Effect of Flux Surface Shape on the Adiabaticity of Fast Ions in Spherical Tori

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1. Introduction

Relatively strong inhomogeneity of the magnetic field $B$ of axisymmetric low-aspect-ratio spherical torus (ST) plasmas is known to cause non-conservation of the magnetic moment $\mu$ of fast ions confined in these devices [1,2]. Here we evaluate non-adiabatic variations of the magnetic moment of fast ions as effected by small-scale (in poloidal angle $c$) perturbations of the flux surface (FS) shape of high beta ST equilibria. These perturbations arise from the combined effects of plasma paramagnetism and large Shafranov shift of noncircular flux surfaces, causing $|B|=B$ to vary non-monotonically as a field line is followed from its inner to the outer mid-plane crossing point. The resulting small-scale modulation of the magnetic field we will call poloidal field (PF) ripples in analogy with the toroidal field ripples. For NSTX with $\beta=23\%$, Fig. 1 displays the EFIT equilibrium $B$-contours in flux coordinates $(r,c)$, where $r$ denotes the FS radius. It can be seen that PF ripples exist at the outboard part of the plasma column ($90^\circ<\chi<270^\circ$ and $r>0.8a$, $\chi=180^\circ$ corresponds to the NSTX equator).

2. Model ST magnetic field

Our consideration is based on a model magnetic field $B_0(\chi)$ with non-circular but poloidally smooth FSs [3]. To introduce the poloidally small-scale flux surface perturbation in accordance with the FS representation $\{R_0(r,\chi), Z_0(r,\chi)\}$ of [3] we use $R=R_0$ and $Z=Z_0+\delta Z=Z_0[1-\delta(r)(1-M(r)\cos^2\chi)^2\cos\chi]$; here $\delta(r)<<1$ and $M(r)>1$ define the magnitude and the poloidal scale of FS perturbation, respectively. At $\delta(r)=\delta(r/a)^2$ and $M(r)\approx 3$, where $\delta_0=\delta(a)$ and $a$ is the plasma radius, the chosen FS perturbation $\delta Z$ corresponds qualitatively to the axisymmetric perturbation $\delta B \sim 0.01 B_0(\chi) \cos(6\chi)$ which fits the equilibrium FSs and
B(χ) in NSTX (with β = 40%) better than B₀(χ). This is demonstrated in Figs. 2, 3 where FSs and B(χ) are displayed for a given flux surface radius r = 0.5 m for unperturbed, perturbed and EFIT magnetic fields. The main difference between B₀ and the EFIT field is observed at the outboard part of the plasma column. It is seen that δZ with δ₀=0.06 and M=2.7 induces additional modulation of B₀(χ) yielding reasonable agreement with the EFIT field. At the same time the model magnetic field is free of numerical poloidal B-oscillations as given by EFIT. Fig. 4 displays the model B contours in NSTX with β=40%. Poloidal ripples exist in this case at poloidal angles (110°-120°) <χ<(240°-250°) and r>0.75a. It can be seen that the poloidal scale of β=40% PF ripples in NSTX is about 80 to 85% of the poloidal scale of ripples in the β=23% case.

3. Effect of small-scale poloidal FS shape perturbations on nonadiabatic jumps of \( \mu \)

We consider first the contribution of the FS shape perturbations to the non-adiabatic variations of the magnetic moment. Fig. 5 demonstrates the time variations of the magnetic moment of 40 keV co-moving deuterons in NSTX (β=40%) for different magnitudes of FS shape perturbations (0≤δ≤0.08) assuming the same initial conditions during more than 8 bounce periods, i.e. \( R_0 = 1.43 \text{m}, Z_0 = Z_{\text{mag}} \) corresponding to the vertical position of magnetic axis, \( V_{\phi_0}/V = 0.62 \) and the normalised magnetic moment \( \lambda_0=\mu_0B(r=0)/E=0.207 \). For strong
FS shape perturbations ($\delta_0 > 0.04$) a significant enhancement of the non-adiabatic variations of $\mu$ is seen, while the effect of PF ripples associated with $\delta_0 \leq 0.02$ is only weak.

The corresponding full gyro-orbits in the $\{R, Z\}$-plane for $\delta_0=0$ and $\delta_0=0.06$ are shown in Fig. 6. According to the analytical estimations [4,1] the non-adiabatic jumps of the magnetic moment per bounce period is $\Delta \mu = \Delta \mu_{\text{max}} \cos(\theta + \alpha)$, where $\Delta \mu_{\text{max}}$ denotes the maximum non-adiabatic variation of $\mu$ per bounce period, $\theta$ is the gyro-phase when the particle has crossed the mid-plane and $\alpha$ a constant.

The numerically calculated non-adiabatic jumps of $\mu$ are depicted in Fig. 7 as a function of $\theta$ using the parameters of Fig. 5 and $\delta_0=0.06$. The observed $\Delta \mu(\theta)$ are seen to be in satisfactory agreement with the analytical predictions confirming the resonant nature of the non-adiabatic jumps of $\mu$.

Finally we consider the contribution of the poloidal field ripples to $\Delta \mu_{\text{max}}$. In Fig. 8 we display the numerically obtained dependence of $\Delta \mu_{\text{max}}$ on the magnitude of the FS shape perturbation, $\delta_0$, for the parameters of Fig. 5. As already mentioned, the dependence

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**Fig. 5:** Variations of $\mu$ of 40 keV co-going deuterons in a NSTX-like $B$-field induced by small-scale FS perturbations.

**Fig. 6:** Gyro-orbits of 40 keV deuterons in the unperturbed and perturbed magnetic field. According to the analytical estimation the jumps of the magnetic moment per bounce period is $\Delta \mu = \Delta \mu_{\text{max}} \cos(\theta + \alpha)$, where $\Delta \mu_{\text{max}}$ denotes the maximum non-adiabatic variation of $\mu$ per bounce period, $\theta$ is the gyro-phase when the particle has crossed the mid-plane and $\alpha$ a constant.

**Fig. 7:** Non-adiabatic jumps of magnetic moment as a function the gyro-phase when the particle has crossed the mid-plane.
\( D_{m\max}(d_0) \) is essentially nonlinear. For \( d_0 \leq 0.02 \) the contribution of FP ripples is weak and \( D_{m\max}(d_0) \approx D_{m\max}(0) \), whereas for \( d_0 > 0.02 \) we obtained \( D_{m\max}(d_0) \sim d_0^2 \). For \( d_0 = 0.06 \) corresponding to the EFIT poloidal modulation of \( B_0 \), and for a flux surface radius \( r = 0.445 m = 0.7a \), the PF ripples enhance the non-adiabatic changes of the magnetic moment of 40 keV deuterons by more than 10 times. The effect considered is expected to play an important role in the confinement and transport of fast ions in spherical tori.

4. Summary

The present investigation demonstrates the substantial effect of small-scale poloidal perturbations of the flux surface shape on the non-adiabaticity of fast ions in spherical tori. In high beta ST plasmas, almost all non-adiabatic variations of the magnetic moment of fast ions are induced by poloidal field ripples. PF ripple induced relative jumps of the magnetic moment, \( \Delta \mu_{\max} / \mu_0 \), of 40 keV deuterons in NSTX are of the order of 10-20% at the plasma periphery \((r/a > 0.7)\). The new PF-ripple effect is expected to play an important role in the confinement and transport of fast ions in spherical tori as well as in stellarators.

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References