shown, that it is possible to perform such an optimization on a cluster of PCs.

An extension of the code to include criteria for Mercier stability and for stability of resistive local modes is possible in the future.

References

[1] V. V. Nemov, S. V. Kasilov, W. Kernbichler and M. F. Heyn. *Phys. Plasmas* **6**, (1999) 4622.

[2] E. Ascasibar et al., Plasma physics and controlled nuclear fusion research *Proceedings of the 15th international conference.* Vienna (Austria). IAEA. Sep 1995. 749

[3] C. Alejaldre et al., *Plasma Phys. Control. Fusion* **41** (December 1999) B109-B117

[4] V. Tribaldos, *Phys. Plasmas* **8** 1229 (2001)

[5] W. Kinnebrock: Optimierung mit genetischen und selektiven Algorithmen, *Oldenbourg* (1994), Wien

[6] J. Schnakenberg: Algorithmen in der Quantentheorie und Statistischen Physik, *Verlag Zimmermann-Neufang* (1995), Ulmen

[7] C. W. Gardiner: Handbook of Stochastic Methods for Physics, Chemistry and the Natural Sciences, *Springer* (1985), Berlin

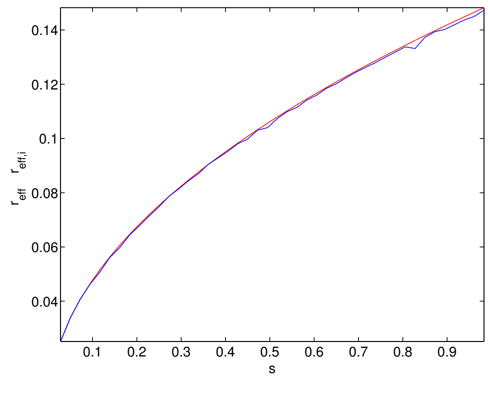

Figure 1: Effective radius calculated as proposed in [1] (red) compared to the effective radius as proposed in Eq. 3 (blue) the TJ-II standard configuration, given in Boozer coordinates.

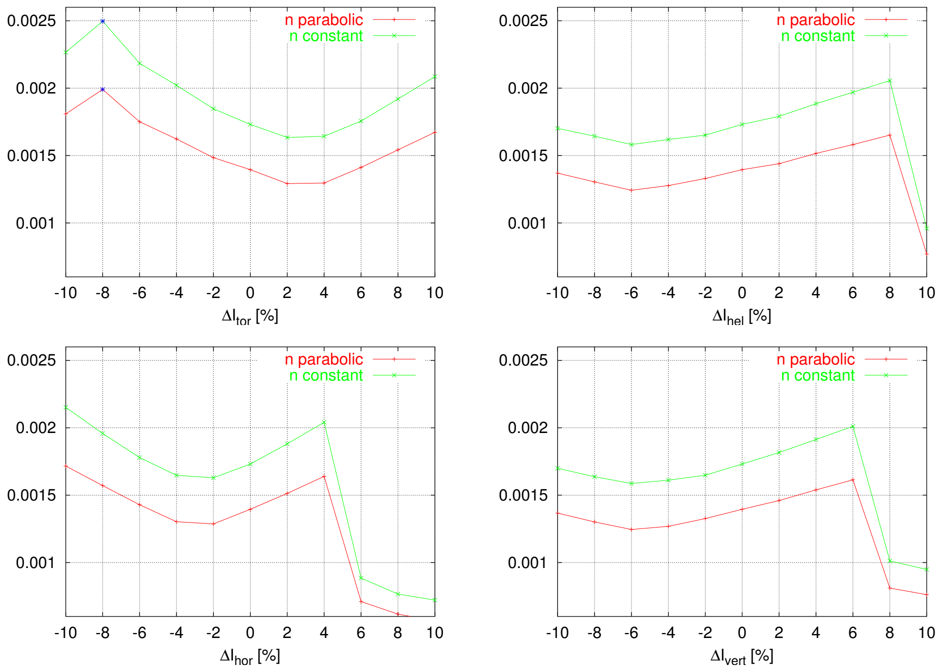

Figure 2: Normalized stored energy (see Eq. 2) vs. change of the corresponding current in percent. In the upper left plot, the current I_{tor} is varied and $I_{hel}, I_{hor}, I_{vert}$ are fixed (other plots accordingly). The energy confinement of the configuration with $\Delta I_{tor} = -8\%$ (blue point in the upper left figure) is about 50% enhanced compared to the standard configuration ($\Delta I_{tor} = 0$).

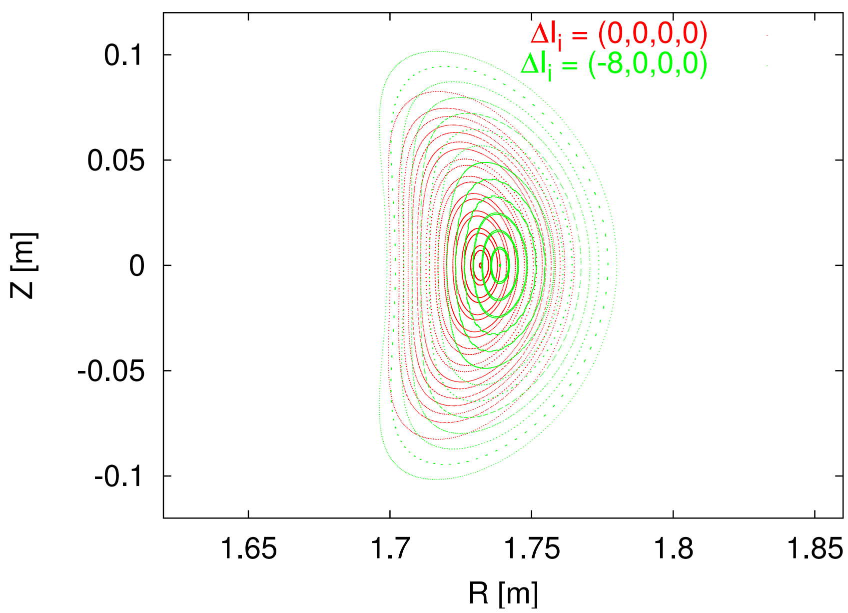

Figure 3: Cross-sections at $\varphi = 0°$ for the TJ-II standard configuration (red) and the configuration, which is marked in the upper left plot in Fig. 2 (green).